Dynamical fixed points in strongly coupled holographic systems

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Based on arXiv:2111.04122

also: arXiv: 1702.01320 (with A.Karapetyan), 1809.08484, 1904.09968, 1912.03566

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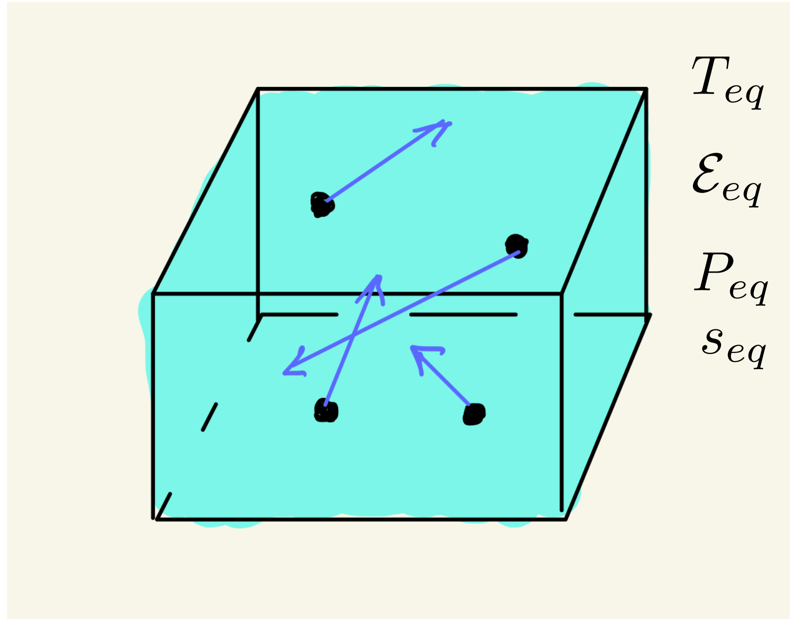
$\Longrightarrow$ Thermodynamic equilibrium



- T_{eq} - the equilibrium temperature
- $\mathcal{E}_{eq}$ - the energy density
- P_{eq} - pressure
- s_{eq} - thermodynamic entropy density

$$\mathcal{F}_{eq} = -P_{eq} = \mathcal{E}_{eq} - s_{eq}\, T_{eq}\,, \qquad d\mathcal{E}_{eq} = T_{eq}\, ds_{eq}$$

$\Longrightarrow$ *Thermodynamic equilibrium* is a late-time attractor of dynamical evolution of isolated interacting quantum system:

$$\lim_{t\to\infty} T_{\mu\nu}(t,\boldsymbol{x}) = \mathrm{diag}\left(\mathcal{E}_{eq}, P_{eq}, \cdots P_{eq}\right)$$

▪ $T_{\mu\nu}$ are the component of the stress-energy tensor of the system at time t and the spatial location $\boldsymbol{x}$

$\Longrightarrow$ We also have a theory — **the hydrodynamics** — that describes the approach to that equilibrium (assuming we are not-far from it):

- Given the local energy density $\mathcal{E}$ and the equilibrium equation of state $P_{eq} = P_{eq}(\mathcal{E}_{eq})$ we define the local pressure P

$$\mathcal{E}(t,\boldsymbol{x}) \equiv T_{00}(t,\boldsymbol{x}) \qquad \Longrightarrow \qquad P(t,\boldsymbol{x}) = P_{eq}\left(\mathcal{E}(t,\boldsymbol{x})\right)$$

- and obtain the local entropy density $s(t,\boldsymbol{x})$ and temperature $T(t,\boldsymbol{x})$

$$\mathcal{E} + P = s\,T\,, \qquad d\mathcal{E} = T\,ds$$

- "not-far from equilibrium" is then

$$T \cdot \left| \frac{\partial_\mu \mathcal{E}}{\mathcal{E}} \right| \ll 1 \qquad \underline{\boldsymbol{and}} \qquad T \cdot \left| \nabla_\mu u^\nu \right| \ll 1$$

where $u^\mu = u^\mu(t, \boldsymbol{x})$ is a local fluid 4-velocity, $u^\mu u_\mu = -1$, used to define the hydrodynamic stress-energy tensor

$$T^{\mu\nu} = \underbrace{\mathcal{E}\ u^\mu u^\nu + P\ \Delta^{\mu\nu}}_{\text{"equilibrium'' part}} \qquad + \qquad \underbrace{\mathcal{T}^{\mu\nu}}_{\text{first}-\text{order dissipative terms}}$$

- $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, $\qquad g_{\mu\nu}$ is the background space-time metric

- $$\mathcal{T}^{\mu\nu} = -\eta\ \sigma^{\mu\nu} - \zeta\ \Delta^{\mu\nu}\ (\nabla \cdot u)$$

where $\sigma^{\mu\nu} \sim \partial^\mu u^\nu$, and $\eta = \eta(\mathcal{E})$, $\zeta = \zeta(\mathcal{E})$ are the shear and the bulk viscosities

$\Longrightarrow$ viscosities are completely determined from the equilibrium thermodynamics (the two-point correlation functions of the equilibrium stress-energy tensor)

$\Longrightarrow$ How do we recover equilibrium thermodynamics?

- assume that $\mathcal{E}$ and P are constant throughout the system and time-independent; the background metric is Minkowski:

$$g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$$

- set $u^\mu = u^\mu_{eq} \equiv (1, \mathbf{0}) \qquad \Longrightarrow$

$$\Delta^{00} = g^{00} + u^0 u^0 = 0\,, \qquad \Delta^{ii} = g^{ii} + u^i u^i = 1\,, \qquad \partial_\mu u^\nu = 0$$

$\Longrightarrow$

$$\mathcal{T}^{\mu\nu} \equiv 0\,, \qquad T^{\mu\nu} = \mathrm{diag}(\mathcal{E}, P, P, P) \equiv \mathrm{diag}(\mathcal{E}_{eq}, P_{eq}, P_{eq}, P_{eq})$$

- In addition, we can introduce the equilibrium entropy current $\mathcal{S}^\mu_{eq}$:

$$\mathcal{S}^\mu_{eq} \equiv s_{eq}\ u^\mu$$

Note:

$$\textbf{no entropy production} \qquad \Longleftrightarrow \qquad \nabla \cdot \mathcal{S} = \frac{ds_{eq}}{dt} = 0$$

i.e., the thermal equilibrium is characterized by the vanishing divergence of the entropy current

$\Longrightarrow$ Back to hydrodynamics (the approach to equilibrium):

- There is no first-principle definition of $\mathcal{S}^\mu$ away from equilibrium; to the first-order in the gradients of the local fluid velocity u^μ,

$$\mathcal{S}^\mu = s\ u^\mu - \frac{1}{T}\ \mathcal{T}^{\mu\nu} u_\nu$$

- from the conservation of the stress-energy tensor,

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow$$

$$T\ \nabla \cdot \mathcal{S} = \zeta\ \left(\nabla \cdot u\right)^2 + \frac{\eta}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}\ \geq\ 0$$

which is manifestly non-negative, provided the viscosities are positive.

$\Longrightarrow$ As one approaches the equilibrium,

$$\lim_{t\to\infty} u^\mu = u^\mu_{eq} = (1, \mathbf{0}) \qquad \Longrightarrow \qquad \lim_{t\to\infty} T\ \nabla \cdot \mathcal{S} = 0$$

i.e., in the approach to equilibrium the entropy production rate vanishes

We can now provide a formal definition of a dynamical fixed point (DFP):

A *Dynamical Fixed Point* is an internal state of a quantum field theory with spatially homogeneous and time-independent one-point correlation functions of its stress energy tensor $T^{\mu\nu}$, and (possibly additional) set of gauge-invariant local operators $\{\mathcal{O}_i\}$,

<u>and</u>

strictly positive divergence of the entropy current at late-times:

$$\lim_{t\to\infty}\left(\nabla\cdot\mathcal{S}\right) > 0$$

$\Longrightarrow$ Apart from the requirement of the strictly non-zero entropy production rate at late times, characteristics of a DFP coincide with that of the thermodynamic equilibrium.

Why?

$\Longrightarrow$ DFP, *i.e.*, the non-vanishing late-time entropy production in **driven** (open) quantum-mechanical systems/QFT:

- time-dependent coupling constants (quantum quenches)
- time-dependent masses
- time-dependent external EM fields, etc

and

- QFTs in cosmological backgrounds, asymptotically de Sitter space-times in particular

$\Longrightarrow$ To study DFPs means to classify the end-of-time dynamics of *massive* QFTs, in cosmologies with dark energy

Outline

- A trivial DFP: thermal states of $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) in de Sitter
  - gauge theory perspective
  - holographic picture
  - de Sitter vacuum 'entanglement' entropy
- Nontrivial DFP
- A taster of results from arXiv:2111.04122 — the zoo of DFPs

$\mathcal{N} = 4$ SYM

- Not our QCD (the theory of strong interactions):
  - different gauge group - $SU(3)$ versus $SU(N)$ (we take $N \to \infty$)
  - QCD has a strong coupling scale (the typical scale in nuclear physics); SYM is *conformal*, *i.e.*, the scale invariant
  - QCD confines (and forms nuclei), SYM is always deconfined

- BUT:
  - similar equation of state at strong coupling in deconfined phase
  - similar transport coefficients:

$$\left.\frac{\eta}{s}\right|_{QCD} \sim (1 \cdots 3) \cdot \left.\frac{\eta}{s}\right|_{\mathcal{N}=4\ SYM}$$

Minkowski vs. de Sitter space-time

- A de Sitter space-time is a special case of FLRW cosmology:

$$ds^2_{closed} = -dt^2 + a(t)^2 dS_3^2 \qquad \text{or} \qquad ds^2_{open} = -dt^2 + a^2(t)\, d\boldsymbol{x}^2$$

$$\text{closed cosmology :} \qquad a(t) = \frac{1}{H}\ \cosh(Ht)$$

$$\text{open cosmology :} \qquad a(t) = e^{Ht}$$

- Minkowski space-time

$$ds^2_{Minkowski} = ds^2_{open}\Big|_{a(t)\equiv 1}$$

■ Note:

$$ds^2_{open} = a(t)^2\left(-\frac{dt^2}{a(t)^2} + d\boldsymbol{x}^2\right) = a^2 \underbrace{\left(-d\tau^2 + d\boldsymbol{x}^2\right)}_{ds^2_{Minkowski}}$$

where we introduced the conformal time

$$\tau = \int^t \frac{dt}{a(t)}$$

$\Longrightarrow$ For a conformal field theory, *e.g.*, $\mathcal{N} = 4$ SYM,

- if $\mathcal{O}_\Delta$ is a primary operator of dimension Δ,

$$\langle \mathcal{O}_\Delta \rangle\Big|_{FLRW} = a^{-\Delta} \, \langle \mathcal{O}_\Delta \rangle\Big|_{Minkowski}$$

- stress-energy tensor is not a primary field:

$$\langle T_{\mu\nu} \rangle\Big|_{FLRW} = a^{-4} \, \langle T_{\mu\nu} \rangle\Big|_{Minkowski} + \text{conformal anomaly}$$

$\Longrightarrow$ for a trace of the stress-energy tensor

$$\langle T^\mu_\mu \rangle\Big|_{FLRW} = a^{-4} \underbrace{\langle T^\mu_\mu \rangle\Big|_{Minkowski}}_{=0} + \frac{c}{24\pi^3} \underbrace{\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)}_{=-12\frac{(\dot{a})^2\ddot{a}}{a^3}}$$

e.g., for $\mathcal{N} = 4$ $SU(N)$ SYM,

$$-\langle T^t_t \rangle\Big|_{FLRW} = \frac{1}{a(t)^4} \mathcal{E} + \frac{3N^2}{32\pi^2} \frac{(\dot{a})^4}{a^4}$$

$$-\langle T^{\boldsymbol{x}}_{\boldsymbol{x}} \rangle\Big|_{FLRW} = \frac{1}{a(t)^4} P + \frac{N^2}{8\pi^2} \left\{ \frac{(\dot{a})^4}{4a^4} - \frac{(\dot{a})^2\ddot{a}}{a^3} \right\}$$

$$\langle T^\mu_\mu \rangle\Big|_{FLRW} = a^{-4} \underbrace{\left(-\mathcal{E} + 3P\right)}_{=0} - \frac{3N^2}{8\pi^2} \frac{(\dot{a})^2\ddot{a}}{a^3}$$

$\Longrightarrow$ Minkowski space-time thermal equilibrium states of $\mathcal{N}=4$ SYM (strong coupling) of temperature T_0:

$$\mathcal{E}_0 = \frac{3}{8}\pi^2 N^2 T_0^4\,, \qquad P_0 = \frac{1}{3}\mathcal{E}_0$$

$\Longrightarrow$ in FLRW cosmology,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4}\,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$

where $T(t)$ is the <u>*effective*</u> temperature

$$T(t) = \frac{T_0}{a(t)}$$

$\Longrightarrow$ Stress-energy tensor in FLRW is covariantly conserved:

$$0 = \langle \nabla^\mu T^\nu_\mu \rangle \qquad \Longleftrightarrow \qquad \frac{d\mathcal{E}(t)}{dt} + 3\frac{\dot{a}}{a}\ \left(\mathcal{E}(t) + P(t)\right) = 0$$

$\Longrightarrow$ entropy density is more tricky...(non-equilibrium, time-dependent)

- In Minkowski space-time:

$$s_0 = \frac{\pi^2}{2} N^2 T_0^3$$

- Assuming the adiabatic expansion in FLRW, the co-moving entropy density, $s_{comoving}$,

$$s_{comoving} \equiv a(t)^3 s(t)$$

is conserved:

$$\frac{d}{dt} s_{comoving} = 0 \qquad \Longrightarrow \qquad s_{comoving} = s_{comoving}\Big|_{t=0} = s_0$$

$\Longrightarrow$

$$s(t) = \frac{\pi^2}{2} N^2 T(t)^3$$

- In expansing FLRW, with $a(t) \to \infty$ as $t \to \infty$,

$$\lim_{t \to \infty} s(t) = 0$$

$\Longrightarrow$ Let's rephrase the de Sitter entropy discussion in the language of the entropy current $\mathcal{S}^\mu$:

- A locally static observer has $u^\mu = (1, \mathbf{0})$
- The entropy current (in Landau frame $\mathcal{T}^{\mu\nu} u_\nu = 0$) is

$$\mathcal{S}^\mu = s\ u^\mu$$

$\Longrightarrow$

$$\nabla \cdot S = \frac{1}{a(t)^3}\ \frac{d}{dt}\left(a(t)^3 s\right) = \frac{1}{a(t)^3}\ \frac{d}{dt} s_{comoving}(t)\ =\ 0$$

That is is why $\mathcal{N} = 4$ SYM (same is true for any conformal theory!) in de Sitter evolved to a trivial DFP

How would a non-trivial DFP arise?

- Imagine that

$$\lim_{t\to\infty} s(t) = s_{ent} \neq 0$$

This limit is natural to call the *vacuum entanglement entropy* density, hence $_{ent}$

- Then,

$$\lim_{t\to\infty} \Big(\nabla \cdot \mathcal{S}\Big) = 3\ H\ s_{ent}$$

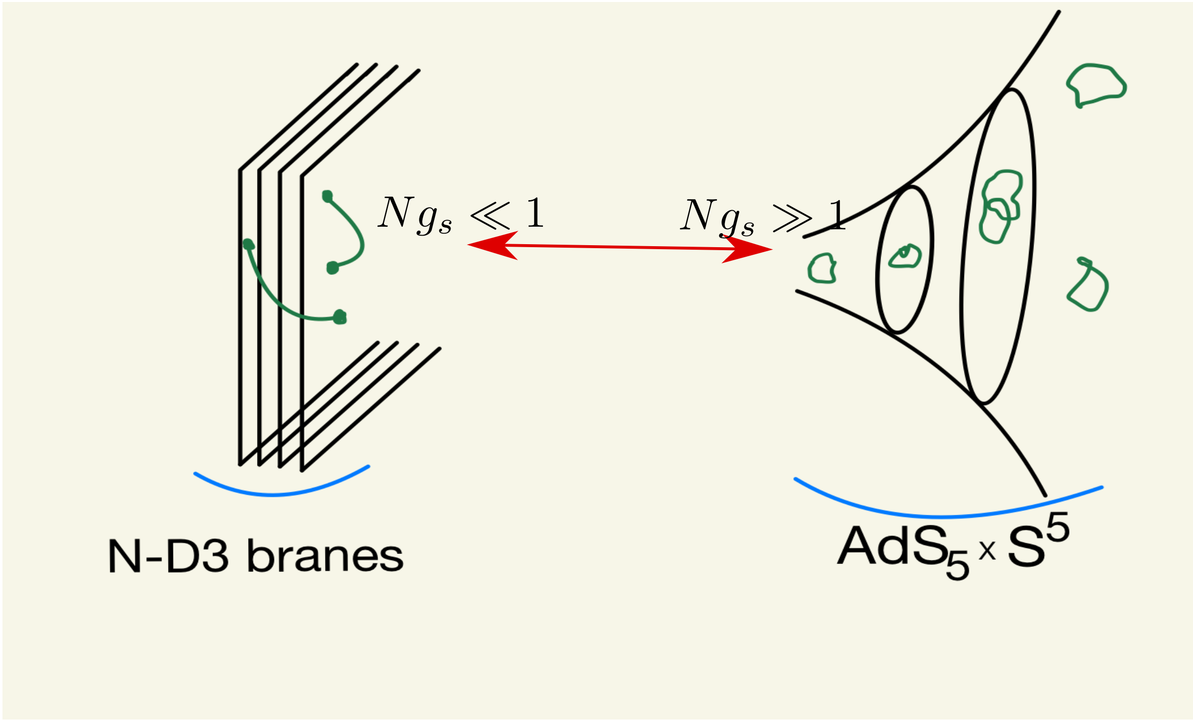
where

$$H = \lim_{t\to\infty} \frac{d}{dt}\ \ln a(t)$$

$\Longrightarrow$ In strongly coupled non-conformal theories with holographic dual

$$s_{ent}\ >\ 0$$

Basic AdS/CFT correspondence in the planar limit



- $Ng_s \ll 1$: weakly coupled open strings, ending on D3 branes in Type IIB SUGRA on $\mathbb{R}^{9,1} \Longleftrightarrow \mathcal{N} = 4$ $SU(N)$ SYM
- $Ng_s \gg 1$: weakly coupled closed strings in Type IIB SUGRA on $AdS_5 \times S^5$

$\Longrightarrow$ Holographic picture for $\mathcal{N} = 4$ SYM in de Sitter

$$S_{\mathcal{N}=4} = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5\xi \sqrt{-g} \left[R + \frac{12}{L^2} \right]$$

$$L^4 = \ell_s^4 \; N g_{YM}^2 \,, \qquad G_5 = \frac{\pi L^3}{2N^2} \,, \qquad 4\pi g_s = g_{YM}^2$$

$\Longrightarrow$ Consider general spatially homogeneous, time-dependent states:

$$ds_5^2 = 2dt \; (dr - A dt) + \Sigma^2 \; d\boldsymbol{x}^2$$

$$A = A(t, r) \,, \qquad \Sigma = \Sigma(t, r)$$

$\Longrightarrow$ We are interested in spatially homogeneous and isotropic states of $\mathcal{N} = 4$ SYM in FLRW, so the bulk metric warp approach the AdS boundary $r \to \infty$ as

$$\Sigma = \frac{a(t)r}{L} + \mathcal{O}(r^0)\,, \qquad A = \frac{r^2}{2L^2} + \mathcal{O}(r^1)$$

Indeed, as $r \to \infty$,

$$ds_5^2 = \frac{r^2}{L^2} \underbrace{\left(-dt^2 + a(t)^2 d\boldsymbol{x}^2\right)}_{\text{boundary FLRW}} + \cdots$$

$\Longrightarrow$ Given the metric ansatz, we can derive derive EOMs (without loss of generality we set $L = 2$):

$$
\begin{aligned}
0 &= (d_+\Sigma)' + 2\Sigma'\ d_+ \ln\Sigma - \frac{\Sigma}{2} \\
0 &= A'' - 6(\ln\Sigma)'\ d_+ \ln\Sigma + \frac{1}{2} \\
0 &= \Sigma'' \\
0 &= d_+^2\Sigma - 2A\Sigma' - (4A\Sigma' + A'\Sigma)\ d_+ \ln\Sigma + \Sigma A
\end{aligned}
$$

where

$$
{}' = \frac{\partial}{\partial r}, \qquad \dot{} = \frac{\partial}{\partial t}, \qquad d_+ = \frac{\partial}{\partial t} + A\frac{\partial}{\partial r}
$$

$\Longrightarrow$ These equations can be solve in all generality for arbitrary $a(t)$:

$$A = \frac{(r+\lambda)^2}{8} - (r+\lambda)\,\frac{\dot{a}}{a} - \dot{\lambda} - \frac{r_0^4}{8a^4(r+\lambda)^2}\,,$$

$$\Sigma = \frac{(r+\lambda)a}{2}$$

where

- r_0 is a single constant parameter
- $\lambda(t)$ is an arbitrary function - the leftover diffeomorphism of the 5d gravitational metric reparametrization $r \to \bar{r} = r - \lambda(t)$:

$$A(t,r) \to \bar{A}(t,\bar{r}) = A(t, r+\lambda(r)) - \dot{\lambda}(t)$$

$$\Sigma(t,r) \to \bar{\Sigma}(t,\bar{r}) = \Sigma(t, r+\lambda(t))$$

$\Longrightarrow$

$$ds_5^2 \;\Longrightarrow\; d\bar{s}_5^2 \qquad = 2dt\,\left(d\bar{r} - \bar{A}dt\right) + \bar{\Sigma}^2\; d\boldsymbol{x}^2$$

$\Longrightarrow$ Identifying

$$\frac{r_0}{2} \equiv T_0$$

$\Longrightarrow$ from holographic computation of the boundary stress energy tensor,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4}\,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$

$$T(t) = \frac{T_0}{a(t)}$$

Precisely as expected from the Weyl transformation of the thermal state from Minkowski to FLRW!

$\Longrightarrow$ Holography buys us more:

- Chesler-Yaffe pioneered numerical studies of EF metrics:

$$ds_5^2 = 2dt\ (dr - Adt) + \Sigma^2\ d\boldsymbol{x}^2$$

- such metrics has an **apparent horizon** (AH) at r_{AH}

$$d_+\Sigma\Big|_{r=r_{AH}} = 0 \qquad \Longrightarrow \qquad r_{AH} = \frac{r_0}{a(t)} - \lambda(t)$$

- causal dependence **must** include

$$r \in [r_{AH}, +\infty)$$

- region

$$r < r_{AH}$$

  is causally disconnected from the holographic dynamics and **must be** excised

- AH is a dynamical horizon

- $$\underbrace{\frac{\Sigma^3}{4G_5}\bigg|_{r=r_{AH}}}_{\text{comoving Bekenstein entropy of the AH}} = \frac{N^2 r_0^3}{128\pi}$$

$$= \underbrace{s_{comoving}}_{\text{SYM comoving entropy density in FLRW}} = a(t)^3 s(t) = \frac{\pi^2}{2} N^2 T_0^3$$

Comments on $t \to +\infty$ dynamics:

- Consider de Sitter background for SYM,

$$a(t) = e^{Ht} \qquad \text{and set} \qquad \lambda(t) = 0$$

- from exact solutions of PDEs:

$$\lim_{t\to\infty} A(t,r) \equiv A_v(r) = \frac{r}{8}(r - 8H)$$
$$\lim_{t\to\infty} \frac{\Sigma(t,r)}{a(t)} \equiv \sigma_v(r) = \frac{r}{2}$$

where $_v$ stands for *vacuum*

- Exactly the same same bulk geometry can be obtained solving ODEs with the metric ansatz

$$ds^2_{5,vacuum} = 2dt\ (dr - A_v dt) + e^{2Ht}\sigma_v^2\ d\boldsymbol{x}^2$$

$$A_v = A_v(r) \qquad \text{and} \qquad \sigma_v = \sigma_v(r)$$

i.e., the late time limit can be taken at the level of PDEs!

- location of the AH is identified from

$$0 = \lim_{t\to\infty} \frac{1}{a(t)}\; d_+\Sigma\Big|_{r=r_{AH}} = (H\sigma_v + A_v\sigma_v')\Big|_{r=r_{AH,v}}$$

- With $\sigma_v = \frac{r}{2}$ and $A_v = \frac{r(r-8H)}{8}$ $\Longrightarrow$

$$r_{AH,v} = 0\,, \qquad \text{while} \qquad A_v = 0 \text{ at } r = r_{A_v} = 8H$$

Remarkable:

- causal evolution requires $r \in [r_{AH,v}, +\infty) = [0, +\infty)$
- $-g_{tt} = 2A$ metric component

(being "outside the Schwarzschild radius of a black hole") must be non-negative $\Longrightarrow r \in [r_{A_v}, +\infty)$

- the part of the geometry $r \in [r_{AH,v}, r_{A_v}]$ disappears upon analytical continuation to Bunch–Davies vacuum or Euclidean vacuum!

$\Longrightarrow$

maybe one of the reasons no previous discussion of s_{ent} in the literature

Non-trivial DFPs: holographic non-conformal models in de Sitter:

- In $\mathcal{N} = 4$ SYM duality we had luxury to study full dynamics (described by PDEs) analytically

- In non-conformal examples (KK reduced from 10-dimensions to 5-dimensions)

$$S_{\rm non-conformal} = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5\xi\sqrt{-g}\,[R + \text{scalars} + \text{scalar potential}]$$

we focus directly on vacuum geometry:

$$ds^2_{5,vacuum} = 2dt\ (dr - A_v dt) + e^{2Ht}\sigma_v^2\ d\boldsymbol{x}^2$$

$$A_v = A_v(r) \qquad \text{and} \qquad \sigma_v = \sigma_v(r) \qquad \text{and} \qquad \text{scalars} = \text{scalars}(r)$$

- We identify location of the AH at late times

$$0 = (H\sigma_v + A_v\sigma_v')\Big|_{r=r_{AH,v}}$$

- compute associated vacuum entanglement entropy:

$$s_{ent,v} \equiv \lim_{t\to\infty} s(t) = \frac{\sigma_v^3}{4G_5}\Bigg|_{r=r_{AH,v}}$$

- from explicit computations of various examples of holography

$$s_{ent,v}^{\mathcal{N}=4 \text{ or CFT}} = 0 \qquad \underline{\text{BUT}} \qquad s_{ent,v}^{\text{non-conformal}} \neq 0$$

Taster from arXiv:2111.04122

$\Longrightarrow$ The model is $d = 2 + 1$ dimensional QFT with a holographic dual:

- Start with a conformal theory $\mathcal{H}_{CFT}$, with the operators

$$\underbrace{T_{\mu\nu}}_{\text{stress}-\text{energy tensor}} \quad , \quad \underbrace{\mathcal{O}_\phi}_{\Delta_\phi=2<d} \quad , \quad \underbrace{\mathcal{O}_\chi}_{\Delta_\chi=4>d}$$

- there is $\mathbb{Z}_2^\phi \times \mathbb{Z}_2^\chi$ discrete symmetry that acts as a parity transformation

$$\phi \leftrightarrow -\phi \qquad \text{and} \qquad \chi \leftrightarrow -\chi$$

- A mass parameter Λ *deformed* the CFT to a massive QFT, explicitly breaking $\mathbb{Z}_2^\phi$ symmetry

$$\mathcal{H}_{CFT} \ \to \mathcal{H}_{CFT} + \Lambda \ \mathcal{O}_\phi \, , \qquad [\Lambda] = 1$$

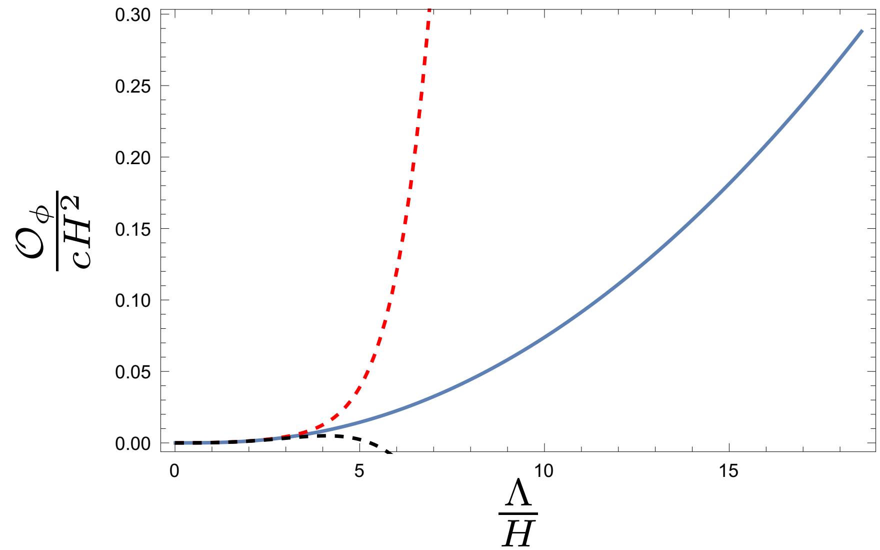
- $\mathbb{Z}_2^\chi$ symmetry can $\langle \mathcal{O}_\chi \rangle \neq 0$ (or not $\langle \mathcal{O}_\chi \rangle = 0$) be spontaneously broken, depending on the Hubble constant:

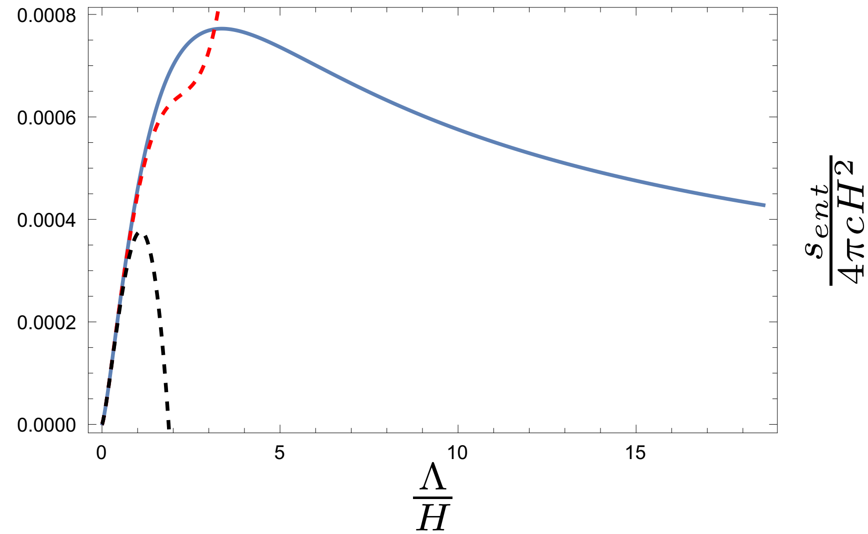
$$ds_4^2 = -dt^2 + e^{2Ht} \ \left(dx_1^2 + dx_2^2\right)$$

In this model we:

- Studied $t \to +\infty$ vacua — DFPs — as a function of $\frac{\Lambda}{H}$
- DFP_s has unbroken $\mathbb{Z}_2^\chi$ symmetry, *i.e.*, $\langle \mathcal{O}_\chi \rangle = 0$
- DFP_b has broken $\mathbb{Z}_2^\chi$ symmetry, *i.e.*, $\langle \mathcal{O}_\chi \rangle \neq 0$
- We studied perturbative stability DFPs — QNMs in BHs

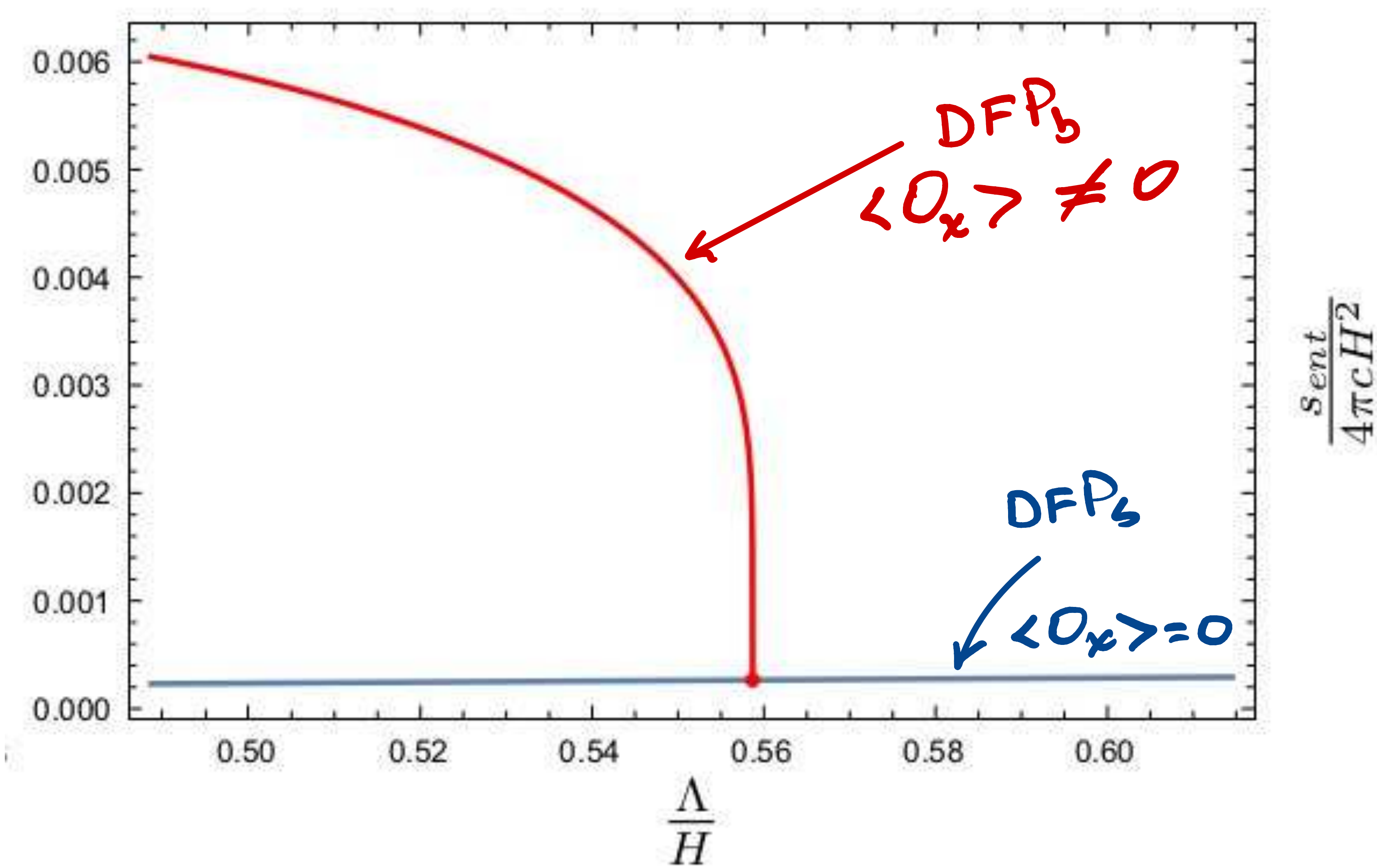
- We developed the evolution code and studied dynamics to confirm:
  - DFP is really an attractor of late-time dynamics
  - verified stability analysis
  - discovered that some perturbatively stable DFP are unstable once the amplitude of perturbation is large; confirmed the role of s_{ent} in classification of attractors



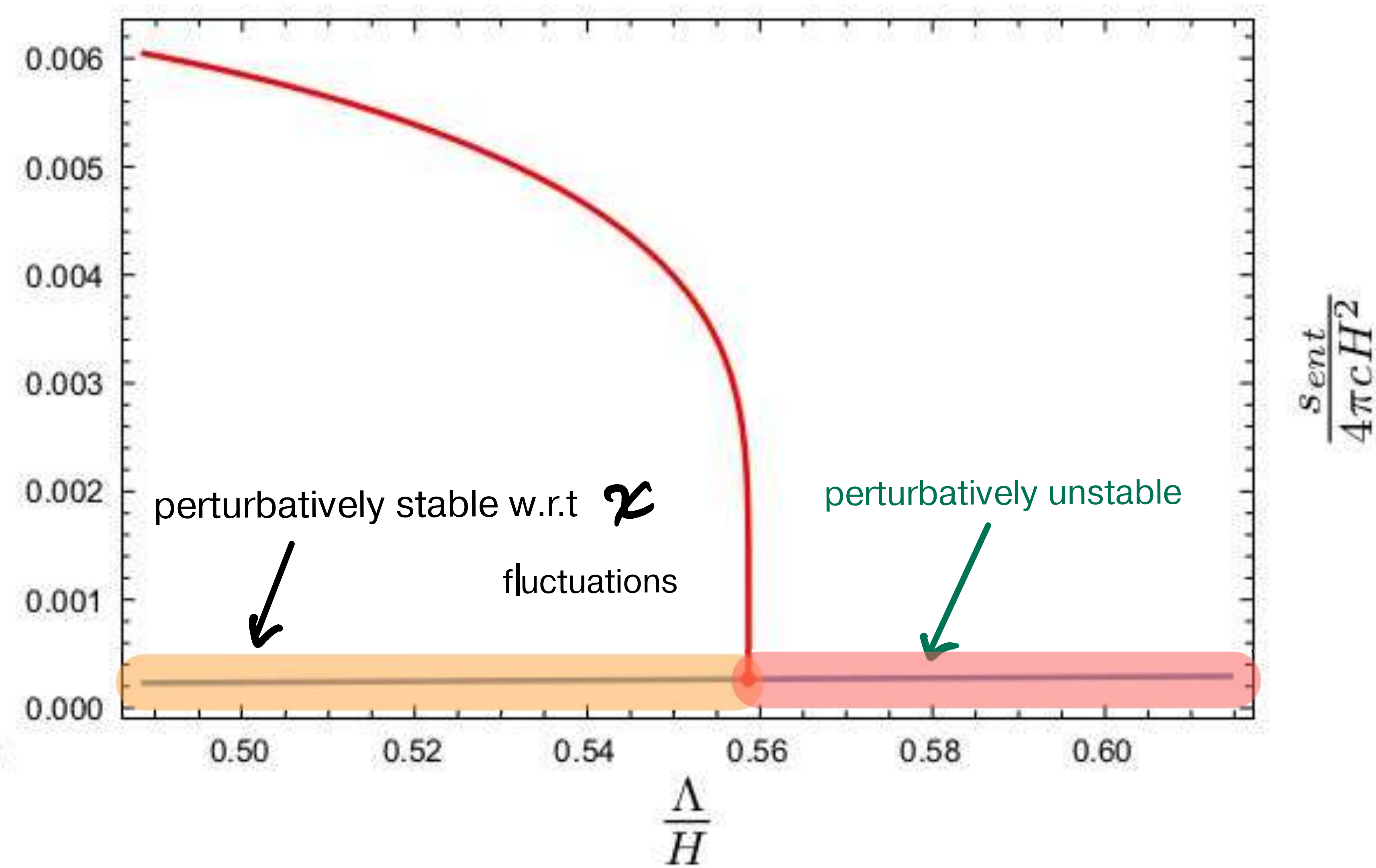


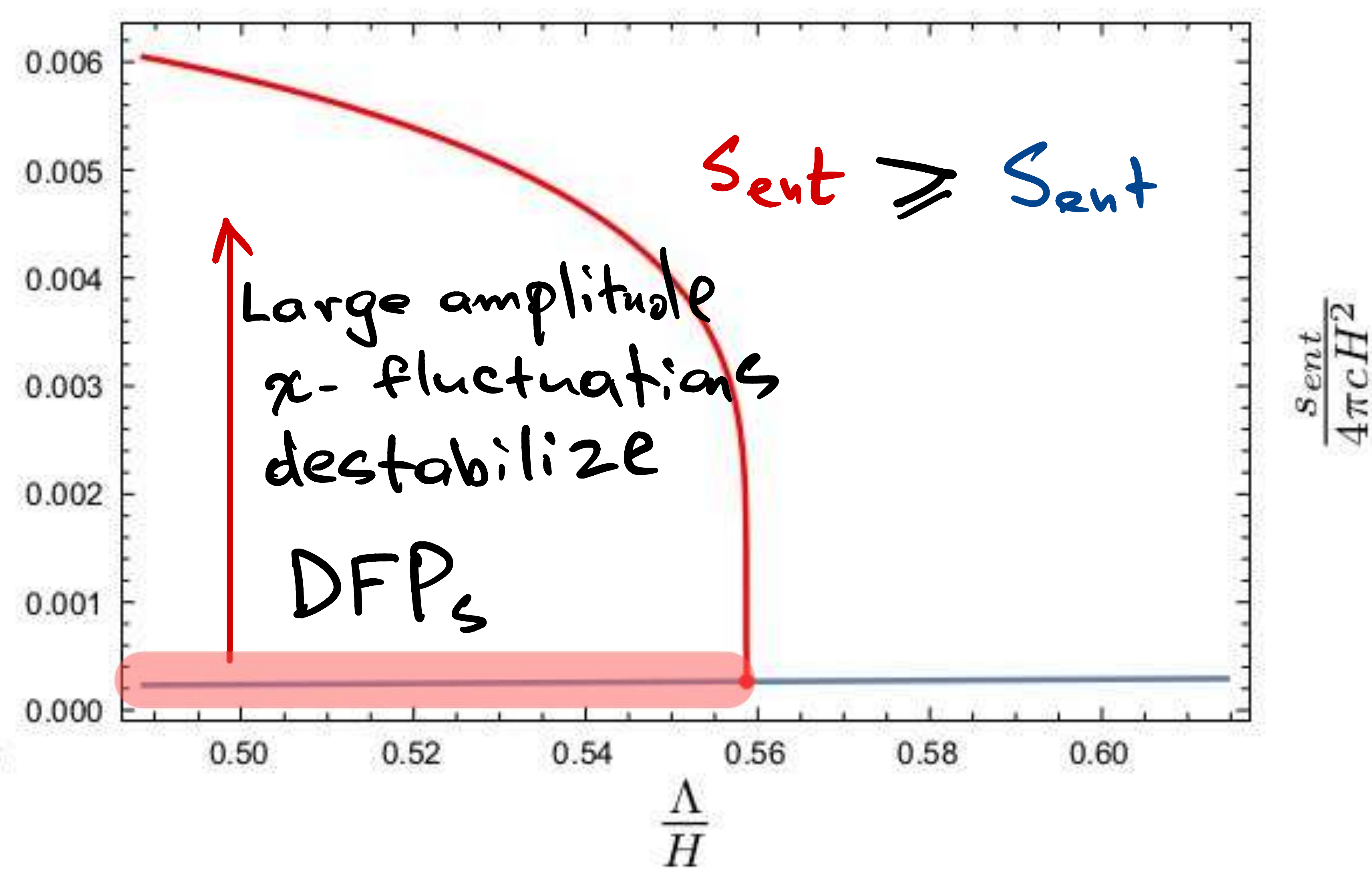
$\langle \mathcal{O}_\phi \rangle$ and s_{ent} in DFP$_s$, *i.e.*, $\langle \mathcal{O}_\chi \rangle = 0$

- c is the central charge of the theory
- Note that $s_{ent} \to 0$ as $\Lambda \to 0$ — recovering the conformal limit of trivial DFP
- Dashed lines are near-conformal perturbation theory (analytics)

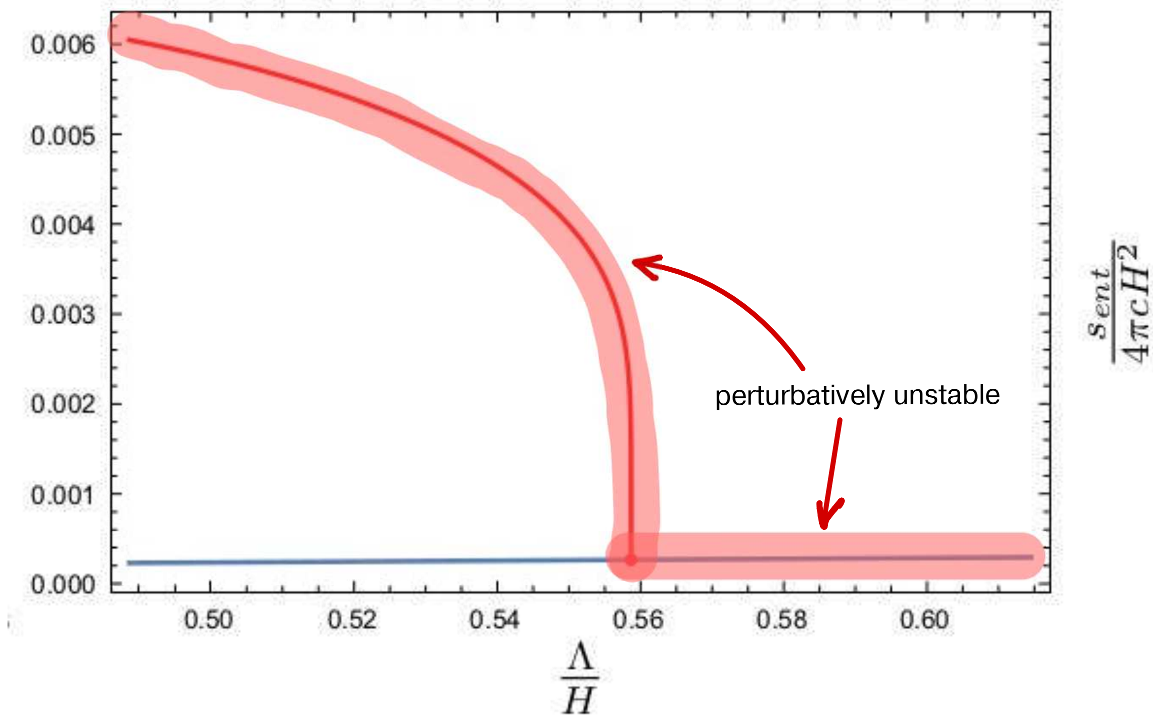


DFP$_b$
$\langle O_x \rangle \neq 0$
DFP$_b$
$\langle O_x \rangle = 0$
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0.50
0.52
0.54
0.56
0.58
0.60
$\frac{\Lambda}{H}$
$\frac{s_{ent}}{4\pi c H^2}$


0.006
0.005
0.004
0.003
0.002
0.001
0.000
0.50
0.52
0.54
0.56
0.58
0.60
$\frac{\Lambda}{H}$
$\frac{s_{ent}}{4\pi c H^2}$
perturbatively stable w.r.t χ
fluctuations
perturbatively unstable



0.006
0.005
0.004
0.003
0.002
0.001
0.000
0.50
0.52
0.54
0.56
0.58
0.60
$\frac{\Lambda}{H}$
$\frac{s_{ent}}{4\pi c H^2}$
$S_{ent} \geqslant S_{ent}$
Large amplitude
x- fluctuations
destabilize
DFPs



Highlighted DFPs, when perturbed, evolve to naked singularities with

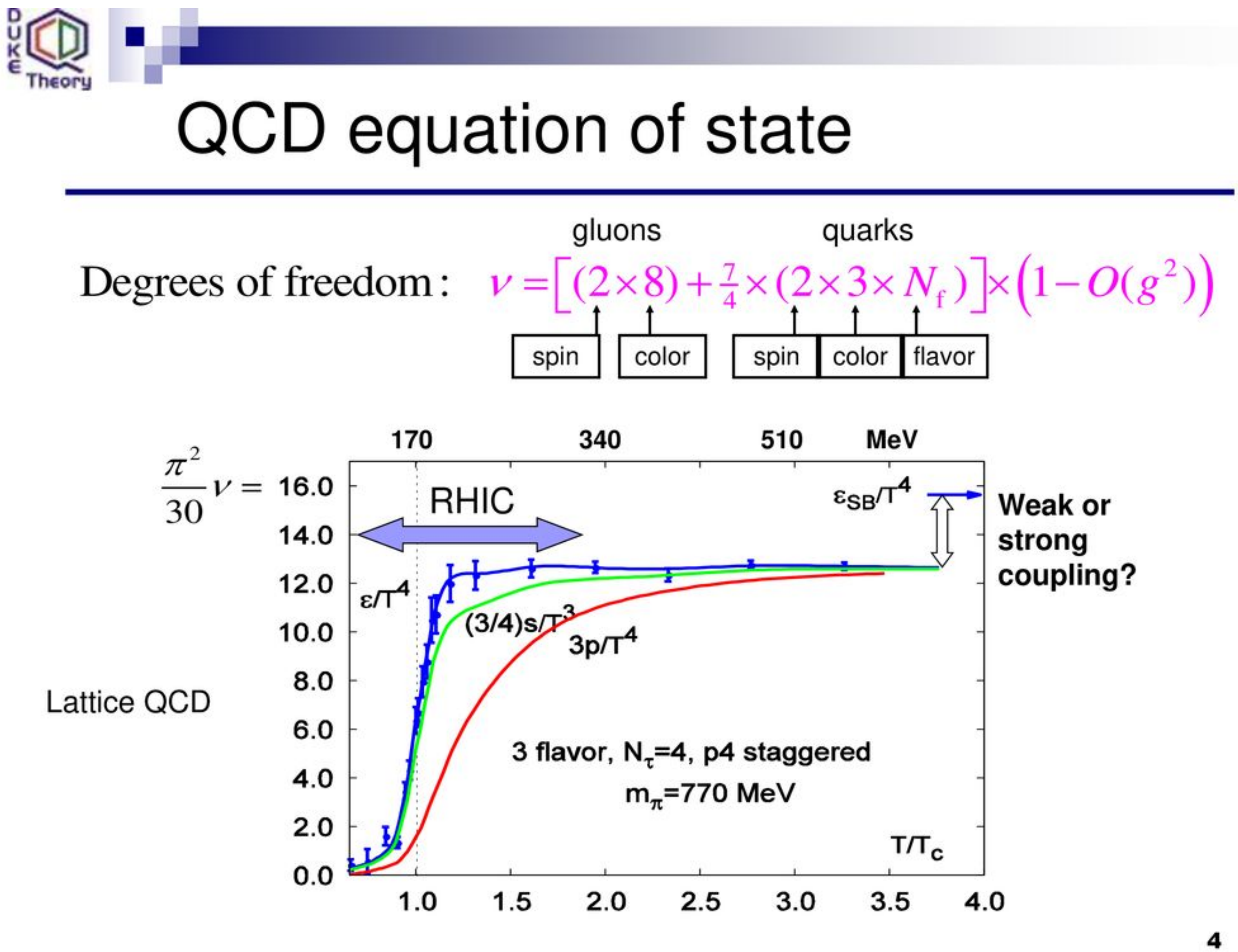
$$\lim_{t \to +\infty} \nabla \cdot S = +\infty$$

Current work:

- Study DFP in 'realistic' QCD-like model:

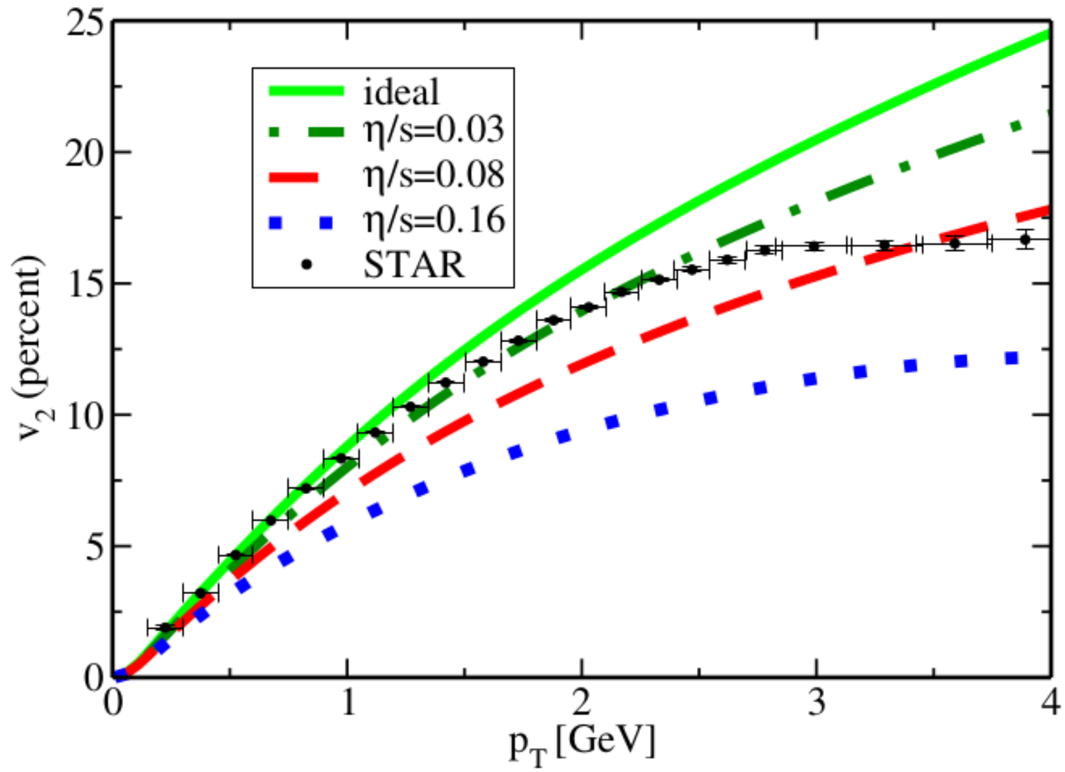
  - top-down string theory holographic example (not a toy)
  - Λ is a strong coupling scale, as in QCD
  - Like QCD, the theory confined
  - Like in QCD, there is chiral symmetry

Extra slides



from: https://slideplayer.com/slide/15105141/

talk by: Berndt Mueller, 2008



from: P.Romatschke and U.Romatschke, Phys.Rev.Lett. 99 (2007) 172301

$$\frac{\eta}{s}\bigg|_{\mathcal{N}=4\ SYM} = \frac{1}{4\pi} \approx 0.0796$$