PHYS 4010/7010: General Relativity
Assignment 4

Due: Wednesday, October 8, 2025 by 10:20 am

1. Particle in a Constant Magnetic Field

A particle with mass m and charge q is interacting with a constant magnetic field $\vec{B} = B\vec{e}_z$. The electric field is assumed to be $\vec{E} = 0$. Solve the covariant equations of motion

$$m\frac{du^\alpha}{d\tau} = \frac{q}{c}F^{\alpha\beta}u_\beta$$

for the initial conditions

$$\begin{aligned}(x^\alpha(\tau = 0)) &= (0, 0, 0, 0)\,,\\ (u^\alpha(\tau = 0)) &= (\gamma_0 c, \gamma_0 v_0, 0, 0)\,,\end{aligned}$$

where we have used $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$. Derive expressions for $x^\alpha(t)$ and $u^\alpha(t)$.

Hints:
i) Use the matrix for $(F^{\alpha\beta})$ derived in class.
ii) Compute first $x^\alpha(\tau)$, $u^\alpha(\tau)$ and then replace the proper time τ by the lab time t.

2. Transformation of $\vec{E}$ and $\vec{B}$

By using a one-dimensional Lorentz-Transformation

$$\left(\Lambda^\alpha_\gamma\right) = \begin{pmatrix} \gamma & -\gamma v/c & 0 & 0 \\ -\gamma v/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

and the relation $F'^{\alpha\beta} = \Lambda^\alpha_\gamma \Lambda^\beta_\delta F^{\gamma\delta}$ we can derive relations between the electric and magnetic fields in different inertial reference frames. Proof that the following transformation formulas are correct

$$\begin{aligned}\vec{E}'_\parallel &= \vec{E}_\parallel, \quad \vec{E}'_\perp = \gamma\left(\vec{E}_\perp + \frac{\vec{v}}{c}\times\vec{B}\right)\\ \vec{B}'_\parallel &= \vec{B}_\parallel, \quad \vec{B}'_\perp = \gamma\left(\vec{B}_\perp - \frac{\vec{v}}{c}\times\vec{E}\right).\end{aligned}$$

Here $\parallel$ corresponds to the x-direction and $\perp$ to the direction perpendicular wrt x.