PHYS 4010/7010: General Relativity
Assignment 3

Due: Wednesday, October 1, 2025 by 10:20 am

1. Equation of Motion for a Constant Force

A rocket in the interstellar space experiences the constant force $F' = mg$ in the rest frame (m is the rest mass). Describe the problem as a one dimensional problem in the x-direction.

a) Compute $x(t)$ and $v(t)$ in the lab inertial reference frame (IS). Assume that the initial conditions are $x(0) = 0$ and $v(0) = 0$.

b) Simplify your formulas for $x(t)$ and $v(t)$ by considering the two limits $t \ll c/g$ and $t \gg c/g$ and discuss the results.

c) Plot the results derived in **a)** and **b)**.

2. Relativistic Hydrodynamics

The pressure tensor $P'^{\alpha\beta}$ in the instantaneous rest frame has the form

$$\left(P'^{\alpha\beta}\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

where we have used the (invariant) *proper pressure* P.

Use the Lorentz-Transformation

$$P^{\alpha\beta} = \Lambda^{\alpha}_{\gamma}\left(-\vec{v}\right)\Lambda^{\beta}_{\delta}\left(-\vec{v}\right)P'^{\gamma\delta}$$

to prove that the pressure tensor $P^{\alpha\beta}$ in an arbitrary inertial reference frame has the form

$$P^{\alpha\beta} = P\left(\frac{u^{\alpha}u^{\beta}}{c^2} - \eta^{\alpha\beta}\right).$$

To perform these calculations you can use the tensor $\Lambda^{\alpha}_{\beta}\left(\vec{v}\right)$ discussed in class for arbitrary $\vec{v}$.