# PHYS 4010/7010: General Relativity
# Assignment 1

Due: Wednesday, September 17, 2025 by 10:20 pm

## 1. Newtonian Gravity: Potential of a Sphere

Calculate the gravitational potential field $\Phi(\vec{r})$ for a homogeneous sphere with a constant density $\rho_0$ and radius $R$. Distinguish between the two different cases $r < R$ and $r > R$.

**a)** Use the formula

$$\Phi\left(\vec{r}\right) = -G \int d^3r' \, \frac{\rho(\vec{r}^{\,\prime})}{|\vec{r} - \vec{r}^{\,\prime}|}$$

in combination with the symmetry assumption $\vec{r} = r\hat{z}$.

**b)** Solve the field equation (Poisson's equation)

$$\Delta\Phi = 4\pi G\rho$$

for the same sphere and compare your results with **1a)**.

**c)** Calculate the gravitational binding energy by evaluating

$$E = \frac{1}{2} \int d^3r \; \rho(\vec{r})\Phi(\vec{r}).$$

## 2. Multipole Expansion in Cartesian Coordinates

The gravitational potential is given by

$$\Phi\left(\vec{r}\right) = -G \int d^3r' \, \frac{\rho(\vec{r}^{\,\prime})}{|\vec{r} - \vec{r}^{\,\prime}|}.$$

Assume that the mass distribution $\rho\left(\vec{r}^{\,\prime}\right)$ is localized so that

$$\rho(\vec{r}^{\,\prime}) \approx 0 \quad \text{if} \quad r' > r_{max}.$$

Approximate the potential for the case $r \gg r_{max}$.

**a)** Proof that the first term in a multipole expansion has the form

$$\Phi_0\left(\vec{r}\right) = -\frac{GM}{r}.$$

**b)** Derive the term for the second order contribution (dipole moment) and express it by using the center of mass $\vec{R}$.

**c)** What is the gravitational dipole moment in the center of mass references frame?