<u>PHYS 7590, Assignment # 5</u>

<u>Problem 1</u>: Define the rapidity

$$\theta := \tanh^{-1}\left(\frac{v}{c}\right).$$

(a) Express the Lorentz transformation matrix A (see page 12 of the lecture notes) in terms of θ.

(b) Express the Einstein velocity addition law (the equation relating u_3' to u_3 and v on page 38) in terms of rapidity. Hint: Let

$$\phi' = \tanh^{-1}\left(\frac{u_3'}{c}\right), \phi = \tanh^{-1}\left(\frac{u_3}{c}\right) \text{ and } \theta = \tanh^{-1}\left(\frac{v}{c}\right);$$

and find ϕ' as a function of ϕ and θ.

<u>Problem 2</u> (The Twin "Paradox"): On their 21st birthday, one twin gets on a moving sidewalk, which carries her to star X at speed $\frac{4}{5}c$; her twin brother stays at home. When the traveling twin gets to star X, she immediately jumps onto the returning moving sidewalk and comes back to Earth, again at speed $\frac{4}{5}c$. She arrives on her 39th birthday (as determined by her watch).

(a) How old is her twin brother (who stayed at home)?

(b) How far is star X? (Give your answer in light years.)

Call the outbound sidewalk system K' and the inbound system K'' (the Earth system is K). All three systems set their master clocks, and choose their origins, so that $z = z' = z'' = 0$ and $t = t' = t'' = 0$ at the moment of departure.

(c) What are the coordinates (z, t) in K of the jump from outbound to inbound sidewalk?

(d) What are the coordinates (z', t') of the jump in K'?

(e) What are the coordinates (z'', t'') of the jump in K''?

(f) If the traveling twin wanted her watch to agree with the clock in K'', how would she have to reset it immediately after the jump? If she did this what would her watch read when she got home? (This would not change her age, of course- she is still 39- it would just make her watch agree with the standard synchronization in K''.)

(g) If the traveling twin is asked the question: "How old is your brother right now?", what is the correct reply (i) just before she makes the jump and (ii) just after she makes the jump? (Nothing dramatic happens to her brother during the split second between (i) and (ii), of course; what does change is his sister's notion of what "right now, back home" means.)

(h) How many earth years does the return trip take? Add this to (ii) from (g) to determine how old she expects him to be at their reunion. Compare your answer to that from (a).

Problem 3: Consider a particle in hyperbolic motion

$$z(t) = \sqrt{b^2 + (ct)^2}; \qquad x = y = 0.$$

(a) Find the proper time τ as a function of t, assuming the clocks are set so that $\tau = 0$ when $t = 0$. Hint: Integrate Equation (31) (with $d\alpha$ replaced by $d\tau$).

(b) Find z and $u := dz/dt$ as functions of τ.

Problem 4: Let $\vec{a}$ and $\vec{a}'$ denote the conventional 3-D accelerations of a particle in reference frames K and K', respectively, with K' moving with velocity $v\hat{k}$ relative to K. Show that

$$a_3' = \frac{a_3}{[\gamma(v)]^3 \left[1 - \frac{vu_3}{c^2}\right]^3}.$$

Problem 5: Suppose you have a collection of particles, all moving in the z direction, with energies $E_1, E_2, E_3, \ldots$ and momenta $p_1, p_2, p_3, \ldots$. Find the velocity of the **center of momentum** frame, in which the total momentum is zero.

Problem 6: In a pair annihilation experiment, an electron with (relativistic) momentum p_e hits a positron at rest. They annihilate, producing two photons. If one of the photons emerges at $60°$ to the incident electron direction, what is its energy?

Problem 7: A particle of rest mass m_0 whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, what is the rest mass of the resulting particle? What is its velocity?

Problem 8: A neutral pion of rest mass m_0 and (relativistic) momentum $p = \frac{3}{4}m_0c$ decays into two photons. One of the two photons is emitted in the same direction as the original pion, and the other in the opposite direction. Find the (relativistic) energy of each photon.

Problem 9: An unstable elementary particle has a mean life of 0.81×10^{-10} sec in a reference frame in which it is at rest; and it has a rest energy of 1,190 MeV. What is this particle's kinetic energy in a laboratory reference frame in which it is observed to travel a distance of 1 mm before decaying?