Schriemer et al. Reply: In our Letter [1], we simply pointed out that the traditional coherent potential approximation (CPA) breaks down in the intermediate frequency regime because of the wave vector dependence of the self-energy. Thus, CPA models, such as the “S-CPA” and “C-CPA” [2,3], are suspect in this frequency range, as they rely on ad hoc variations of the traditional CPA approach to determine the properties of the effective medium (e.g., requiring the scattering amplitude from the basic scattering unit embedded in the effective medium to vanish only in the forward direction, or to vanish in the forward direction and one other direction). Soukoulis et al. [4] correctly point out that their so-called energy density CPA models (E-CPA) [2,5] attempt to circumvent these problems by introducing a different condition to determine the properties of the effective medium, namely the assumption of energy density homogenization. While this assumption is appealing in its simplicity, it is important to check its accuracy by careful comparison with either experiment or a more rigorous theory.

We also pointed out [1] that their E-CPA model for acoustic waves [2] predicts considerably less structure in the frequency dependence of the energy velocity \( v_e \) than is found in both our experiments and theory. This is explicitly shown in Fig. 1, which compares our experimental data and theory with the E-CPA. It is clear that the dotted curve, representing the E-CPA, is in poor agreement with our data for \( v_e \), especially near the first large dip near \( k_w a \approx 2 \) [1,6]. By contrast, our theory is in excellent agreement with our data (Fig. 1).

The fact that the E-CPA does not agree quantitatively with our experimental results [1] at the volume fraction \( \phi \) of 0.63 has nothing to do with the \( \phi \) dependence of the velocity dispersion. We have already established [7,8], through measurements of the phase \( (v_p) \) and group \( (v_g) \) velocities, that the pronounced structure in the frequency dependence seen at high values of \( \phi \) [1,9] is progressively reduced as \( \phi \) decreases; furthermore, this behavior is in excellent agreement with our model based on a spectral function approach. By contrast, the E-CPA, while showing the same general trends, is not quantitatively accurate; for example, at \( \phi = 0.23 \), it predicts unrealistically large values of \( v_g \) compared with our results for \( v_p \) and \( v_g \). We note that the differences between the predictions of these two models are consistent with the large departures from energy density homogenization found in our theory.

Finally, we emphasize that our spectral function approach is more than just a plausible “ansatz.” It is well established that the Green’s function \( G(\omega,k) \) determines the amplitude response of a system, from which \( v_p \) and \( v_g \) can be obtained. In our calculations, these ballistic velocities are determined by identifying the frequencies and wave vectors of the excitation modes from the peaks in the spectral function, \(-\text{Im}(G(\omega,k))\), giving results that are in excellent agreement with experiment [8,9]. One of the main points of our Letter was to demonstrate that the energy velocity of diffusing acoustic waves, while nominally an “energy density” quantity, is nevertheless given directly by \( v_e \) divided by a factor that accounts for the additional propagation delay experienced by the scattered waves; this factor is also determined by the amplitude response of the system and explains the difference between \( v_e \) and \( v_g \) in simple physical terms. Thus, it is not surprising that our spectral function approach makes predictions that agree quantitatively, not just qualitatively, with the measurements of \( v_e \), as well as with \( v_p \) and \( v_g \).

H. P. Schriemer, M. L. Cowan, J. H. Page, Ping Sheng, Zhengyou Liu, and D. A. Weitz

1Department of Physics & Astronomy
University of Manitoba Winnipeg, MB, Canada R3T 2N2
2Department of Physics
The Hong Kong University of Science & Technology
Clear Water Bay, Kowloon, Hong Kong
3Department of Physics and Astronomy
University of Pennsylvania
Philadelphia, Pennsylvania 19104-6396

Received 16 June 1998 [S0031-9007(99)08435-5]
PACS numbers: 43.35.+d, 43.20.+g, 62.30.+d

[6] A similar discrepancy was found for electromagnetic waves, where the E-CPA predicts a much flatter variation in \( v_e \) than is seen experimentally (see Ref. [5]).