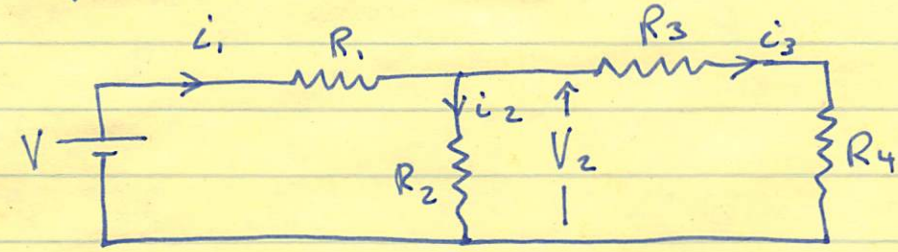
Phys 2610 (2017) Pre-lab exercise 1 solution

a)



$$R_{eff} = R_1 + R_{\parallel} = R_1 + R_2 \parallel (R_3 + R_4) = R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

$\left(\text{where } R_{\parallel} = \left(\frac{1}{R_2} + \frac{1}{R_7}\right)^{-1}, \text{ and } R_7 = R_3 + R_4\right)$

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Using nominal values $R_1 = 1\,k\Omega$, $R_2 = 3.3\,k\Omega$, $R_3 = 2.2\,k\Omega$, and $R_4 = 10\,k\Omega$,

$$R_7 = 12.200\,k\Omega, \quad R_{\parallel} = 2.597\,k\Omega$$

to give $R_{eff} = 3.597\,k\Omega$

b) Using $R_{eff} = R_1 + R_{11}$, $\delta R_{eff}^2 = \delta R_1^2 + \delta R_{11}^2$

Now, $\frac{1}{R_{11}} = \frac{1}{R_2} + \frac{1}{R_7}$ $\left(R_{11} = \frac{R_2 R_7}{R_2 + R_7} = \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} \right)$

To use the simple rule, the terms must be independent, so

$$\delta(1/R_{11})^2 = \delta(1/R_2)^2 + \delta(1/R_7)^2$$

but $\frac{\delta(1/x)}{(1/x)} = \frac{\delta x}{x}$ since relative errors add (in quadrature) for division

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so $\delta x = x^2 \delta(1/x)$ and $\delta(1/x) = \frac{\delta x}{x^2}$, giving

$$\delta R_{11}^2 = R_{11}^4 \, \delta(1/R_{11})^2 = R_{11}^4 \left(\delta(1/R_2)^2 + \delta(1/R_7)^2 \right)$$

$$= R_{11}^4 \left(\frac{\delta R_2^2}{R_2^4} + \frac{\delta R_7^2}{R_7^4} \right)$$, and

$$\delta R_{eff}^2 = \delta R_1^2 + R_{11}^4 \left(\frac{\delta R_2^2}{R_2^4} + \frac{\delta R_3^2 + \delta R_4^2}{(R_3+R_4)^4} \right)$$ (using $R_7 = R_3 + R_4$)

With the above nominal values, and with 0.25% uncertainty, $\delta R_1 = 0.0025 R_1 = 2.5\Omega$, $\frac{\delta R_2}{R_2} = 0.0025$, $\delta R_3 = 5.5\,\Omega$, $\delta R_4 = 25\,\Omega$.

Substituting gives $\delta R_{eff} = 5.8\Omega$ so $R_{eff} = 3.597 \pm .006\,k\Omega$, for a per cent error $\frac{\delta R_{eff}}{R_{eff}} \times 100 = 0.17\%$ //

Note that the same result for δR_{11} can be obtained using the general formula:

$$R_{11} = \left(\frac{1}{R_2} + \frac{1}{R_7} \right)^{-1}$$

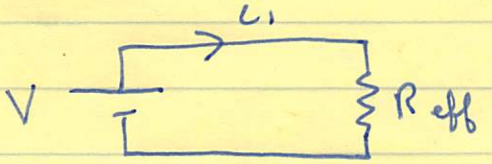
$$\rightarrow \delta R_{11} = \left(\frac{\partial R_{11}}{\partial R_2} \right)^2 \delta R_2^2 + \left(\frac{\partial R_{11}}{\partial R_7} \right)^2 \delta R_7^2$$

Here $\frac{\partial R_{11}}{\partial 2} = (-1) \left(\frac{1}{R_2} + \frac{1}{R_7} \right)^{-2} (-1) R_2^{-2} = \frac{R_{11}^2}{R_2^2}$,

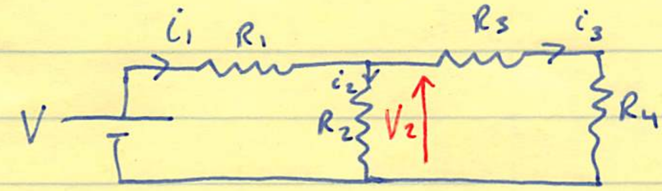
$\frac{\partial R_{11}}{\partial R_7} = \frac{R_{11}^2}{R_7^2}$, so

$$\delta R_{11}^2 = R_{11}^4 \left(\frac{\delta R_2^2}{R_2^4} + \frac{\delta R_7^2}{R_7^4} \right)$$

Note also that combining uncertainties for the numerator & denom. in $R_{11} = R_2 R_7 / (R_2 + R_7)$ tends to exaggerate the uncertainty because the numerator & denominator are <u>not</u> independent.

c) 

$$\Rightarrow \quad i_1 = \frac{V}{R_{eff}}$$



The voltage across R_2 is $V_2 = i_1 R_{\parallel} = \dfrac{V}{R_{eff}}\left(\dfrac{R_2(R_3+R_4)}{R_2+R_3+R_4}\right)$

so $$i_2 = \frac{V_2}{R_2} = \frac{V}{R_{eff}} \frac{(R_3+R_4)}{(R_2+R_3+R_4)}$$

and $$i_3 = \frac{V_2}{R_3+R_4} = \frac{V}{R_{eff}}\left(\frac{R_2}{R_2+R_3+R_4}\right)$$