UNIVERSITY OF MANITOBA

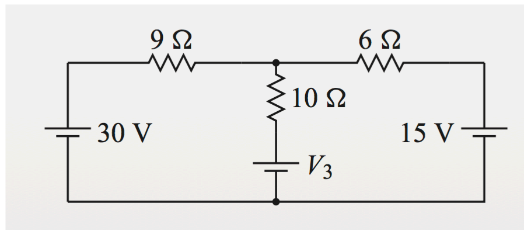
April 13, 2016 FINAL EXAM

DEPARTMENT & COURSE NO.: PHYS 2610 TIME: 3 hours

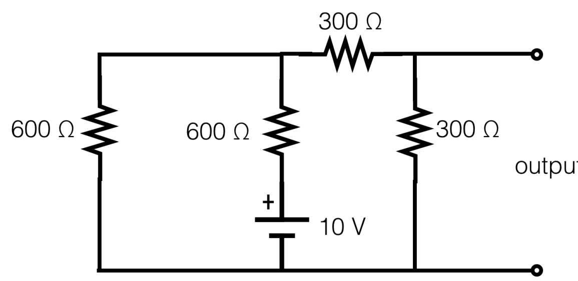
EXAMINATION: Circuit Theory and Introductory Electronics EXAMINER: W Ens

Answer all questions. All questions except 7 are worth 10 marks. Number 7 is worth 20 marks.

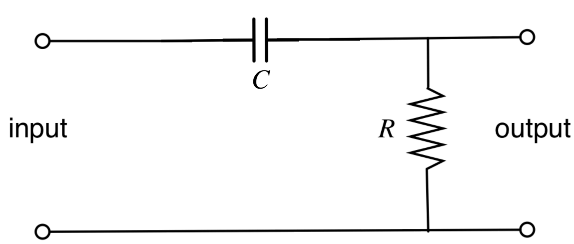
1. For the circuit shown below, find the value of V_3 such that the current in the 10 Ω resistor is zero.



2. Find the Thevenin equivalent circuit for the circuit shown below.



3. In the RC circuit shown below, determine the output voltage as a function of time if the input is stepped from zero to V at time $t = 0$. What is the current at very long times?

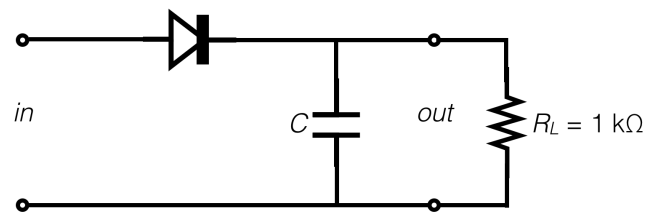


4. For the circuit of question 3, determine the output voltage if the input is given by $v_{in} = V\cos\omega t$. What is the phase shift? Is this a high-pass or a low-pass filter? What is the breakpoint frequency, where the gain drops by 3 dB from its maximum value?

5. For the circuit of question 3 with $R = 1$ kΩ and $C = 0.10\ \mu$F, determine what load connected to the output gives maximum power transfer, if the input voltage is sinusoidal with a frequency of 1000 Hz.

continued…

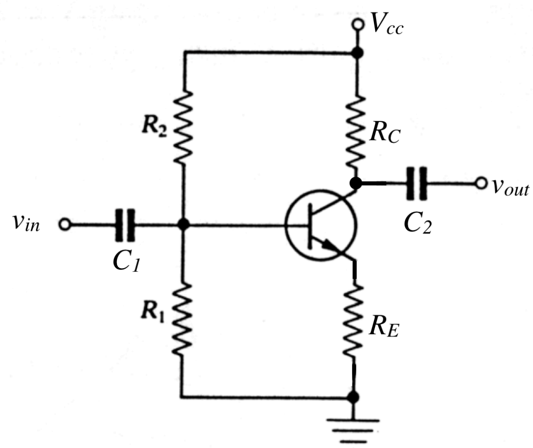
6. In the rectifier circuit shown below, the input voltage is sinusoidal with a peak voltage of 100 V and a frequency of 50 Hz. Sketch the output voltage (across R_L) for (a) no capacitor, (b) $C = 10\ \mu$F, and (c) $C = 100\ \mu$F. Overlay the 3 outputs on the same axes. Calculate the peak-to-peak ripple for (c). Treat the diode as an ideal diode.

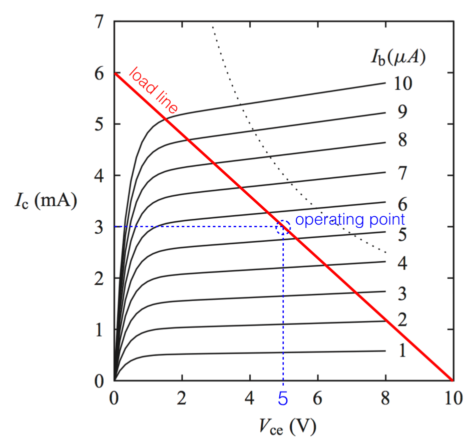


7. The common emitter amplifier shown below is configured to give the load line and operating point as indicated on the characteristic curves.

(a) Select values of V_{cc}, R_c, R_E, R_1, and R_2 that would give this load line and operating point and a gain of -6.

(b) What value of C_1 would cause the gain to drop by 3 dB at 100 Hz, ignoring other contributions to the attenuation at this frequency? Assume $r_{BE} = 2\ \text{k}\Omega$.





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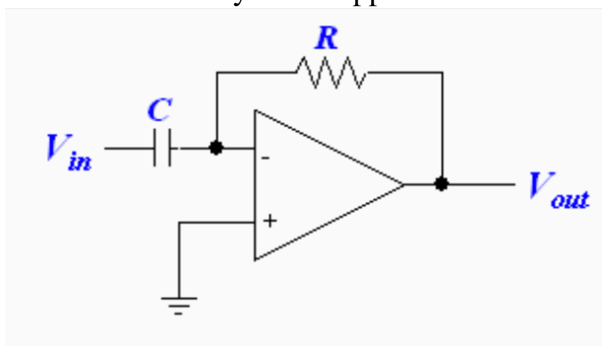
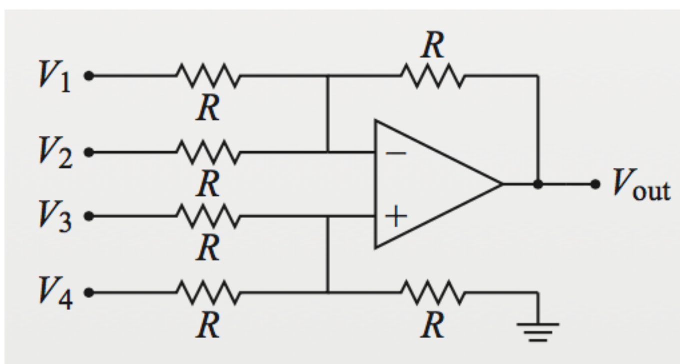
8. Show that the output of the circuit below is approximately proportional to the derivative of the input, and give the condition for the validity of the approximation.

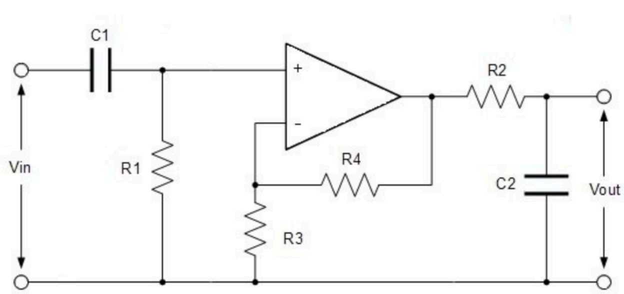


Draw the circuit for a passive differentiator, and give the condition for its validity.

9. Find an expression for the output voltage of the circuit shown below in terms of the 4 input voltages and the resistance *R*. Simplify the result as much as possible.



10. Briefly explain the function of the op amp in a band pass filter like the one shown below.



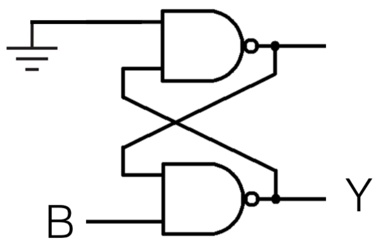
continued…

11. Using diodes, npn transistors, and resistors, give an example of a circuit diagram for 3 of the following gates: (a) OR, (b) NOR, (c) AND, (d) NAND, and (e) NOT. (3 diagrams are expected)

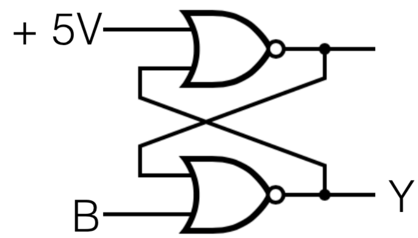
12. A half-adder takes two inputs (A, B) and provides two outputs ($A \oplus B, A \cdot B$). Show how to implement a half adder using only NOR gates.

13. (a) In the NAND RS flip flop below, what is the relationship between B and Y? (b) In the NOR RS flip flop below, what is the relationship between B and Y?

(a)



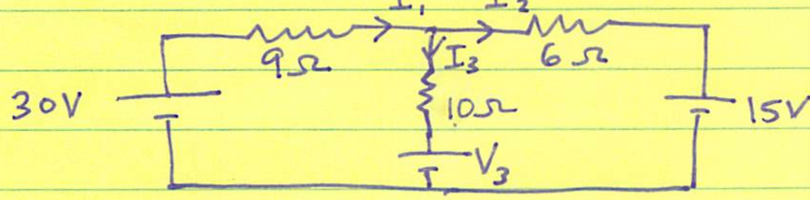
(b)



The End

Phys 2610 (2017) Final Exam solutions

(1)



$I_3 = 0 \Rightarrow I_1 = I_2 = I$

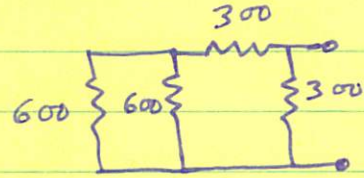
Then KVL around outer loop: $30V - I(9\Omega + 6\Omega) - 15V = 0$

$\rightarrow I = \frac{15V}{15\Omega} = 1A.$

KVL on left side: $30V - I\,9\Omega - V_3 = 0$

$\rightarrow V_3 = 30V - 9V = \boxed{21V}$

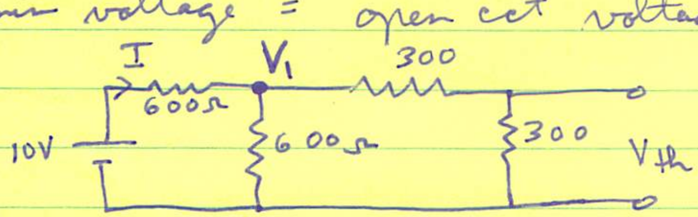
(2) Thevenin resistance from



$R_{th} = 300\Omega \,//\, (300\Omega + 600\Omega \,//\, 600\Omega)$

$= 300\Omega \,//\, (300\Omega + 300\Omega) = 300\Omega \,//\, 600\Omega = \boxed{200\Omega}$

Thevenin voltage = open cct voltage. Redrawing

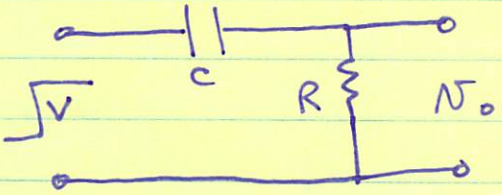


$I = \frac{10V}{(600 + 600//600)\Omega}$

$= \frac{10V}{900\Omega} = .0111A.$

Then $V_1 = 10V - I(600\Omega) = 3\tfrac{1}{3}V \rightarrow V_{th} = \frac{V_1}{2} = \boxed{1.67V}$

(3)



KVL: $V - q/C - iR = 0$

$$\Rightarrow \frac{di}{dt} + \frac{1}{RC} i = 0$$

Solution is: $i = A e^{-t/RC}$

At $t=0$, $v_c = 0$, so $iR = V \Rightarrow A = \frac{V}{R} \Rightarrow i = \frac{V}{R} e^{-t/RC}$

Then the output is $v_o = iR = \boxed{V e^{-t/RC}}$

For $t \to \infty$, $\boxed{i \to 0}$

(4) The complex impedance is

$$Z = R + \frac{1}{j\omega C} = |Z| e^{j\varphi} \quad \text{with } |Z| = \sqrt{R^2 + (1/\omega C)^2}$$
$$\text{and } \tan\varphi = (1/\omega RC)$$

so the current is (the real part of): $i = \frac{v_{in}}{Z} = \frac{V}{|Z|} e^{j(\omega t - \varphi)}$

and the output voltage is (the real part of):

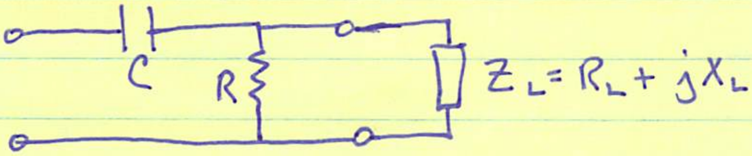
$$v_{out} = iR = \boxed{\frac{V}{\sqrt{1 + (1/\omega RC)^2}} e^{j(\omega t - \varphi)}}$$

The phase shift is $-\varphi = \boxed{-\arctan(1/\omega RC)}$

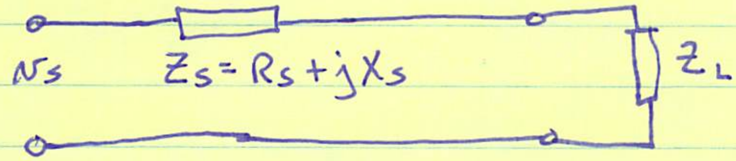
As $\omega \to \infty$, $v_{out} \to v_{in}$, so this is a **high pass filter**

At the breakpoint freq. ω_B, $v_{out} \to v_{in}/\sqrt{2} \Rightarrow$
$\sqrt{1 + (1/\omega RC)^2} \to \sqrt{2} \Rightarrow \boxed{\omega_B \to 1/RC}$

(5)



Max power transfer occurs when $Z_L = Z_s^*$ where Z_s is the Thevenin equivalent series impedance:



So, $R_L = R_s$
$X_L = -X_s$

The equivalent impedance is obtained by shorting the voltage source so

$$Z_s = R \,//\, \frac{1}{j\omega C} = \frac{R/j\omega C}{R + 1/j\omega C} = \frac{R}{1 + j\omega RC}\left(\frac{1 - j\omega RC}{1 + j\omega RC}\right)$$

$$= \frac{R - j\omega R^2 C}{1 + (\omega RC)^2}$$

Using $R = 1\,k\Omega$, $C = 0.10\,\mu F$, and $\omega = 2\pi(1000\,Hz)$, gives

$$Z_s = (614 - j385)\,\Omega$$

so $R_L = R_s =$ **$614\,\Omega$**

and $X_L = -X_s =$ **$+385\,\Omega$** $\rightarrow$ e.g. $\omega L = 385\,\Omega$

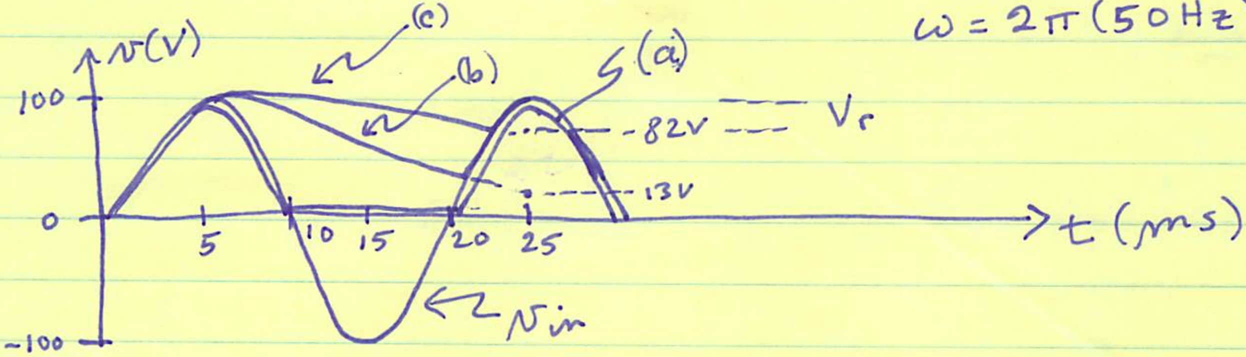
$\rightarrow L = \frac{385\,\Omega}{\omega} =$ **$61\,mH$**

(6)

$v_{in} = V_p \cos \omega t$

$V_p = 100V$

$\omega = 2\pi(50Hz)$



(a) No capacitor: $v_{out} = v_{in}$ for $v_{in} > 0$

$= 0$ for $v_{in} < 0$

(b) $C = 10\mu F$: $R_L C = 1k\Omega \cdot 10\mu F = 0.010s = 10ms$

$v_{out} = 100V e^{-t'/10ms}$ where $t' = 0$ at max v_{in}

$v_{out}(t' = 20\text{ ms}) = 13V$

(c) $C = 100\mu F$: $R_L C = 100ms$

$v_{out} = 100V e^{-t'/100ms}$

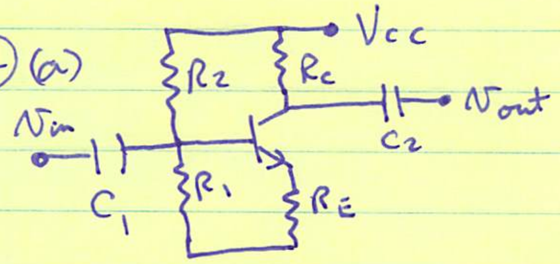
$v_{out}(t' = 20ms) = 82V$

$V_{ripple} \cong V_p - 82V = \boxed{18V}$

Also $V_r = v_{out}(t' = 0) - v_{out}(t' - T)$

$\cong V_p(1 - e^{-T/\tau}) \cong \frac{V_p T}{RC} = \boxed{20V}$

7 (a)



(i) From the load line

$$I_C = \frac{V_{CC}}{R_C+R_E} - \frac{V_{CE}}{R_C+R_E}$$

$\boxed{V_{CC} = 10V}$ and $\frac{V_{CC}}{R_C+R_E} = 6\,mA$

$$\rightarrow R_C + R_E = \frac{10V}{6\,mA} = \underline{1.67\,k\Omega}$$

(ii) For a gain of −6, $a = \frac{-R_C}{R_E} = -6 \rightarrow R_C = 6R_E$

$\therefore 6R_E + R_E = 1.67\,k\Omega \rightarrow \boxed{R_E = 238\,\Omega} \rightarrow \boxed{R_C = 1.4\,k\Omega}$

(iii) Operating point: $V_{CE} = 5V$, $I_C = I_E = 3mA$, $I_B = 5.5\mu A$

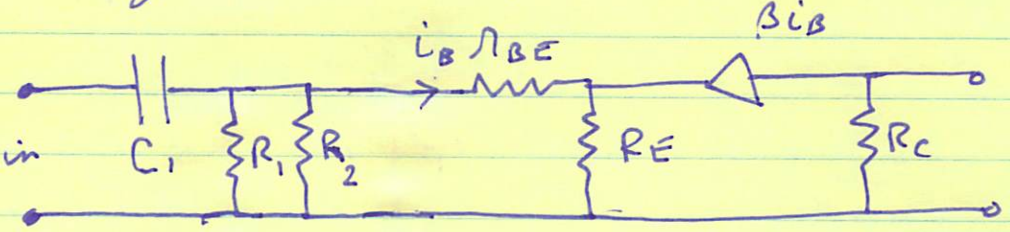
$$V_B = V_{BE} + I_E R_E = 0.6V + (3mA)\,238\,\Omega = 1.31V$$

but $V_B = I_1 R_1 = 20 I_B R_1 \rightarrow R_1 = \frac{V_B}{20 I_B} = \boxed{11.9\,k\Omega}$

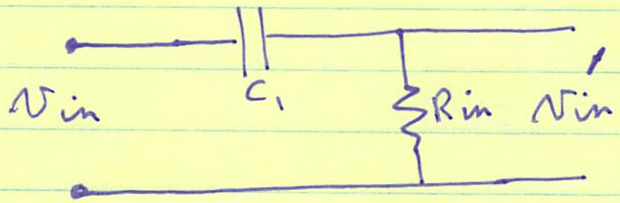
(taking $I_1 = 20 I_B$)

$$V_{CC} - I_2 R_2 = V_B \rightarrow R_2 = \frac{V_{CC} - V_B}{I_1 + I_B} = \boxed{75.2\,k\Omega}$$

(7) (b) To find the attenuation by C_1, consider it in series with the ac equivalent circuit for the amplifier



The input resistance of the amplifier forms a high pass filter with C_1:



For the above circuit $R_{in} = R_1 // R_2 // (r_{BE} + \beta R_E)$

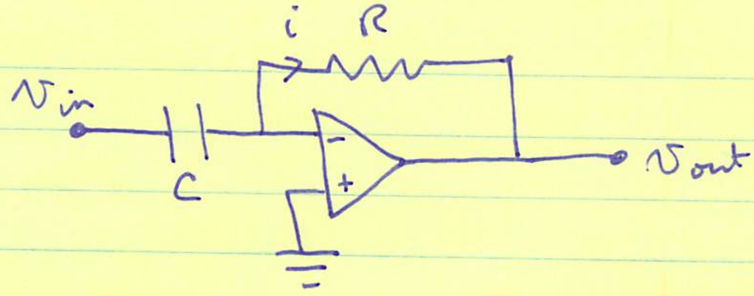
$$\cong R_1 // R_2 = 10.3\ k\Omega$$

Attenuation by 3 dB occurs when $\omega = \dfrac{1}{R_{in} C_1}$

so $$C_1 = \frac{1}{\omega_B R_{in}} = \frac{1}{2\pi (100\ Hz)(10.3\ k\Omega)}$$

$$\boxed{C_1 = 154\ nF}$$

(8)



Since $v_- = v_+$, $v_- = 0$ (virtual ground)

Then $v_{out} = -iR$

But $v_{in} = q/C$ so $\frac{dv_{in}}{dt} = \frac{1}{C} i \Rightarrow i = C \frac{dv_{in}}{dt}$

$\therefore$

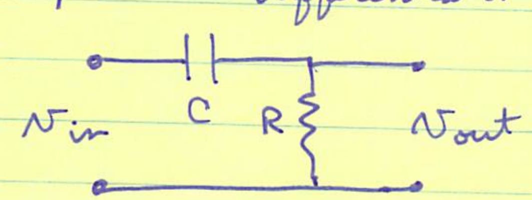
$$\boxed{v_{out} = -RC \frac{dv_{in}}{dt}}$$

This is valid for an ideal op amp where $a \to \infty$ or where

$$a \gg \frac{v_{out}}{v_{in}} \Rightarrow a \gg \frac{R}{1/\omega C}$$

or $\boxed{\omega RC \ll a}$

A passive differentiator is

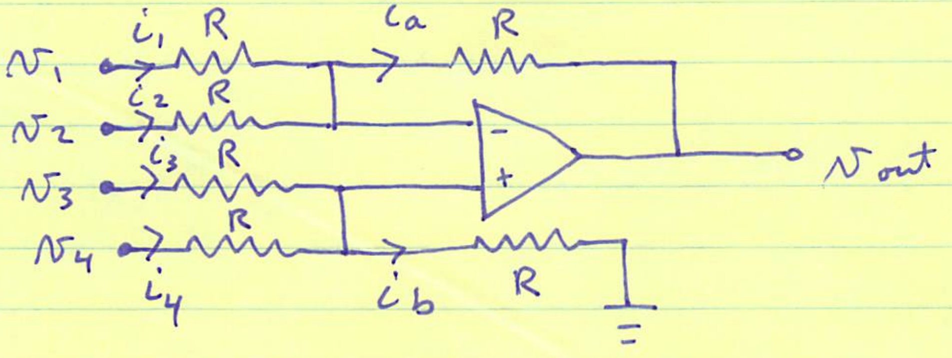


Here, $v_{out} = iR$
and $v_{in} = q/C + iR$

so $v_{out} = RC \frac{dv}{dt}$ if $q/C \gg iR$
or $v_{out} \ll v_{in}$
or $R \ll 1/\omega C$

$\Rightarrow$ $\boxed{\omega RC \ll 1}$

(9)



Since current into op amp is negligible,

$i_1 + i_2 = i_a$ and $i_3 + i_4 = i_b$

$$\frac{v_1 - v_-}{R} + \frac{v_2 - v_-}{R} = \frac{v_- - v_{out}}{R} \longrightarrow v_1 + v_2 = 3v_- - v_{out}$$

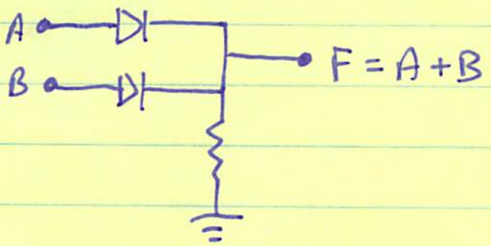
and $$\frac{v_3 - v_+}{R} + \frac{v_4 - v_+}{R} = \frac{v_+}{R} \longrightarrow v_3 + v_4 = 3v_+$$

Using $v_+ = v_-$, and taking the difference,

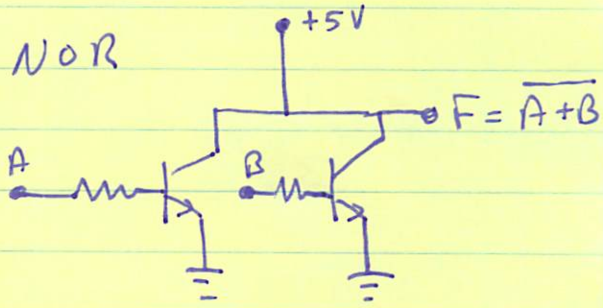
$$v_{out} = v_3 + v_4 - (v_1 + v_2)$$

(10) The op amp acts as a buffer, preventing the second filter from affecting the performance of the first, which is usually calculated without a load. The op amp passes the voltage signal (usually with unity gain), but presents no load (infinite impedance) to the first filter.

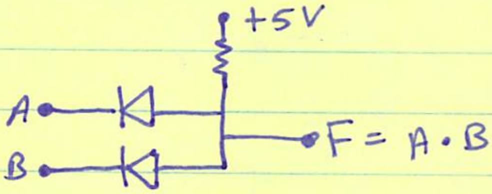
(11) (a) OR



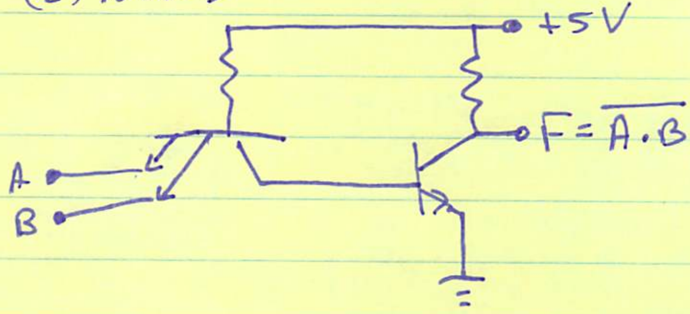
(b) NOR



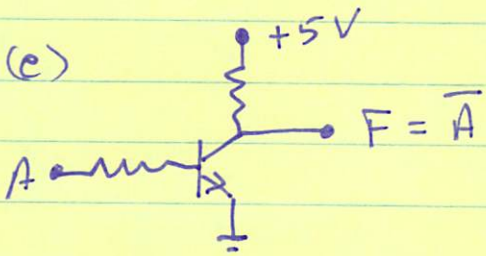
(c) AND



(d) NAND

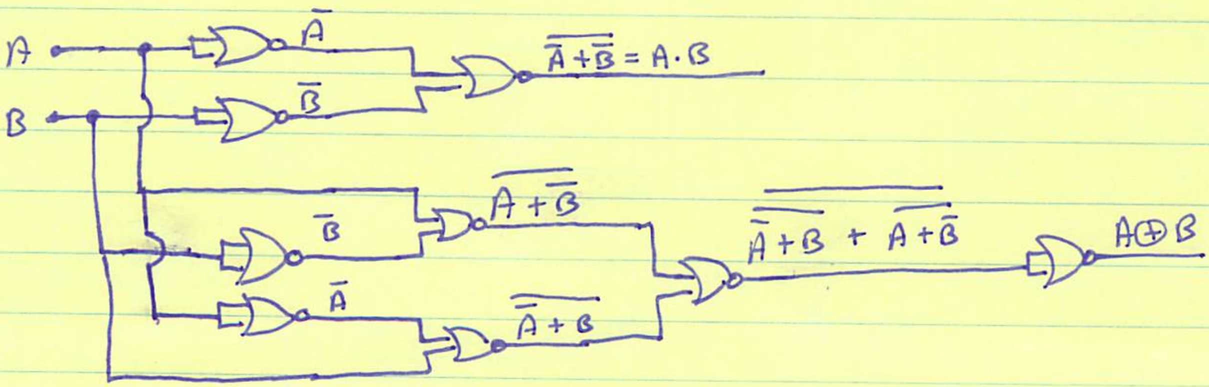


(e)



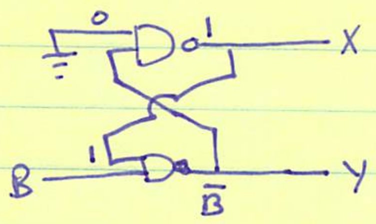
(12) $A \oplus B = A \cdot \overline{B} + \overline{A} \cdot B$ $\qquad$ $A \cdot B = \overline{\overline{A} + \overline{B}}$

$$= \overline{\overline{\overline{\overline{A} + B} + \overline{A + \overline{B}}}}$$



This is the most direct sol'n, but simpler circuit results after more algebra.

(13) (a)

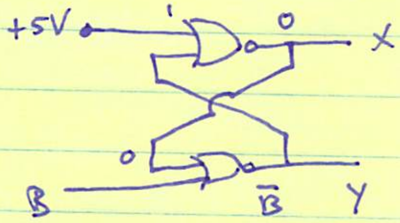


$y = \overline{B}$

ground $\Rightarrow$ $X = 1$ always.

$\Rightarrow$ $Y = \overline{B \cdot 1} = \overline{B}$

(b)



+5V $\Rightarrow$ $X = 0$ always

$\Rightarrow$ $Y = \overline{B + 0} = \overline{B}$