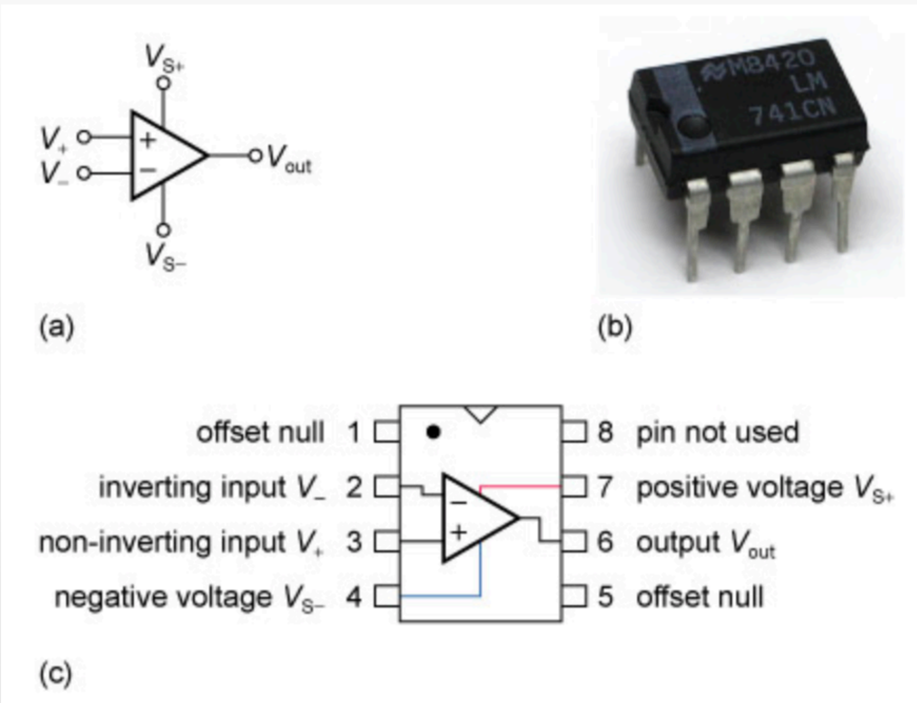
Chapter 5

Operational Amplifiers

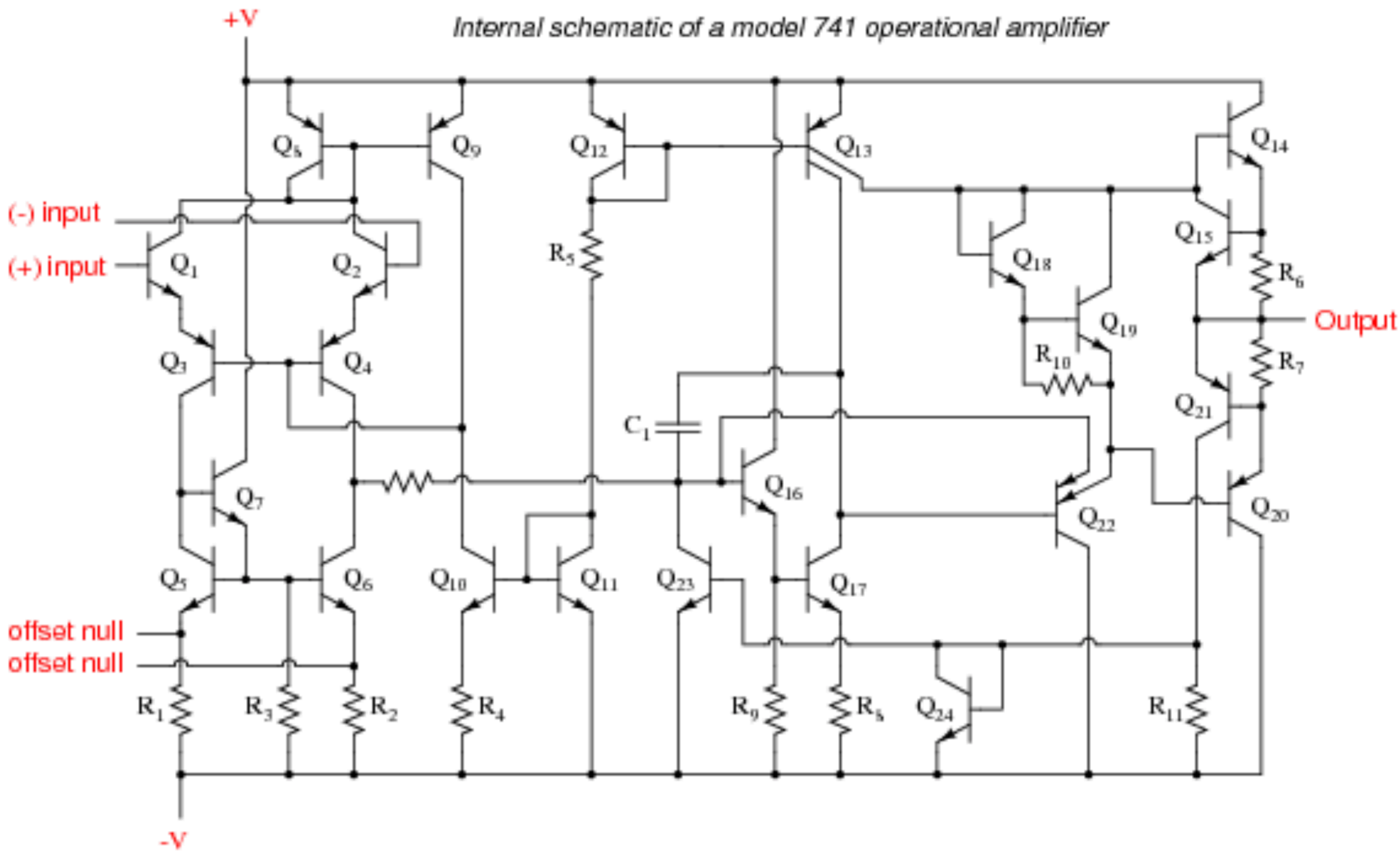
An **operational amplifier** (often **op-amp** or **opamp**) is a DC-coupled high-gain electronic voltage amplifier with a differential input and, usually, a single-ended output



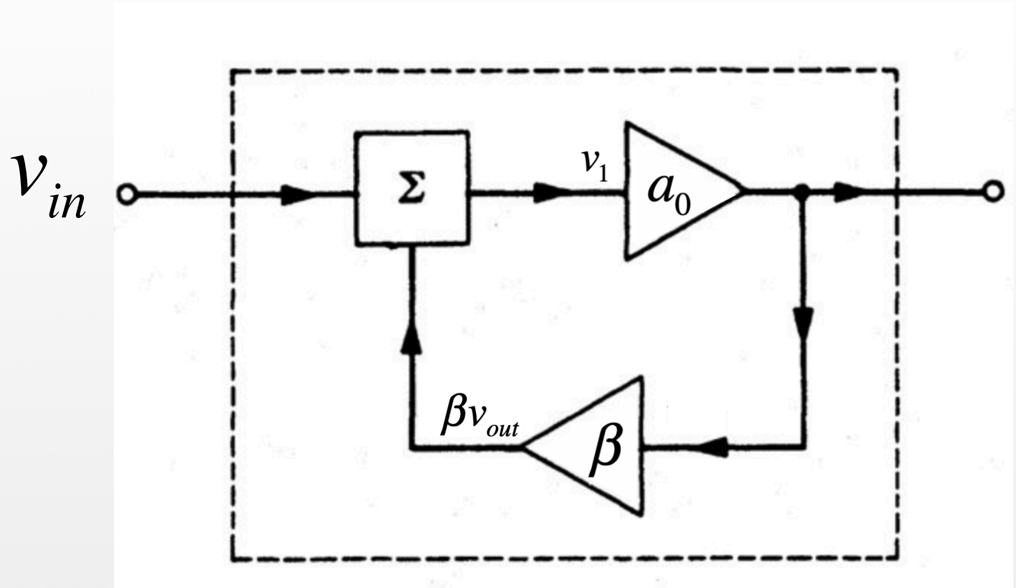
The 741 Op Amp was first introduced in 1968 and quickly became popular due to its ease of use.

- 1MHz Bandwidth
- 0.5V/us Slew Rate
- 1mV Input Offset Voltage
- 200V/mV Gain
- 90dB CMRR
- 15V Supply voltage
- Large Input Voltage Range
- No Latch-up
- High Gain
- Short-circuit Protection
- No Frequency Compensation Required.

The internal schematic diagram for a model 741 op-amp is shown in Figure below.



1) Negative voltage feedback



$$v_{out} = a_0 v_1$$

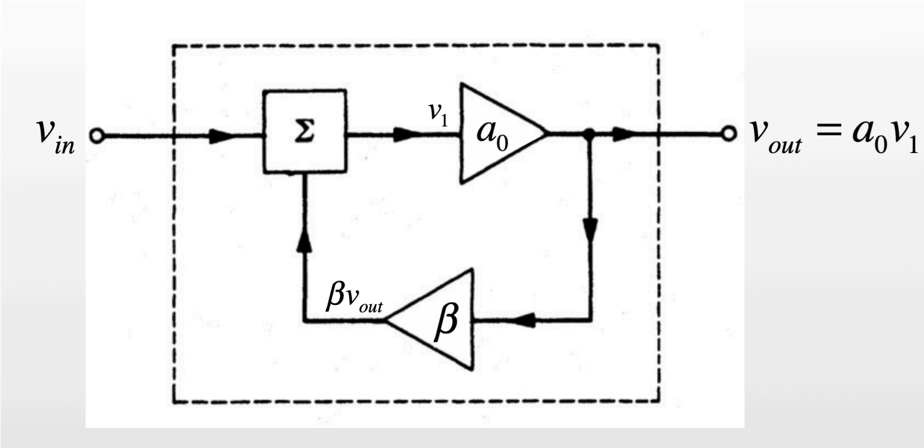
$$|a_o| \gg 1$$

$$|\beta| < 1 \quad \text{(attenuation)}$$

$$a_o\beta < 0 \quad \text{negative feedback}$$

(a) Closed loop gain

$$\left.\begin{aligned} v_{out} &= a_0 v_1 \\ v_1 &= v_{in} + \beta v_{out} \end{aligned}\right\} \quad v_{out} = a_o\left(v_{in} + \beta v_{out}\right) \xrightarrow[\div v_{in}]{} \frac{v_{out}}{v_{in}} = a_o\left(1 + \beta \frac{v_{out}}{v_{in}}\right)$$



$$\frac{v_{out}}{v_{in}} = a_o\left(1+\beta\frac{v_{out}}{v_{in}}\right)$$

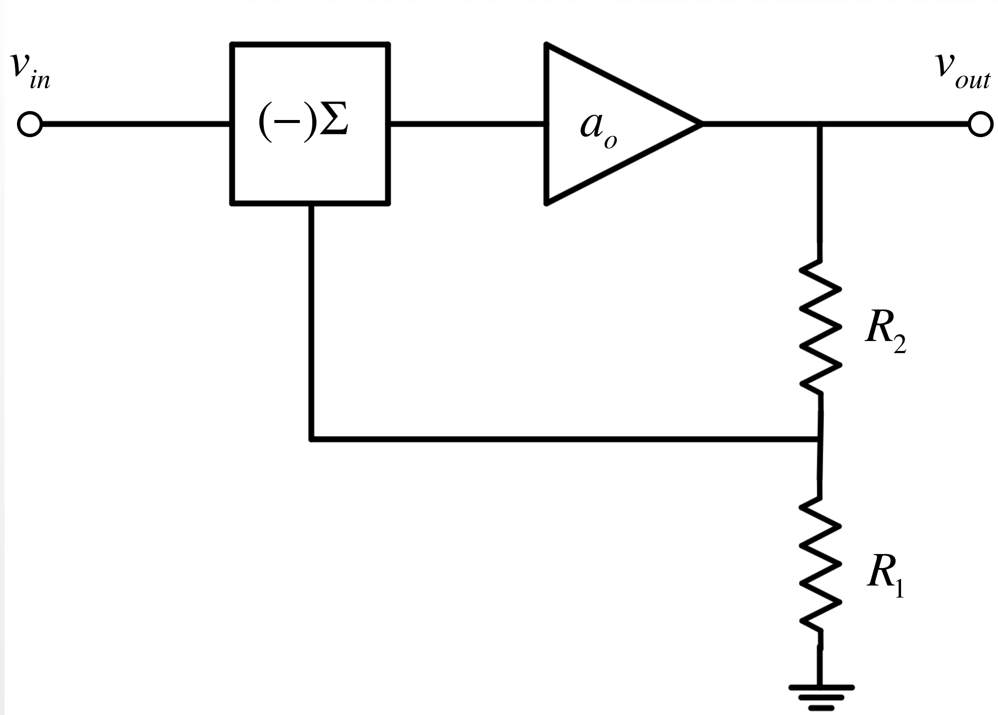
but the *closed loop gain* (or gain of the system) is $a' = \frac{v_{out}}{v_{in}}$ so $a' = a_o + \beta a_o a'$

or

$$a' = \frac{a_o}{1-\beta a_o}$$

- If $\beta a_o \rightarrow 1$ system unstable (oscillates)
- If $\beta a_o < 0$, $|a'| < |a_o|$
- If $|\beta a_o| \gg 1$, $a' \cong \frac{-1}{\beta}$ indep't of a_o

Example



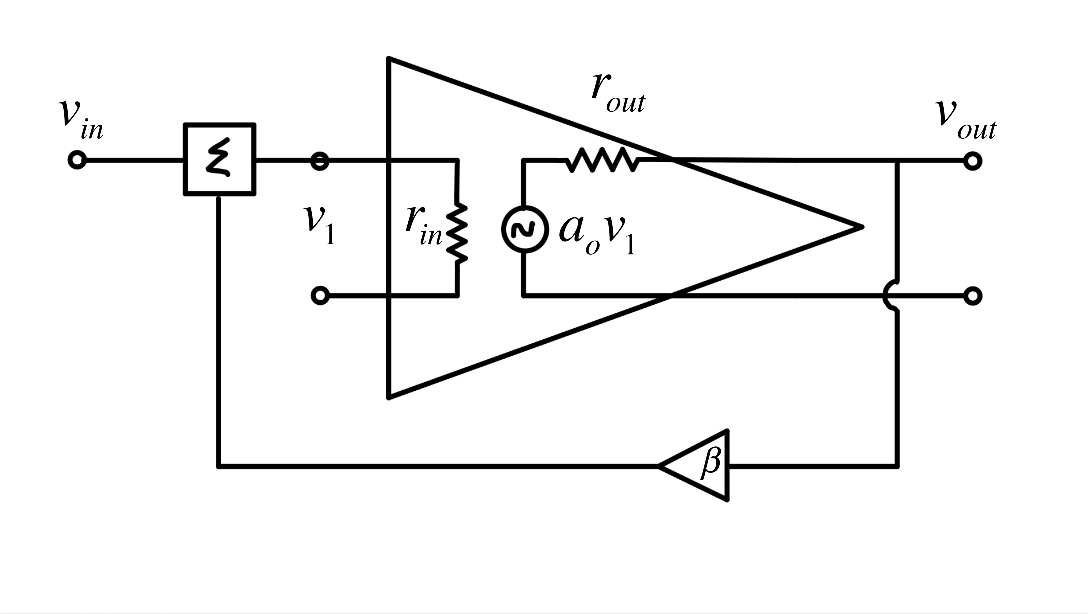
$$\beta = \frac{-R_1}{R_1 + R_2} \qquad a' = \frac{R_1 + R_2}{R_1}$$

Consider $\beta = -1\%$ $a' \cong \frac{-1}{\beta} = 100$

a_o	$a' = \frac{a_o}{(1-\beta a_o)}$
5000	98.3
10,000	99.0
20,000	99.6
10^5	99.9

10% fluctuation in a_o results in < one part in 10^4 change in a'

b) Input impedance $r_{in}{}' = \dfrac{v_{in}}{i_{in}}$



$$v_1 = v_{in} + \beta v_{out}$$

but $v_1 = i_{in} r_{in}$

and $v_{out} = \dfrac{a_o}{1 - a_o \beta} v_{in}$

so $v_{in} = i_{in} r_{in} - \frac{\beta a_o}{1-\beta a_o} v_{in}$

Dividing by i_{in} gives $r_{in}{}' = r_{in} - \frac{\beta a_o}{1-\beta a_o} r_{in}{}'$

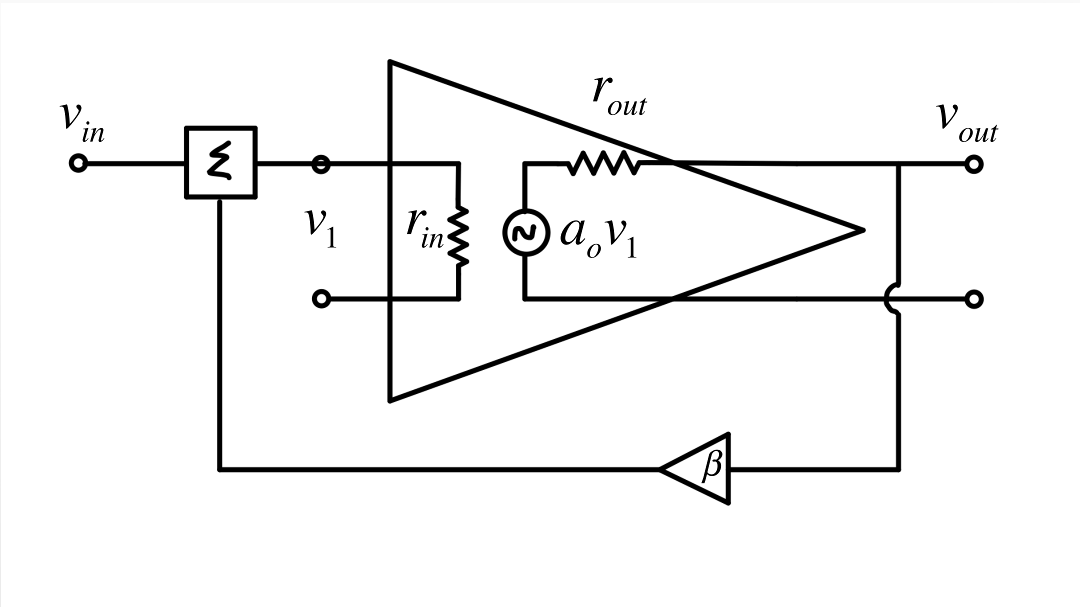
and solving for $r_{in}{}'$

$$r_{in}{}' = (1 - \beta a_o) r_{in}$$

Input impedance is increased by $|\beta a_o|$

c) Output impedance

$$r_{out}' = \frac{v_{out}(open)}{i_{out}(short)}$$



$$v_{out}(open) = \tfrac{a_o}{1-\beta a_o} v_i$$

$$i_{out}(short) = \frac{a_o v_1(short)}{r_{out}}$$

but when the output is zero,

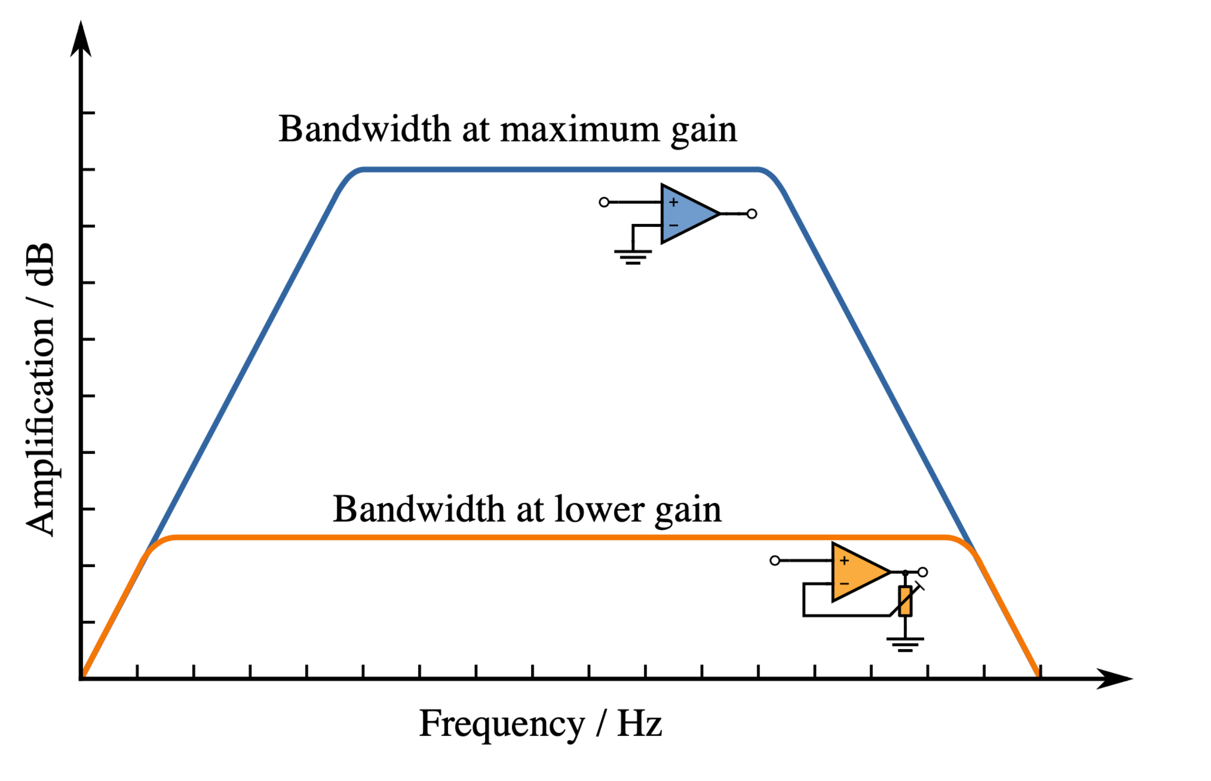
$$v_1 = v_{in} + \beta(0) = v_{in}$$

so,

$$r_{out}' = \frac{r_{out}}{1-\beta a_o}$$

Output impedance reduced by $|\beta a_o|$

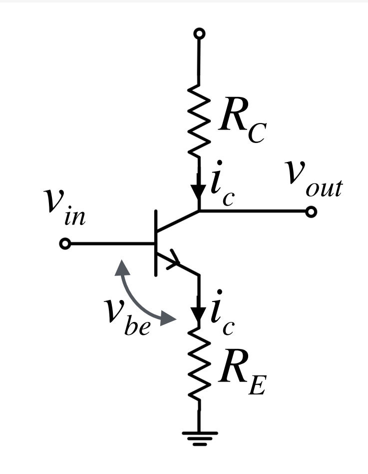
d) Bandwidth



Negative feedback increases bandwidth

d) Examples of feedback

CE amplifier:



Recall, $v_{out} = -i_c R_C$ $\quad i_c = \beta_t i_b$ $\quad v_{be} = i_b r_{be}$

so the gain of the transistor (for signal across *be*) is

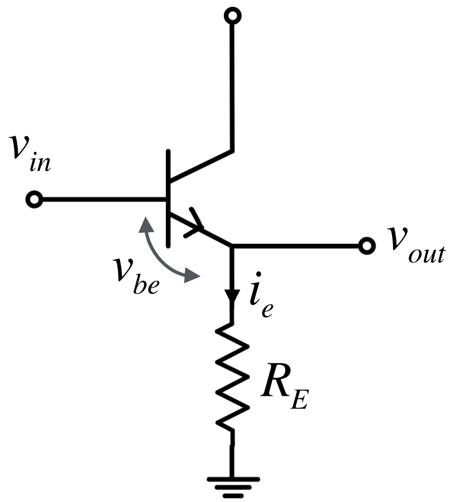
$$a_o = \frac{v_{out}}{v_{be}} = -\frac{\beta_t R_c}{r_{be}} \qquad |a_o| \gg 1$$

but $\quad v_{be} = v_{in} - i_c R_E \; = v_{in} + \left(\frac{R_E}{R_C}\right) v_{out}$

We had $\quad v_1 = v_{in} + \beta v_{out}$ $\qquad$ Here $\quad \beta = \frac{R_E}{R_C}$ $\quad$ is positive, but a_o is negative, so feedback is negative

so $\quad a' = \frac{-1}{\beta} = \frac{-R_C}{R_E}$ $\quad$ as obtained from direct analysis of the equivalent circuit

Emitter follower:



Here, $v_{out} = i_e R_E$ $\quad i_e = \beta_t i_b$ $\quad v_{be} = i_b r_{be}$

so the gain of the transistor (for signal across *be*) is

$$a_o = \frac{v_{out}}{v_{be}} = \frac{\beta_t R_E}{r_{be}} \gg 1$$

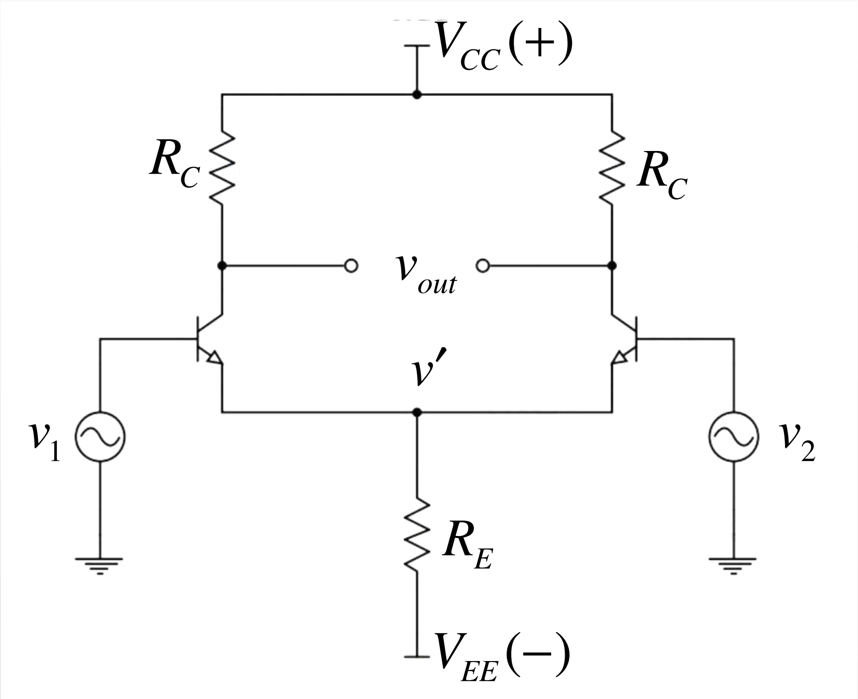
but $v_{be} = v_{in} - i_e R_E \quad = v_{in} - v_{out}$

We had $v_1 = v_{in} + \beta v_{out}$ Here $\beta = -1$ is negative and a_o is positive, so feedback is negative

$$a' = \frac{-1}{\beta} = 1$$

as obtained from direct analysis of the equivalent circuit

2) Difference amplifier



$$v_{out1} = a(v_1 - v') \qquad v_{out2} = a(v_2 - v')$$

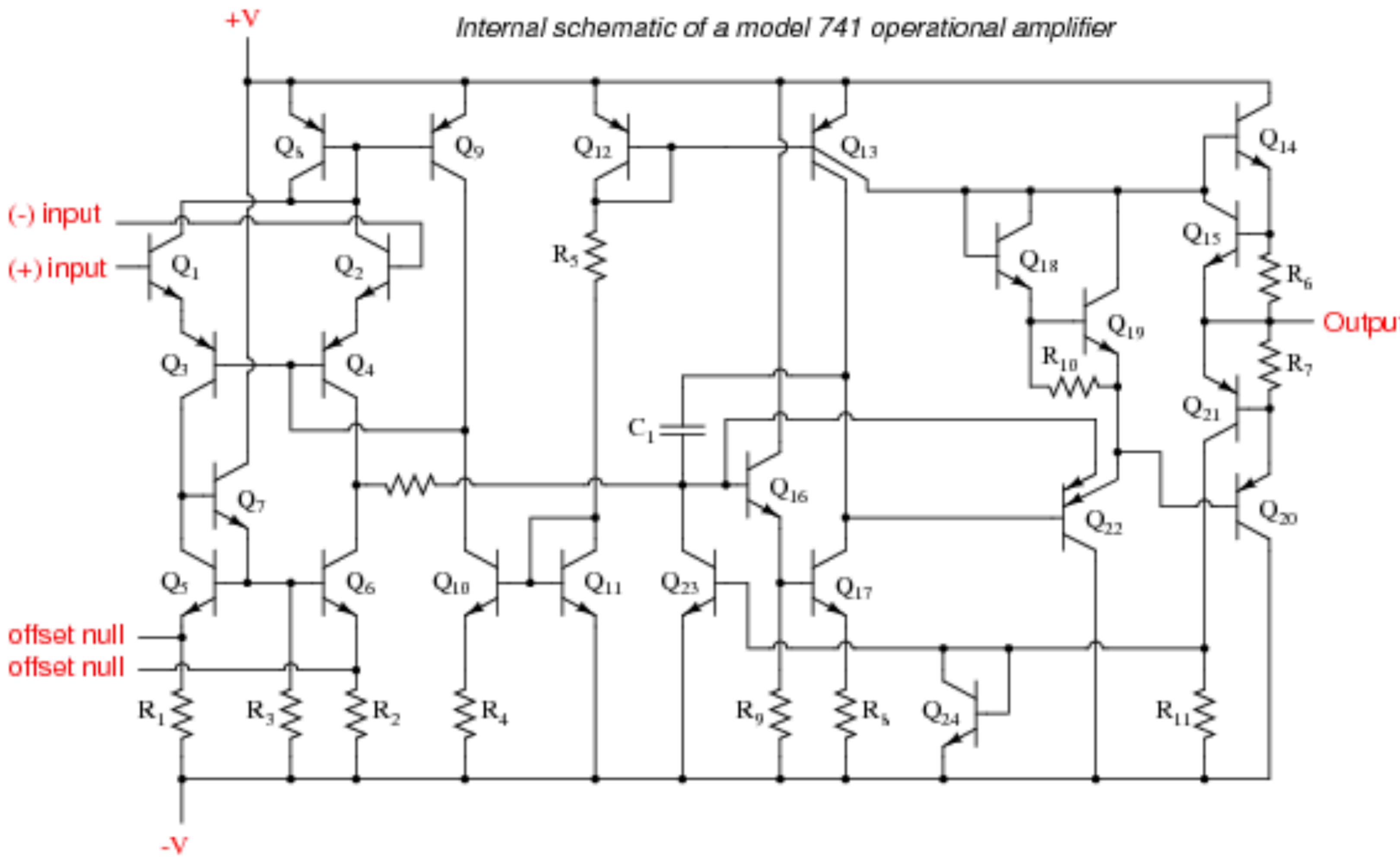
a represents the transistor gain of *be* signal

$$v_{out} = v_{out2} - v_{out1} = a(v_2 - v' - v_1 + v')$$

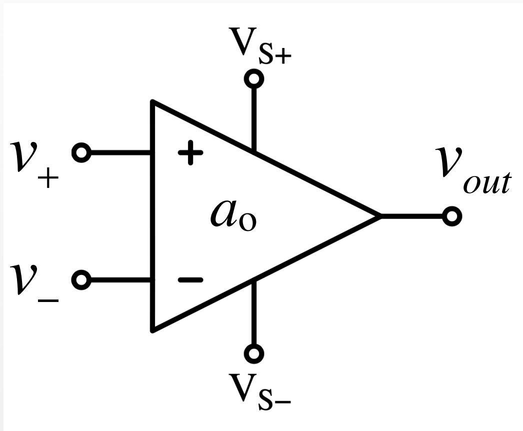
$$v_{out} = a(v_2 - v_1)$$

- identical transistors
- inputs at dc ground; no coupling capacitors
- difference amplified —> common signal rejected
- R_E does not reduce gain

The internal schematic diagram for a model 741 op-amp is shown in Figure below.



3) Ideal operational amplifier

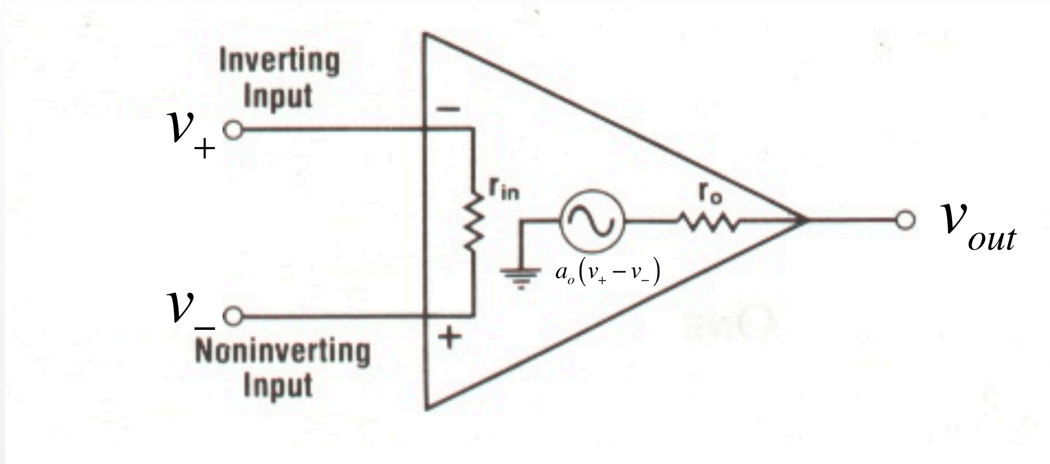


$V_{s+/-}$ omitted in most circuit diagrams

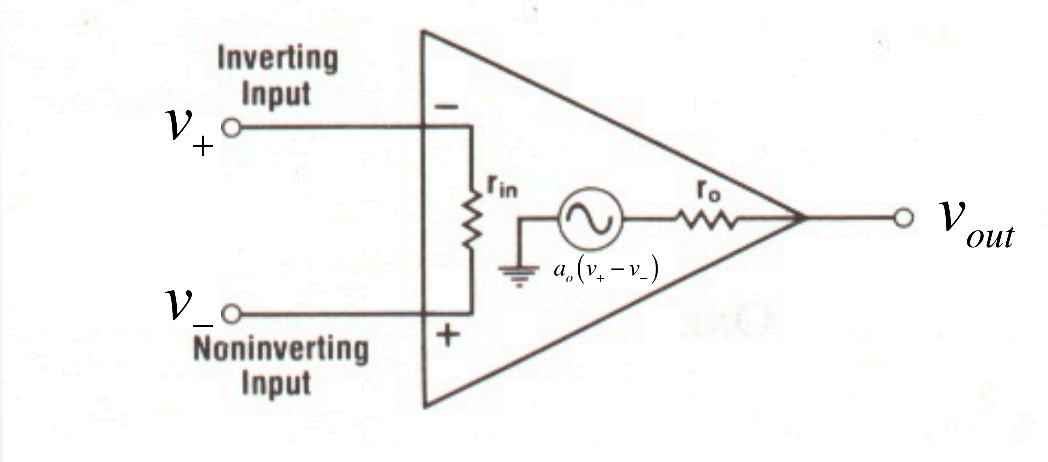
a_o = differential (open loop) gain
v_{out} in phase with v_+ (non-inverting input)
v_{out} out of phase with v_- (inverting input)

$$v_{out} = a_o\left(v_+ - v_-\right)$$

Equivalent circuit



	Ideal	Typical
a_o	∞	10^5 - 10^9
a_{CM}	0	< 1
CMRR	∞	10^5-10^{12}
r_{in}	∞	MΩ - > GΩ (FET)
r_{out}	0	100 -1000 Ω



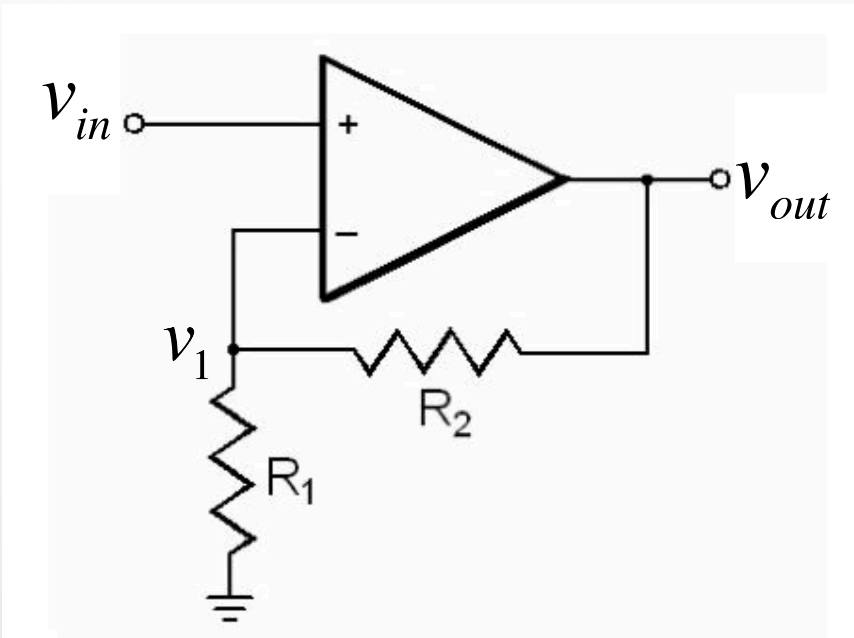
Rules (approximations) for analyzing op amp circuits

1. Current into either input is zero
2. Differential voltage is zero

$$v_+ \cong v_-$$

4) Non-inverting amplifier

a) Voltage gain



Rule 1 $\rightarrow v_1 = \dfrac{v_{out} R_1}{R_1 + R_2}$

(current equal in both resistors)

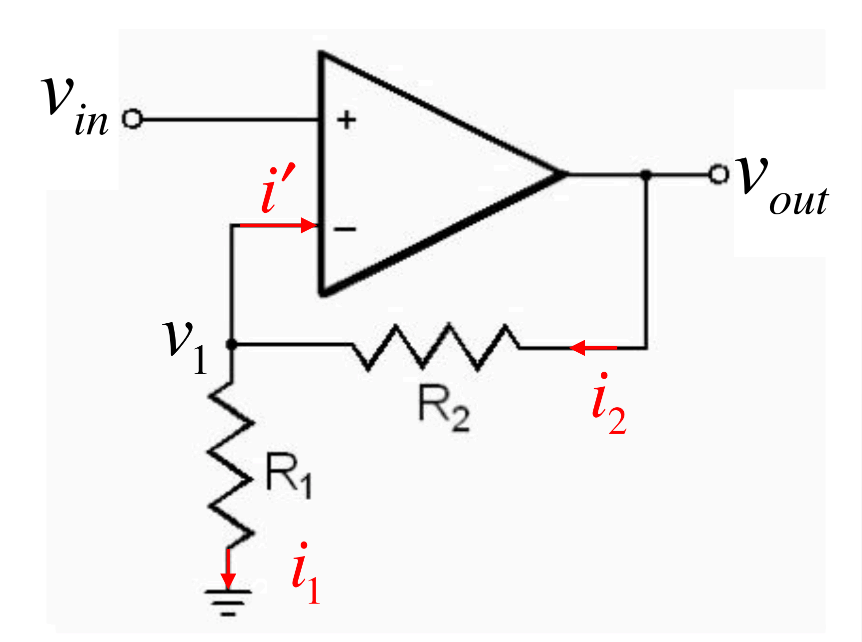
Rule 2 $\rightarrow v_1 = v_{in}$

so $v_{in} = \dfrac{v_{out} R_1}{R_1 + R_2}$

$$a = \frac{v_{out}}{v_{in}} = \frac{R_1 + R_2}{R_1}$$

$$\left(= \frac{-1}{\beta} \right)$$

A less approximate analysis:



Still assume $i_2 \gg i'$, so $i_1 = i_2$

so $\dfrac{v_{out} - v_1}{R_2} = \dfrac{v_1}{R_1}$

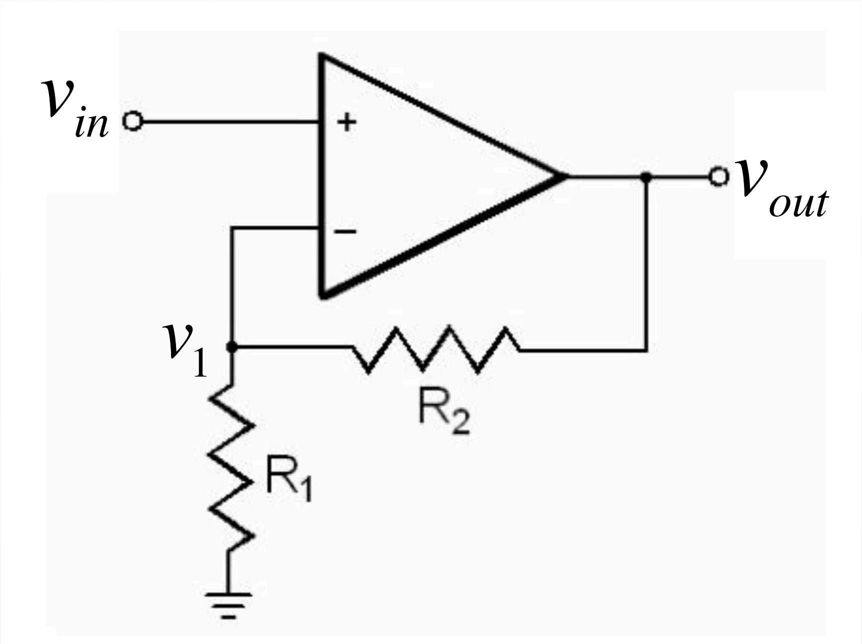
but $v_{out} = a_o(v_{in} - v_1) \quad \rightarrow -v_1 = \dfrac{v_{out}}{a_o} - v_{in}$

Substitute and divide through by v_{in} , to give: $\dfrac{a + \frac{a}{a_o} - 1}{R_2} = \dfrac{-\frac{a}{a_o} + 1}{R_1}$

Solve for $$a = \frac{v_{out}}{v_{in}} = \frac{a_o\left(R_1 + R_2\right)}{R_1\left(a_o + 1\right) + R_2}$$

For $a_o \gg 1, \quad a = \dfrac{R_1 + R_2}{R_1}$

b) Input Impedance



Recall, for negative feedback,

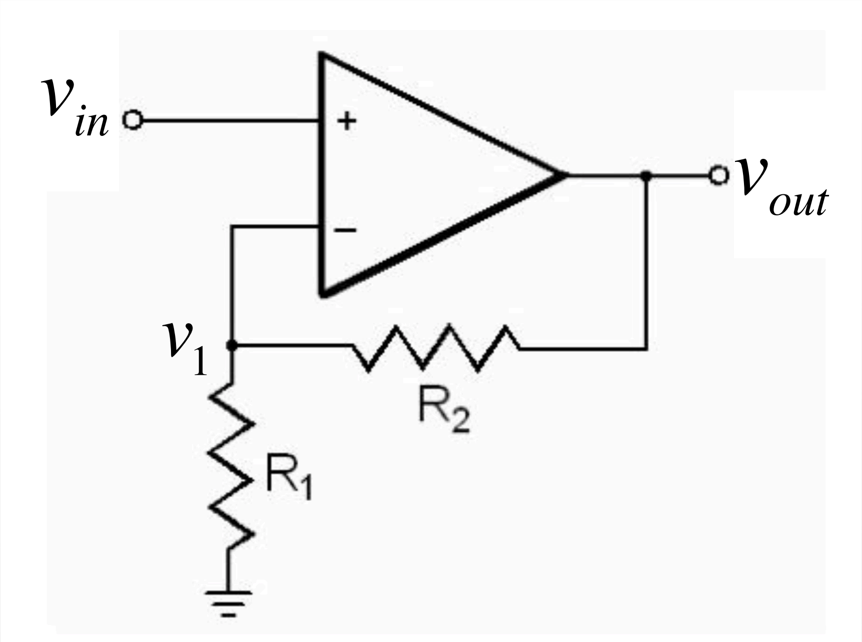
$$r_{in}' = (1 - \beta a_o) r_{in}$$

$$r_{in}' \cong -\beta a_o r_{in}$$

- effectively infinite

$$\beta = -\frac{R_1}{R_1 + R_2}$$

c) Output Impedance



Recall, for negative feedback,

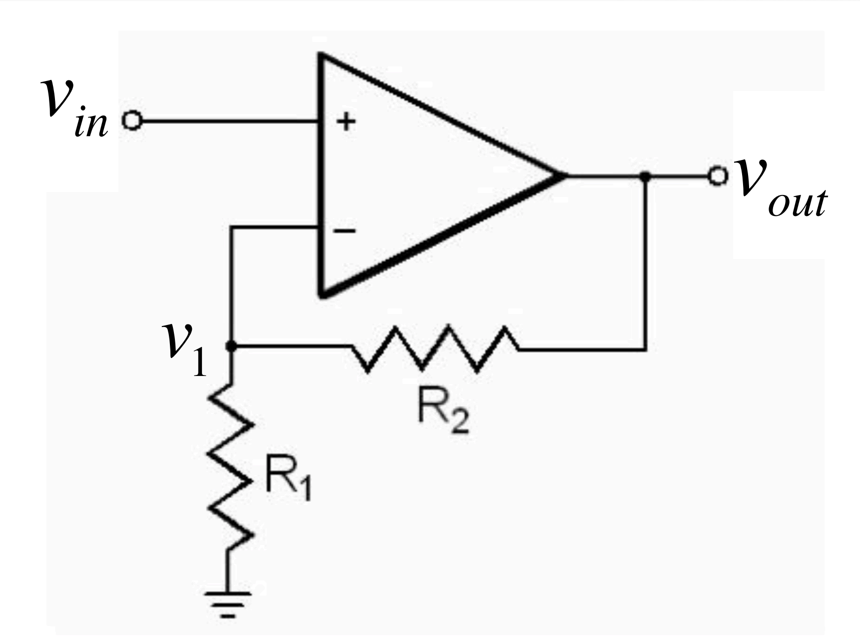
$$r_{out}' = \frac{r_{out}}{1-\beta a_o}$$

$$r_{out}' \cong -\frac{r_{out}}{\beta a_o}$$

~ 1 or a few Ω

$$\beta = -\frac{R_1}{R_1 + R_2}$$

d) Summary non-inverting amplifier



$$a = \frac{v_{out}}{v_{in}} = \frac{R_1 + R_2}{R_1}$$

typically 1 to 100

stable

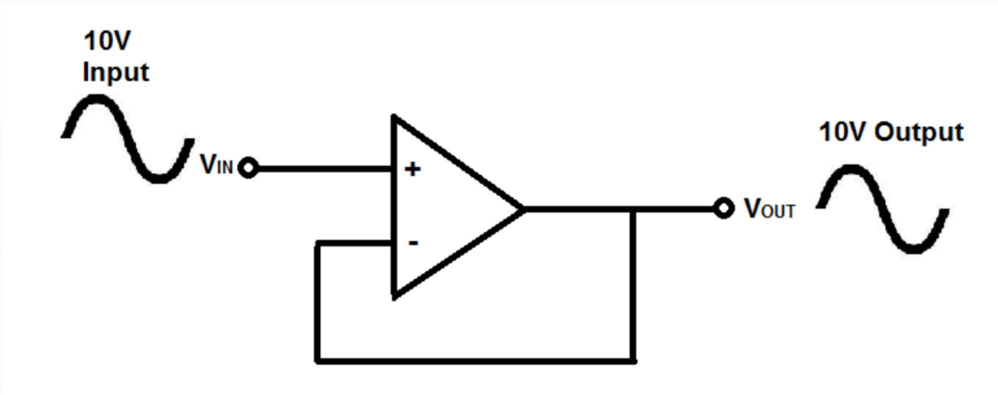
$$r_{in}' \cong -\beta a_o r_{in}$$

v. high

$$r_{out}' \cong -\frac{r_{out}}{\beta a_o}$$

v. low

d) Voltage follower



$$v_{out} = v_{in} \rightarrow a = 1$$

$$R_1 = \infty, \quad R_2 = 0 \quad \rightarrow a = \frac{v_{out}}{v_{in}} = \frac{R_1 + R_2}{R_1} = 1$$

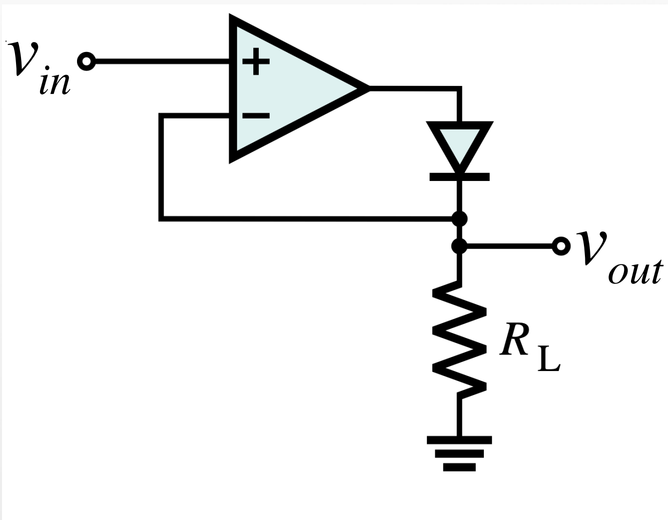
$$\beta = -1 \quad \rightarrow r_{in}' \cong -\beta a_o r_{in} = a_o r_{in}$$

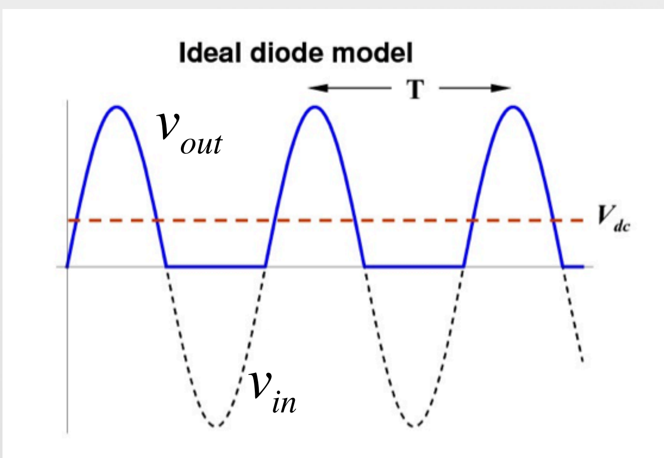
$$\rightarrow r_{out}' \cong -\frac{r_{out}}{\beta a_o} = \frac{r_{out}}{a_o}$$

Buffer:
- unity gain
- high input impedance; does not load earlier circuit
- low output impedance;
  - later circuit does not affect output

7) Ideal Rectifier

(a) Ideal diode (half-wave rectifier)

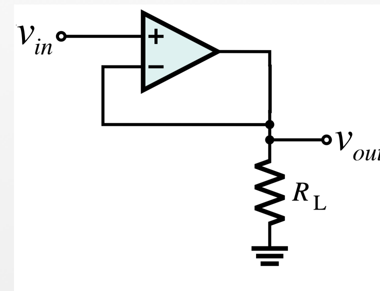




- $v_{in} > 0 \quad \rightarrow \quad v_{out} > V_t$

  $\rightarrow$ neg. feedback

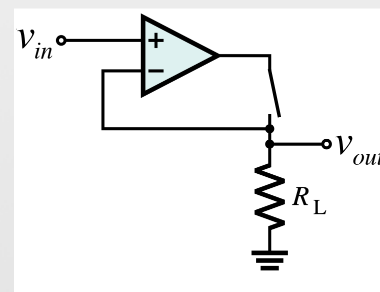
  $\rightarrow \quad v_{out} = v_{in}$



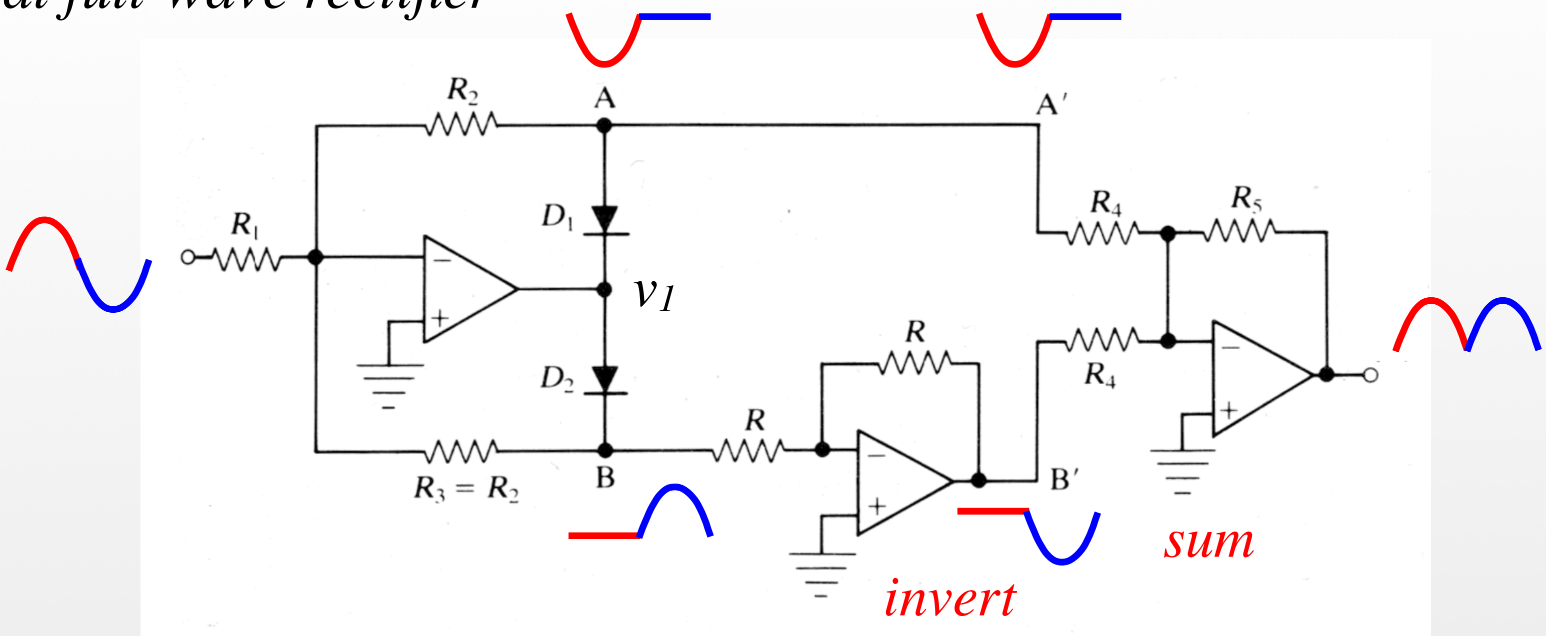
- $v_{in} < 0 \quad \rightarrow \quad v_{out} < V_t$

  $\rightarrow$ no feedback

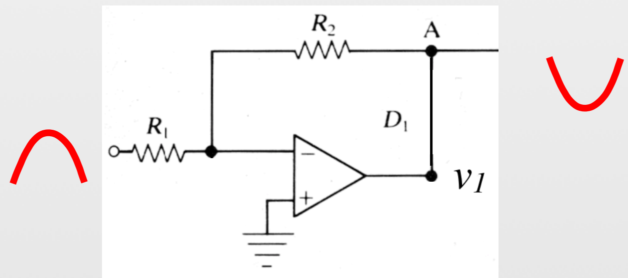
  $\rightarrow \quad v_{out} = 0$



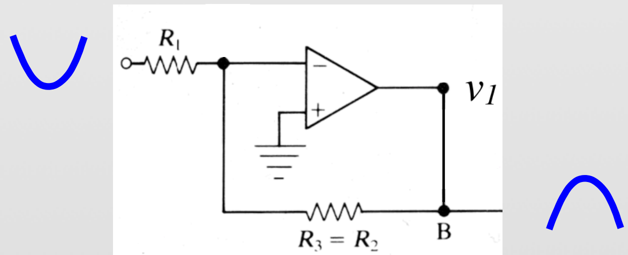
(b) Ideal full-wave rectifier



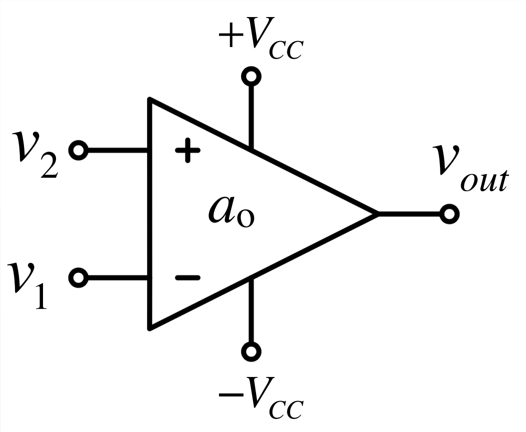
• positive input $\rightarrow v_1 < 0$ $\rightarrow$ D_1 on, D_2 off

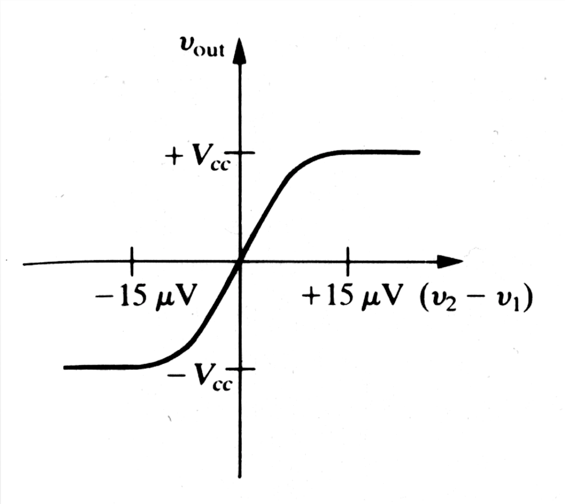


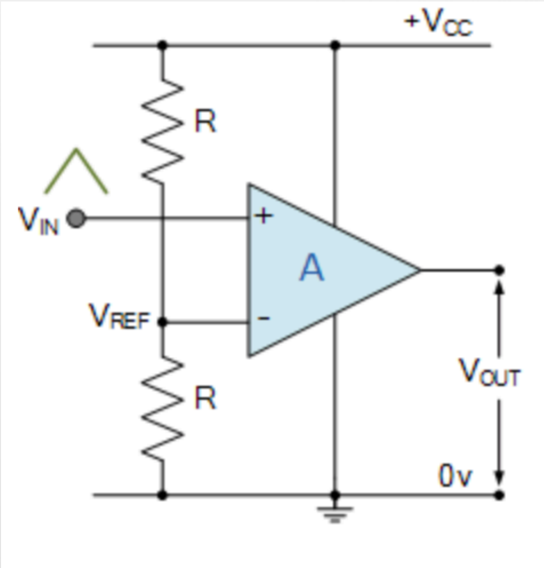
• negative input $\rightarrow v_1 > 0$ $\rightarrow$ D_1 off D_2 on

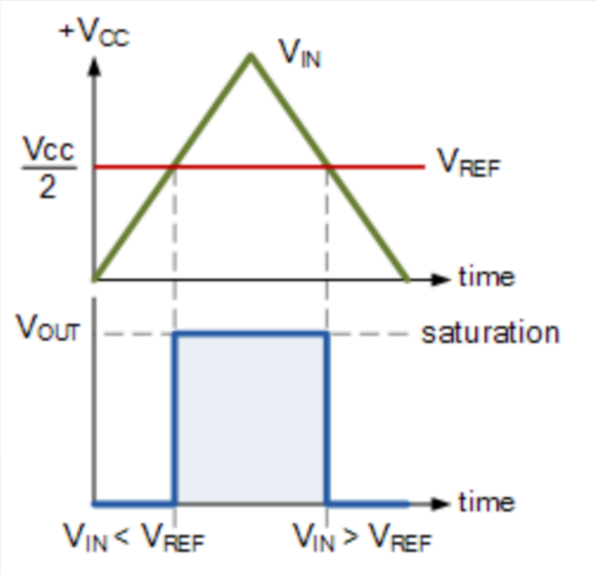


8) Comparator (discriminator)

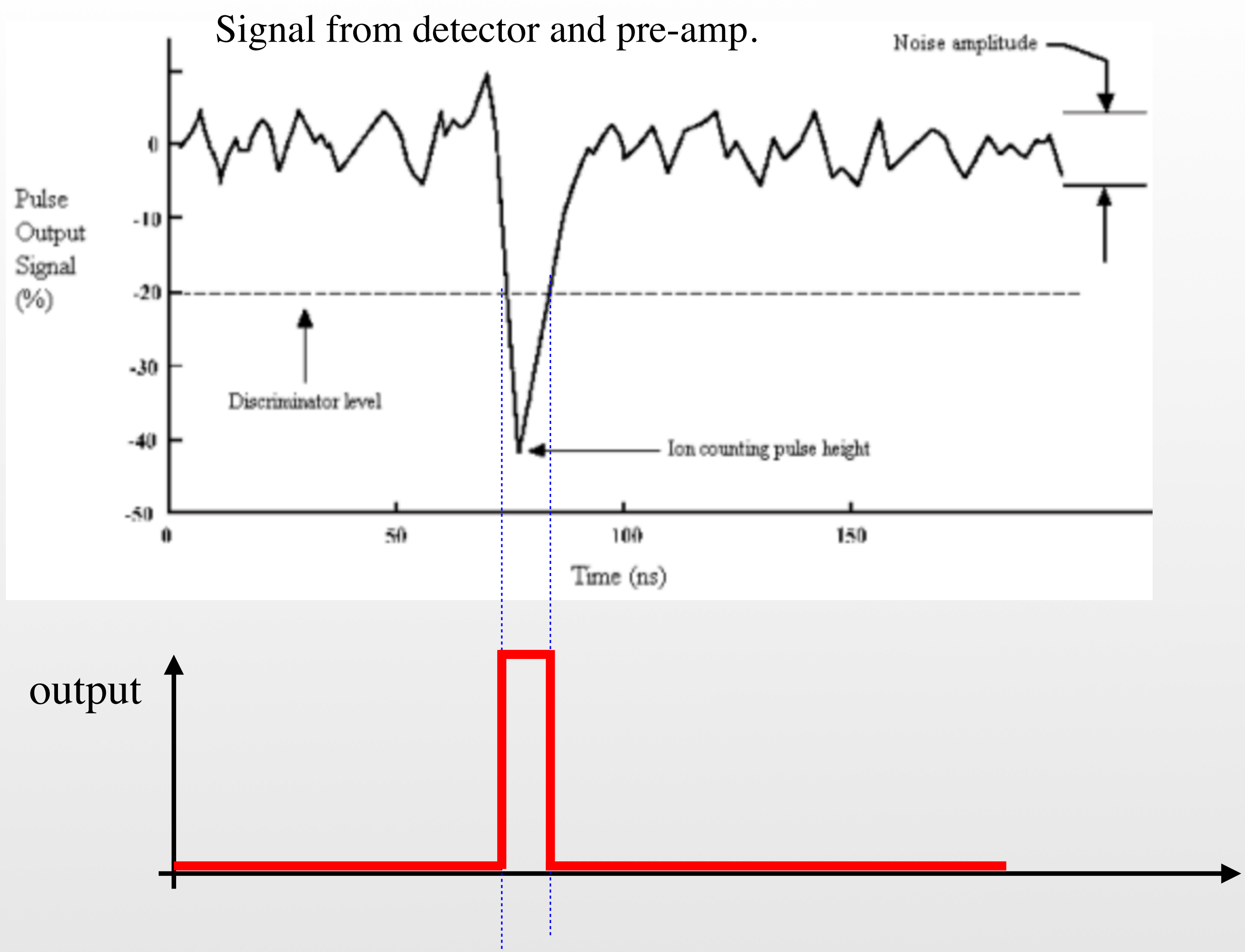








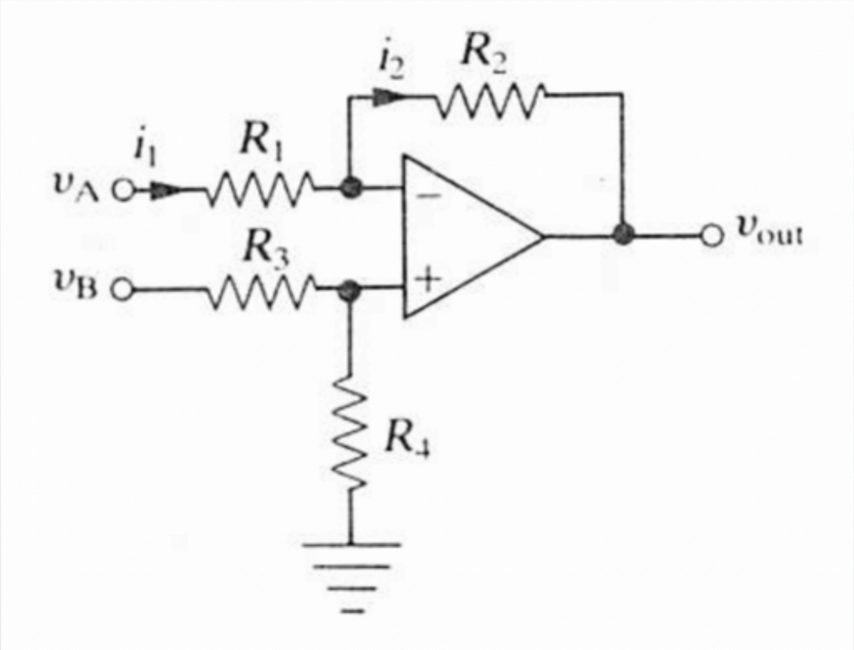
When input exceeds a reference (or threshold), output toggles to saturation



A discriminator threshold is set above noise pulses, but below signal pulses. (Output pulse width is fixed by additional circuitry.)

9) Difference Amplifier

(a) Simple difference amplifier (finite gain using feedback)

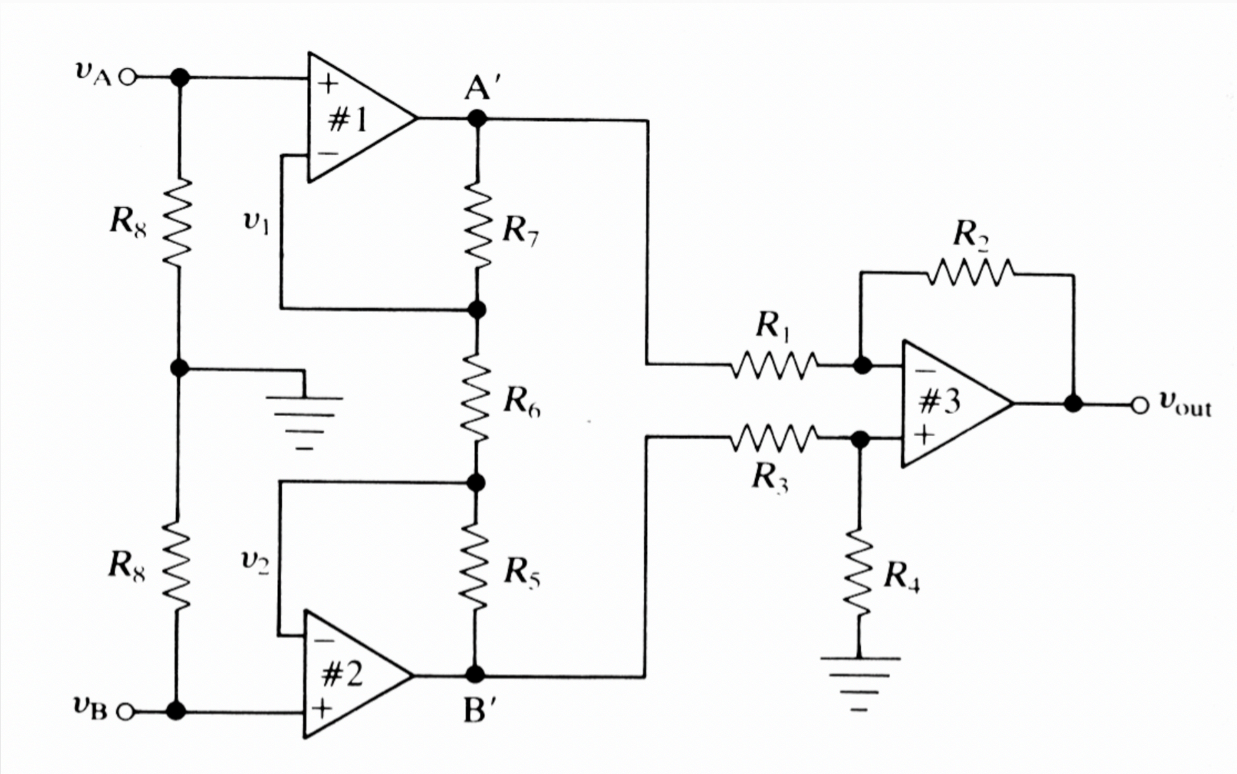


$$v_{out} = \frac{R_2}{R_1}(v_B - v_A)$$

if $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Provides noise rejection (common mode) for weak signals transmitted over long cables. (e.g.)

(b) Instrumentation amplifier



$$a = \frac{v_{out}}{v_A - v_B} = -\frac{R_2}{R_1}\left(1 + \frac{2R}{R_6}\right)$$

if $R_5 = R_7 = R$

and $\frac{\boldsymbol{R_1}}{\boldsymbol{R_2}} = \frac{\boldsymbol{R_3}}{\boldsymbol{R_4}}$

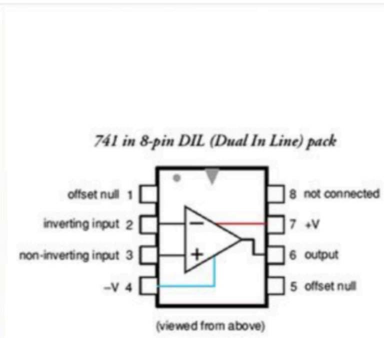
Difference amplifier with high input impedance

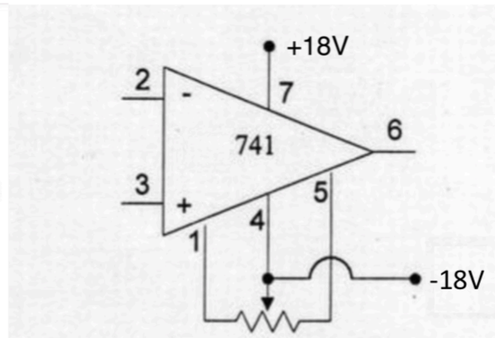
- both inputs are essentially buffered by voltage followers
- R_8 can be arbitrarily high (even infinite)

11) Practical op amp considerations *(details in the text)*

(a) Offset Null

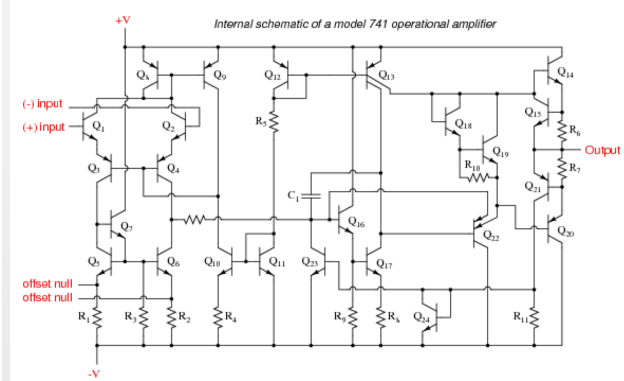
Asymmetries between the internal circuits ==> output saturates for both inputs grounded.
- circuit provides for null adjustment on the pinouts





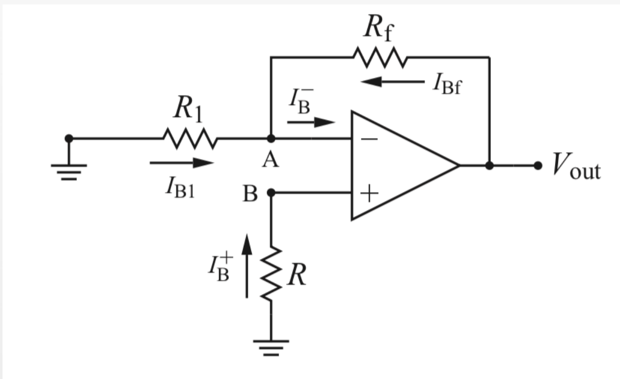
10kΩ potentiometer
(Voltage offset null adjust)

The internal schematic diagram for a model 741 op-amp is shown in Figure below.



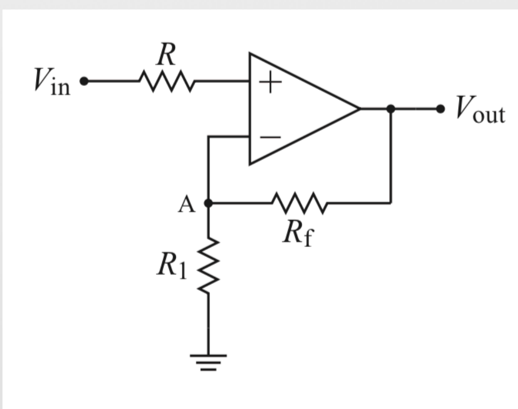
(b) Bias currents

Small bias currents (< 500 nA) must flow into the op amp inputs, so the positive input cannot be grounded in the inverting amplifier. A compensating resistor approximately equal to the parallel combination of the input and feedback resistors should be used. Usually, this is very close to the input resistance.



$$R = R_1 // R_f \cong R_1$$

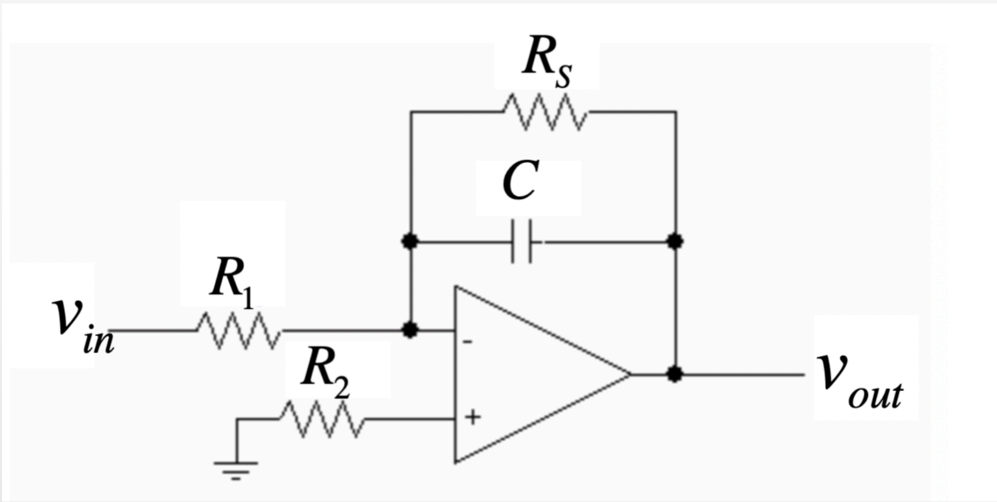
Similar considerations for the non-inverting amplifier suggest a compensating resistor at the non-inverting input:



$$R = R_1 // R_f \cong R_1$$

(c) Practical integrator

Because of drift or assymetry in the op amps, the capacitor in an integrator gradually acquires a dc charge, eventually saturating when the voltage reaches V_{cc}. This can be prevented by connecting a resistor across the capacitor which is large enough so ac operation is not appreciably affected, but small enough to prevent dc charging.



(d) Frequency response

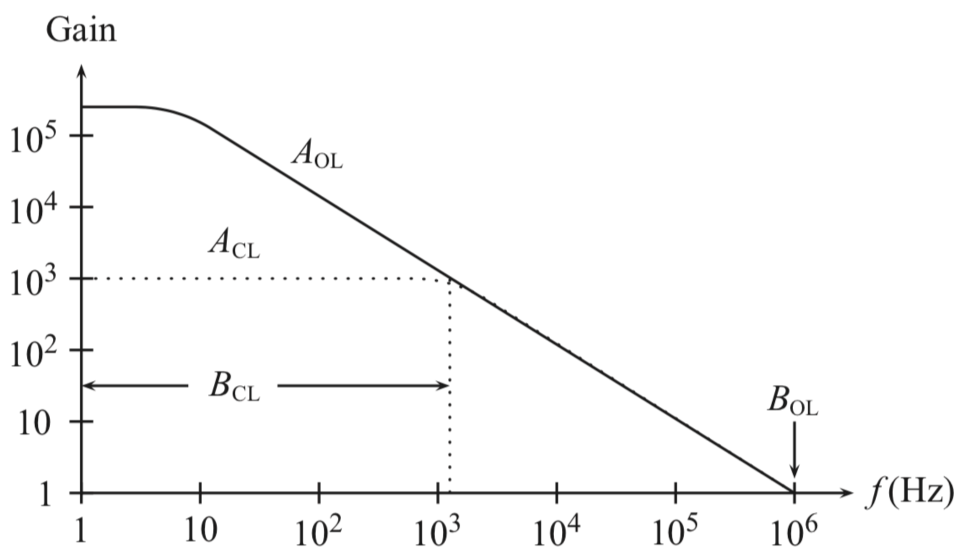


Figure 6.17 Frequency response of the 741 op-amp.

Open loop gain drops from about 6 Hz.

When the infinite gain approximation loses validity, the closed loop gain will also drop according to the more accurate gain equation:

$$a = \frac{-a_o R_2}{R_1 + R_2 + a_o R_1}$$