Instructions: Answer all questions. Be as concise as possible with descriptive answers. Define variables clearly with reference to a labeled diagram. Ask for assistance if you require a formula that is not explicitly provided on the sheet.

## 1. Potato Circuits

a) An electrolytic cell battery can be constructed out of a potato and two dissimilar metal electrodes - copper and zinc - as shown in the diagram. A DMM is used to measure the potential difference across the terminals of the potato battery, with the following results:

| Test condition: | DMM reading: |
| :--- | :--- |
| Open circuit | 0.83 V |
| $1 \mathrm{k} \Omega$ resistor connected across terminals | 0.13 V |



Draw the potato battery test circuit, and analyze the above data to determine a value of the potato cell's output resistance. Why is the potato battery not a very practical source of EMF?
b) To diagnose the potato as a circuit element ( P ), the zinc nail is replaced with a second copper wire, and the potato is connected in series with a $10 \mathrm{k} \Omega$ resistor.

An ideal ac function generator applies a sinusoidal potential difference $\varepsilon(t)$ across the "P-R" series circuit, and the signals are studied with a digital oscilloscope.

Draw a diagram of the $\mathrm{P}-\mathrm{R}$ test circuit, specifying the 'scope probe connections to display $\varepsilon(\mathrm{t})$ on channel 1 and and the potato waveform $\mathrm{V}_{\mathrm{p}}(\mathrm{t})$ on channel 2.

c) A screen capture of the potato circuit test signals at an applied frequency of 10 kHz is shown in the figure. The scope is triggered on channel $1=\varepsilon(\mathrm{t})$, which crosses zero in the center of the screen before channel $2, V_{p}(t)$ does.

What is the RMS value of $V_{p}(t)$ ?
From the data, evaluate the phase of $V_{p}$ relative to $\varepsilon$ at this frequency.
d) Assume that the potato can be modeled as a resistor $R_{p}$ and capacitor $C_{p}$ in parallel with each other.


If $\varepsilon(t)=\varepsilon_{o} \cos (\omega t)$ and $\mathrm{V}_{\mathrm{p}}(\mathrm{t})=v_{o} \cos (\omega t+\phi)$, use the complex impedance method to predict the magnitude and sign of the phase shift $\phi$ as a function of $\omega$. What do you expect for $\phi$ at very low and very high frequencies?
e) Use your prediction of part d) to estimate the capacitance, and hence the characteristic RC response time of the potato.

## 2. Grab bag - short problems

## a) RLC circuit:

i) Consider the circuit shown in the diagram; it has been in this configuration for a long time.


What are the current $i_{1}$ and the voltage across the capacitor?
ii) Next, suppose the battery is replaced by an ac function generator applying a potential difference $\varepsilon(t)=\varepsilon_{o} \cos \omega t$. What is the circuit impedance in the limit $\omega \rightarrow \infty$ ? What are the amplitudes of the current $i_{1}$ and capacitor voltage waveforms in this case?

## b) Spray can current:

A spray can sits on a grounded metal countertop as shown. When the nozzle is depressed, a stream of tiny charged droplets is emitted uniformly from the nozzle, into a cone of $1 / 2$ angle $\alpha$. A distance D from the nozzle, there is a small target of area $A$, which intercepts some of the spray.

i) If the current emitted from the spray can has a constant value $i_{o}$, what is the current incident on the small target? (Ignore the effect of gravity on the droplets!)
ii) Next, suppose that the can emits a puff of spray as indicated in the figure. The capacitor is initially uncharged. Sketch the voltage across the capacitor as a function of time.


## c) Transistor:

A schematic of the npn bipolar junction transistor and its biasing circuit is shown in the figure. $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ are dc power supplies (not explicitly shown).
i) What, if anything, is the difference between the two n-type regions? Is this important for the operation of the device?
ii) Draw a sketch of the transistor, label the emitter, base and collector regions as E, B, C respectively, and indicate the polarities and approximate / reasonable values of $V_{1}$ and $V_{2}$ in order to operate the transistor in the 'linear' region, i.e. so that it could be used as a current amplifier.

Label all currents, showing their directions clearly, and state a relationship between them. What is meant by 'linear' in this context?
iii) Briefly state how the transistor conducts current, in view of the biasing arrangements for $V_{1}$ and $V_{2}$, pointing out only the most important features.

## 3. Two kinds of light:

a) Red diode

A measurement of the I-V curve for a red LED under forward bias is shown in the figure below:

i) Use these data to estimate the resistance of the diode when it is conducting current.
ii) The LED is connected in series with a $1 \mathrm{k} \Omega$ resistor, and a 5 V potential difference is connected across them both. Draw the circuit, and write down an equation for the load line.
iii) Draw a sketch of the I-V curve, and also draw on it the load line for these circuit parameters. Note the operating point clearly on your diagram. Evaluate the current in the LED at the operating point, and also the power dissipated in the LED.

## b) Light Switches and logic

In household wiring applications, it is convenient to be able to switch on and off a light fixture from two different locations. This can be accomplished with a pair of 3-way switches that are connected to each other, as illustrated in the diagram.

i) Denote the switches as S1 and S2, and assume they are set as indicated initially, with S1 off and S2 on, and the light is on.

Show how a person could turn off the light using S1 and then turn it on and off again using S2. Summarize the 4 possible states of S1, S2, light using a logic table with $0=$ off and $1=\mathrm{on}$.
ii) What logical function do these switches perform?
iii) Using only AND, OR and INVERT logic elements, design and draw the diagram for a logic circuit that would accept TTL signals from the two switches and output the resultant state of the light fixture.

## 4. Notebook Resonator

a) Draw a circuit diagram to illustrate a resistor, inductor and capacitor connected in series and subjected to an ideal ac function generator applying an EMF $\varepsilon(\mathrm{t})=\mathrm{v}_{\mathrm{o}} \cos \omega \mathrm{t}$.

What is the relation between the potential difference across each circuit element and the current flowing through it? Show how these relations can be used, together with complex analysis, to define the impedance $Z(\omega)$ for each of the 3 elements.
b) Write down an expression for the total impedance Z of the LCR series circuit, and express it in the form $\mathrm{Z}=|\mathrm{Z}| e^{j \phi}$, specifying both the magnitude and the phase.
c) At what frequency is the amplitude of the current a maximum? What is the phase of the current with respect to the ac generator at this frequency?
d) A physics student, bored with studying for exams, decides to build a LCR series circuit out of items that she finds on her desk:

- The spiral coil holding her notebook together will be the inductor - it has 50 turns of wire, with a diameter of 1 cm .
- The notebook cover is $20 \mathrm{~cm} \times 30 \mathrm{~cm}$ in area, and the thickness of the book is 0.5 cm . Some aluminum foil wrapped around the front and back covers of the book will turn it into a capacitor.

- Her pencil is 15 cm long, with a 2 mm diameter graphite core - this will be the resistor.

Work out numerical values for $\mathrm{L}, \mathrm{C}$ and R for these items, making use of the formulae and data below.
What is the expected resonant frequency f in Hz ?
For an applied EMF with amplitude $v_{o}$, what is the amplitude of the potential difference across the resistor at resonance? If $v_{o}=1 \mathrm{~V}$, what is the maximum current at resonance?
e) Suggest how the operation of this circuit might be detected remotely, e.g. if it is energized at the resonant frequency in the back of the classroom and the professor wants to know if it is turned on without looking at it.

Data, etc:
Resistivity of graphite: $\rho=5 \times 10^{-6} \Omega . \mathrm{m}$
Dielectric constant of paper: $\kappa=3$; capacitance: $\mathrm{C}=\kappa \varepsilon_{0} \mathrm{~A} / \mathrm{d}, \varepsilon_{o}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$
Magnetic field, ideal solenoid: $B=\mu_{o} N I / \ell, \mu_{o}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$

## The End!

