Phys 2610 (2019) Pre-lab 4 solutions

Using $R = 100\,\Omega,\ 1\,k\Omega,\ 2\,k\Omega$, $L = 25\,mH$, $C = 2.2\,nF$

a) Calculate the circuit resonant angular frequency ω_0 (in rad/s), and the resonant temporal frequency f_0 (in Hz) for which the current is maximum. Find the value of Q for each resistor value.

Resonance: $\omega_0 = 1/\sqrt{LC} = \boxed{134.8\ k\,rad/s}$

$f_0 = \omega_0/2\pi = \boxed{21.46\ kHz}$

Q: $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{3371\,\Omega}{R} = \boxed{33.7,\ 3.37,\ 1.68}$ (for $R = 100\,\Omega, 1\,k\Omega, 2\,k\Omega$)

b) The complex impedance of the circuit is given by $z = |Z|e^{j\theta}$, where

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \quad \text{and} \quad \tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}.$$

What are the values of the phase at low, resonant and high frequencies?

At resonance, $\omega = \omega_0 = 1/\sqrt{LC} \rightarrow \tan\theta = 0 \rightarrow \boxed{\theta = 0}$

For $\omega \rightarrow 0$, $\tan\theta \rightarrow -\infty \rightarrow \boxed{\theta \rightarrow -\pi/2}$

For $\omega \rightarrow \infty$, $\tan\theta \rightarrow +\infty \rightarrow \boxed{\theta \rightarrow +\pi/2}$

c) The voltage across the resistor is $v_R = iR = \frac{v}{z}R$, so the real signal is

$$v_R = V_R\cos(\omega t + \alpha), \quad \text{where} \quad V_R = V_0\frac{R}{|Z|}, \quad \text{and} \quad \alpha = -\theta.$$

What are the expected amplitude and phase at resonance?

At resonance, $|Z| \rightarrow R$, and $\theta \rightarrow 0$, so

$V_R = \frac{V_0 R}{|Z|} = \boxed{V_0}$, and $\alpha = -\theta = \boxed{0}$

d) The voltage across the capacitor is $v_C = iZ_C = \frac{v}{z}z_C$. Using the expression for z from (b) and $z_C = \frac{1}{j\omega C} = \frac{1}{\omega C e^{j\pi/2}}$, we can write

$v_C = V_C\cos(\omega t + \beta)$, where $V_C = V_0\frac{1}{\omega C|Z|} = \frac{V_0}{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}}$, and $\beta = -\left(\theta + \frac{\pi}{2}\right)$

(i) What is the ratio of amplitudes V_C/V_0 at resonance for each value of R? How can this be greater than 1?

(ii) Determine the low, resonant, and high frequency values of β. Sketch a graph of β versus ω from low to high frequency, marking the resonance condition clearly.

(i) $\frac{V_c}{V_0} = \frac{1}{\omega C|Z|}$

At resonance, $\omega = \omega_0 = 1/\sqrt{LC}$, and $|Z| = R$, so

$\frac{V_c}{V_0} = \frac{1}{\omega_0 RC} = Q = \boxed{33.7,\ 3.37,\ 1.68}$ (for $R = 100\,\Omega, 1\,k\Omega, 2\,k\Omega$)

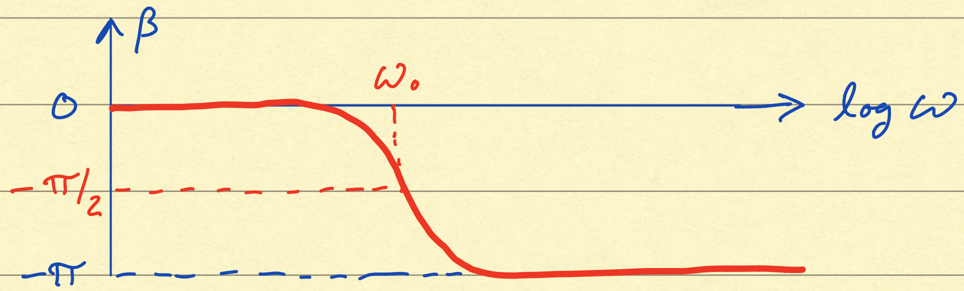
(larger than 1 because $\frac{V_L}{V_0}$ is also > 1 and π out of phase)

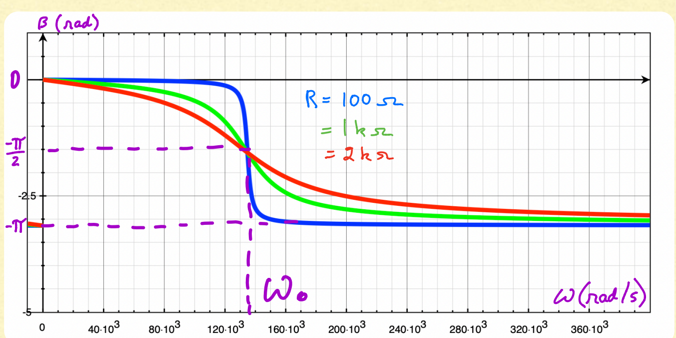
(ii) $\beta = -(\theta + \pi/2)$, so using (b) above,

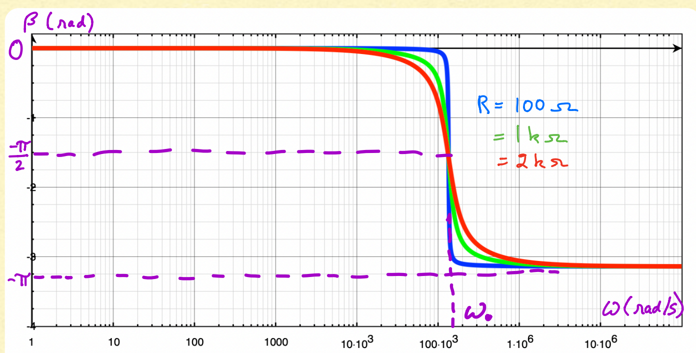
at resonance, $\theta = 0$, $\beta = -\pi/2$

for $\omega = 0$, $\theta = -\pi/2$, $\beta = 0$

for $\omega \to \infty$, $\theta \to \pi/2$, $\beta \to -\pi$







e) Defining "capacitor resonance" to be the condition for which the amplitude ratio V_C/V_0 is a maximum, give the expression for the value of ω at this resonance. What is the value of this resonance, and what is its value in the $R = 0$ limit? Calculate the amplitude ratio at this capacitor resonance for the 3 resistor values.

For maximum V_C, $\omega = \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \boxed{\sqrt{\omega_0^2 - \frac{R^2}{2L^2}}}$

Using the above components, $\omega' = \boxed{134.8,\ 131.8,\ 122.4 \text{ krad/s}}$

(for $R = 100, 1000, 2000\ \Omega$)

In the $R \to 0$ limit, $\omega' = \omega_0 = \boxed{134.8 \text{ krad/s}}$

Amplitude ratio: $\frac{V_C}{V_0} = \frac{1}{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}}$ For the above values of ω and R,

$\frac{V_C}{V_0} = \boxed{33.7,\ 3.41,\ 1.76}$ (for $R = 100, 1000, 2000\ \Omega$ + $\omega = 134.8, 131.8, 122.4$ krad/s)

compared to $\frac{V_C}{V_0} = 33.7,\ 3.37,\ 1.68$ (for $R = 100, 1000, 2000\ \Omega$ + $\omega = \omega_0 = 134.8 \times 10^3$ rad/s)