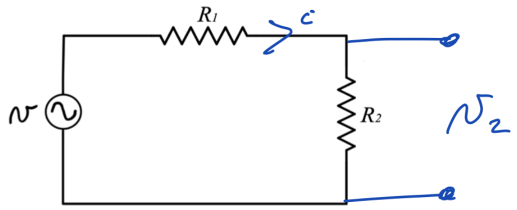
Phys 2610 (2019) Assignment 2 solutions

1. For the circuit below, if v is a sinusoidal voltage with an amplitude of 8 V, R_1 = 5.6 kΩ, and R_2 = 2.8 kΩ, calculate the peak, the rms, and the average voltage across R_2. What is the maximum instantaneous power dissipated by the R_2? What is the average power dissipated by R_2? What is the average power supplied by the signal generator?



Take $v = V \sin \omega t$

For R_2

Voltage divider eq'n : $v_2 = v \dfrac{R_2}{R_1+R_2} = \underbrace{\dfrac{V R_2}{R_1+R_2}}_{V_2} \sin \omega t$

Peak voltage: $V_2 = \dfrac{R_2}{R_1+R_2} = \boxed{2.67\text{V}}$

rms voltage: $V_{rms} = \dfrac{V_2}{\sqrt{2}} = \boxed{1.89\text{V}}$

Average voltage: $\langle v_2 \rangle = \dfrac{1}{T}\displaystyle\int_0^{T=2\pi/\omega} V_2 \sin \omega t \, dt = \boxed{0}$

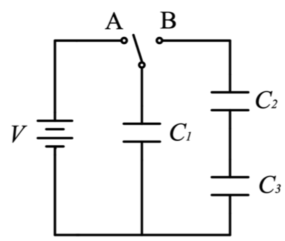
Max inst. power: $p_2 = \dfrac{v_2^2}{R_2} \rightarrow p_{2\,max} = \dfrac{v_{2\,max}^2}{R_2}$

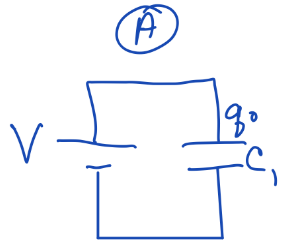
$= \dfrac{V_2^2}{R_2} = \boxed{2.55\,\text{mW}}$

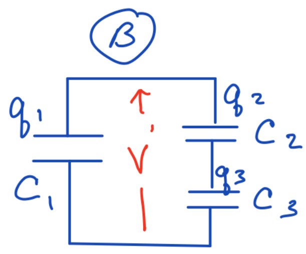
Ave power: $P_{2\,ave} = \dfrac{V_{2\,rms}^2}{R_2} = \boxed{1.28\,\text{mW}}$

Signal Generator: Ave power: $P = \dfrac{V_{rms}^2}{(R_1+R_2)} = \dfrac{V^2}{2(R_1+R_2)} = \boxed{3.81\,\text{mW}}$

2. In the circuit below, the capacitors are initially uncharged. The switch is first put in position A for a long time, and then switched to position B. Find the final charges on the 3 capacitors after equilibrium is established.







Position A: $V = q_0 / C_1$

Position B: Conservation of charge $\Rightarrow q_0 = q_1 + q_2$ and $q_2 = q_3$

Also, $V' = q_1/C_1 = q_2/C_2 + q_3/C_3 = q_2\left(\frac{1}{C_2} + \frac{1}{C_3}\right)$

$$\therefore \quad \frac{q_0 - q_2}{C_1} = q_2\left(\frac{1}{C_2} + \frac{1}{C_3}\right) \longrightarrow \frac{q_0}{C_1} = q_2\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\rightarrow \boxed{q_2 = q_3 = V\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}} \quad \text{or} \quad \boxed{q_2 = q_3 = V \frac{C_1C_2C_3}{C_1C_2 + C_1C_3 + C_2C_3}}$$

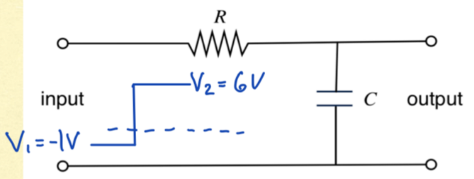
Then, $q_1 = q_0 - q_2 = VC_1 - V\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$

$$\rightarrow \boxed{q_1 = V\left(C_1 - \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}\right)}$$

or

$$\boxed{q_1 = V\left(\frac{C_1^2 (C_2 + C_3)}{C_1C_2 + C_1C_3 + C_2C_3}\right)}$$

3. Determine the output voltage as a function of time for the circuit shown below if the input is a step function from – 1 V to +6 V, for $R = 1\ \mathrm{M\Omega}$, and $C = 100\ \mu\mathrm{F}$. Sketch the voltage to scale as a function of time from zero to 4 time constants.



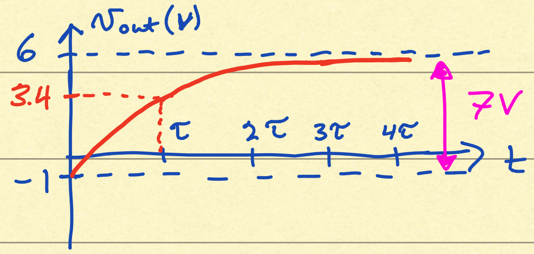
$$RC = 10^6\ \Omega \cdot 100 \times 10^{-6}\ \mathrm{F}$$
$$\tau = 100\ \mathrm{s}$$

For $t = 0$, $v_{out} = -1\mathrm{V}$. For $t \to \infty$, $v_{out} \to +6\ \mathrm{V}$

By inspection, $v_{out} = 7\mathrm{V}\left(1 - e^{-t/RC}\right) - 1\mathrm{V}$

$$= 6\mathrm{V} - 7\mathrm{V}\, e^{-t/RC}$$

$$\boxed{v_{out} = 6\mathrm{V}\left(1 - \tfrac{7}{6}\mathrm{V}\, e^{-t/100\mathrm{s}}\right)}$$



For $t = \tau$, $v_{out} = 7\mathrm{V}(1 - 1/e) - 1\mathrm{V} = 3.4\mathrm{V}$

Also,

From K.L., $V_2 - iR - q/C = 0 \longrightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{V_2}{R} \to q = V_2 C + A e^{-t/RC}$

Since $q = V_1 C$ at $t = 0$, $V_1 C = V_2 C + A \Longrightarrow A = (V_1 - V_2) C$

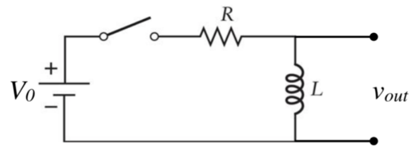
So, $q = V_2 C + (V_1 - V_2)\, e^{-t/RC}$

$$\Longrightarrow \quad v_{out} = \frac{q}{C} = V_2 + (V_1 - V_2)\, e^{-t/RC}$$

$$= V_2\left(1 - \frac{V_2 - V_1}{V_2}\, e^{-t/RC}\right) = V_2\left(1 - \frac{V_2 - V_1}{V_2}\, e^{-t/RC}\right)$$

$$\Longrightarrow \quad \boxed{v_{out} = 6\mathrm{V}\left(1 - \tfrac{7}{6}\mathrm{V}\, e^{-t/100\mathrm{s}}\right)}$$ as above

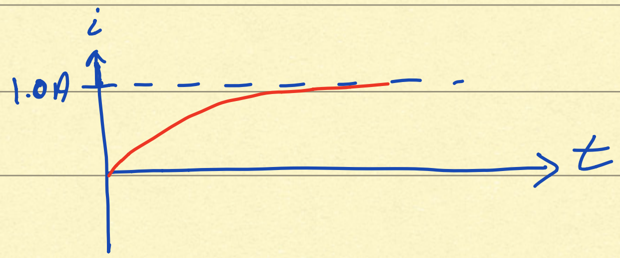
4. For the circuit below, with $V_0 = 10$ V, $R = 10\ \Omega$, and $L = 100$ mH, what is the current after the circuit reaches equilibrium? How long does it take for the current to increase from 0.1 A to 0.9 A?



$$\tau = \frac{L}{R} = \frac{100\times10^{-3}\,H}{10\,\Omega} = 10\,ms$$

For $t \to \infty$, $\frac{di}{dt} \to 0 \Rightarrow \boxed{I_{max} = \frac{V_0}{R} = 1.0\,A}$

For $t = 0$, $i = 0$.



By inspection, $i = I_{max}\left(1 - e^{-tR/L}\right)$

Solve for t: $-\left(\frac{i}{I_{max}} - 1\right) = e^{-tR/L}$

$$\ln\left(1 - \frac{i}{I_{max}}\right) = -tR/L$$

$$t = -\frac{L}{R}\ln\left(1 - \frac{i}{I_{max}}\right)$$

For $i/I_{max} = .1$, $t_1 = -10\,ms\,\ln(.9) = 1.05\,ms$

For $i/I_{max} = .9$, $t_2 = -10\,ms\,\ln(.1) = 23.0\,ms$

So $\boxed{\Delta t = t_2 - t_1 = 22.0\,ms}$