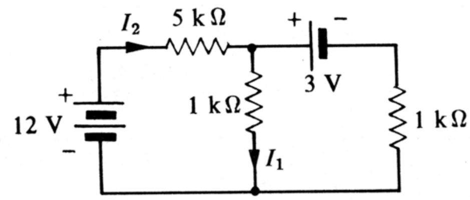
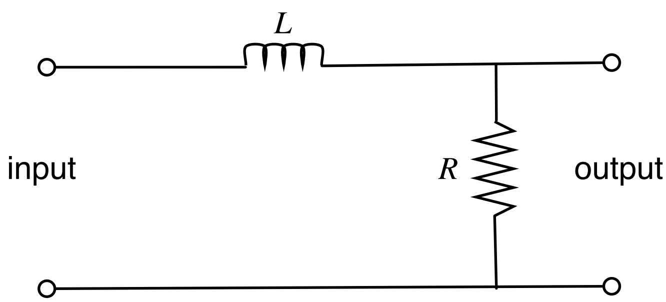
Answer 5 of the 6 questions. All questions are of equal value.

1. Calculate I_1 and I_2. What is the power delivered by the 3-V battery?



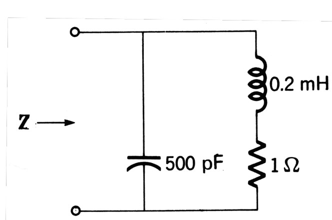
2. A high impedance voltmeter is used to measure the output voltage of an unknown network, giving a result of 4.5 V. When a 300 Ω resistor is connected across the output, the voltage is reduced to 1.25 V. What are the Thevenin equivalent voltage and resistance of the unknown circuit?

3. In the RL circuit below, determine the output voltage as a function of time if the input is stepped from zero to V_0 at time $t = 0$. What is the current at very long times?



4. Determine the output voltage for the RL circuit of question 3 if $v_{in} = V_0 \cos \omega t$. What is the phase shift? Is this a high-pass or a low-pass filter? What is the breakpoint frequency, where the gain drops by 3 dB from its maximum value?

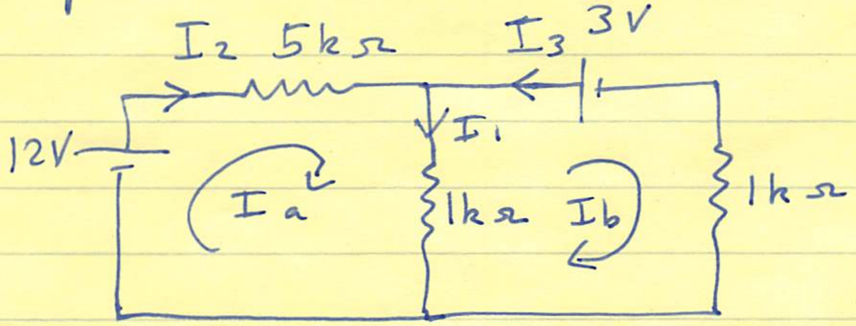
5. Calculate the magnitude of the impedance at resonance for the following circuit.



6. Give the expressions for the magnitude and phase of the impedance of a series RLC circuit, and sketch a graph of both as a function of frequency, indicating the resonance frequency and the vertical scales.

Phys 2610 (2017) Midterm Solutions

1



$12V - I_a(5k\Omega) - (I_a - I_b)1k\Omega = 0$ loop a
$-3V - I_b(1k\Omega) - (I_b - I_a)1k\Omega = 0$ loop b

Rearrange: $I_a(6k\Omega) + I_b(-1k\Omega) = 12V$ ①
$I_a(-1k\Omega) + I_b(2k\Omega) = -3V$ ②

2·① + ②: $I_a(11k\Omega) = 21V \Rightarrow I_a = 1.91mA$

① + 6·②: $I_b(11k\Omega) = -6V \Rightarrow I_b = -0.56mA$

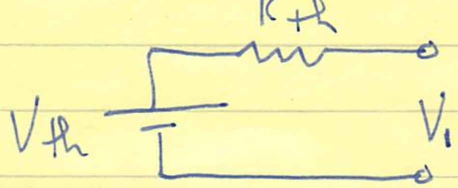
Then $I_2 = I_a = \underline{1.91mA}$

$I_1 = I_a - I_b = \underline{2.45mA}$

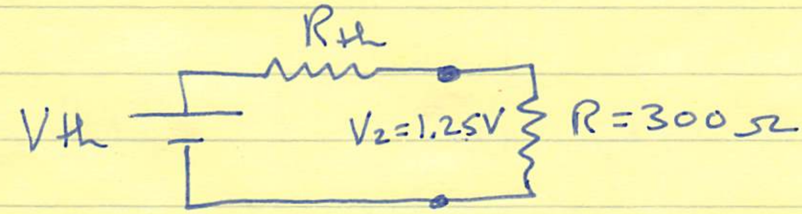
Power from 3V battery: $P = VI_3 = V(-I_b)$

$= \underline{1.63mW}$

2



$V_1 = V_{th} = \underline{4.5V}$

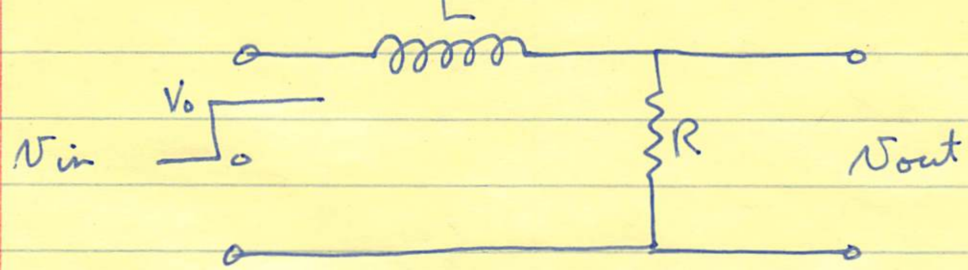


$$V_2 = \frac{V_{th} R}{R + R_{th}} \longrightarrow R_{th} = \frac{V_{th} R}{V_2} - R$$

$$= \left(\frac{V_{th}}{V_2} - 1\right) R = \left(\frac{4.5V}{1.25V} - 1\right) 300\,\Omega$$

$$= \underline{780\,\Omega}$$

3



For $t \to \infty$, inductor impedance $\to 0$, so

$$i = \frac{V_0}{R} \quad \text{and} \quad v_{out} = V_0$$

For $t = 0$, v_L is max + $i = 0 \Rightarrow v_{out} = 0$

Using KVL: $V_0 - L\frac{di}{dt} - iR = 0$

$$\to i(t) = A e^{-t/\tau} + i(\infty) \qquad (\tau = L/R)$$

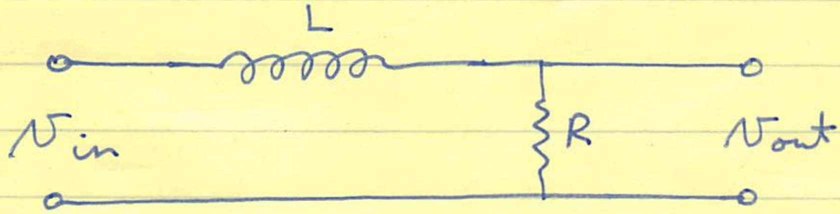
$$\to i(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$$ from above boundary cond'ns

$$\to v_{out} = V_0\left(1 - e^{-t/\tau}\right)$$

or using $v_{out} = V_1 + V_2 e^{-t/\tau}$ $\qquad (\tau = L/R)$

$$= V_0\left(1 - e^{-t/\tau}\right)$$ from above boundary cond'ns.

4



Voltage divider: $\quad v_{out} = \left(\dfrac{R}{R + j\omega L}\right) v_{in}$

$$\rightarrow v_{out} = \frac{R}{\sqrt{R^2 + (\omega L)^2}\, e^{j\theta}} V_0 e^{j\omega t}$$

$$\text{where } \tan\theta = \omega L / R$$

$$\rightarrow v_{out} = \frac{V_0}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} e^{j(\omega t - \theta)}$$

Phase shift is $\alpha = -\theta = -\arctan(\omega L / R)$

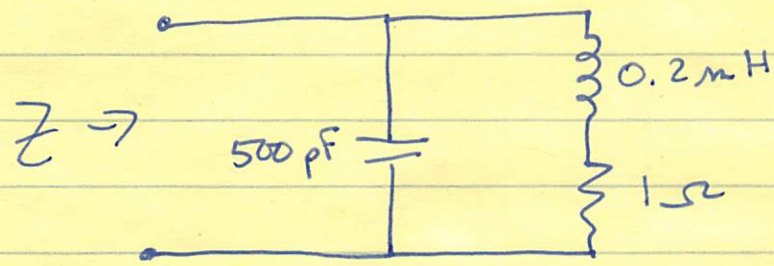
This is a low pass filter since for $\omega \rightarrow 0$,

$$v_{out} \rightarrow v_{in}$$

Break point frequency $\Rightarrow \left|\dfrac{v_{out}}{v_{in}}\right| = \dfrac{1}{\sqrt{2}}$

$$\Rightarrow \frac{\omega L}{R} = 1 \Rightarrow \omega_B = \frac{R}{L}$$

5



$$Z = (1/j\omega C) \,//\, (R + j\omega C)$$

$$= \frac{(1/j\omega C)\,(R + j\omega C)}{R + j\omega L + 1/j\omega C} = \frac{(1/\omega C)\,(\omega L - jR)}{R + j(\omega L - 1/\omega C)}$$

At resonance, $\omega = \omega_0 = 1/\sqrt{LC}$

$$\Rightarrow \omega L = 1/\omega C = \sqrt{\frac{L}{C}} = \sqrt{4.0 \times 10^5}\ \Omega$$

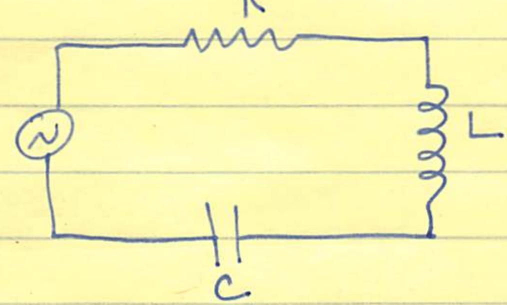
Then $$Z(\omega_0) = \frac{(4.0 \times 10^5)\,\Omega^2 - j\sqrt{4.0 \times 10^5}\ \Omega^2}{1\,\Omega}$$

or

$$Z(\omega_0) = \sqrt{(400\,k\Omega)^2 + 4.0 \times 10^5\ \Omega^2}$$

$$= \underline{400\,k\Omega} \qquad (400.0005\,k\Omega)$$

6



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = |Z| e^{j\theta}$$

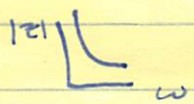
with $|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

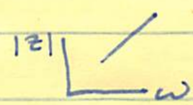
& $\tan\theta = \dfrac{(\omega L - 1/\omega C)}{R}$

For $\omega \to 0$, $|Z| \to \infty$, $\tan\theta \to -\infty$, $\theta \to -\pi/2$

For $\omega \to \infty$, $|Z| \to \infty$, $\tan\theta \to +\infty$, $\theta \to \pi/2$

For $\omega = \omega_0 = 1/\sqrt{LC}$, $|Z| = R$, $\tan\theta = 0 \Rightarrow \theta = 0$

Also, for $\omega \to 0$, $|Z| \cong 1/\omega C$ 

& for $\omega \to \infty$, $|Z| \cong \omega L$ 

So