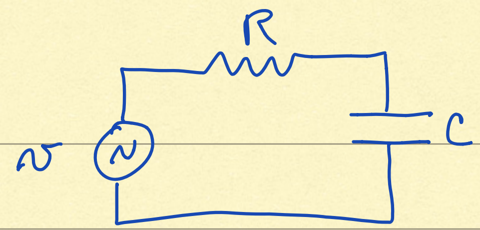
Phys 2610 (2019) Prelab 3 solutions

Pre-Lab exercises:

a) Working with the complex waveforms $\tilde{v}(t) = V_0 e^{j\omega t}$, $v(t) = Re(\tilde{v})$, use the complex impedance formalism to show that the current in the circuit is given by:

$i(t) = \frac{V}{\sqrt{R^2+(1/\omega C)^2}} \cos(\omega t - \theta)$, where $\tan\theta = -1/\omega RC$.



$\tilde{v} = V_0 e^{j\omega t}$

The complex impedance is

$\tilde{z} = R - \frac{j}{\omega C} = |Z| e^{j\theta}$ where $\tan\theta = -1/\omega RC$

and $|Z| = \sqrt{R^2 + (1/\omega C)^2}$

Then the (complex) current is

$$\tilde{i} = \frac{\tilde{v}}{Z} = \frac{V_0 e^{j\omega t}}{|Z| e^{j\theta}} = \frac{V_0}{|Z|} e^{j(\omega t - \theta)}$$

and the real part is

$$\boxed{i = \frac{V_0}{\sqrt{R^2 + (1/\omega C)^2}} \cos(\omega t - \theta) \quad \text{with } \theta = -\arctan(1/\omega RC)}$$

b) Based on the solution to part a), find expressions for the potential differences across the resistor, $v_R(t) = V_R \cos(\omega t + \alpha)$, and capacitor, $v_C(t) = V_C \cos(\omega t + \beta)$, showing the phases ($\alpha$ and β) with respect to the input voltage. The capacitor values will be examined in this experiment.

Resistor Voltage: $v_R = iR = \frac{V_0 R}{Z} \cos(\omega t - \theta)$

so $v_R = V_R \cos(\omega t + \alpha)$ with $\alpha = -\theta = -\arctan(-1/\omega RC)$

$= \boxed{\arctan(1/\omega RC)}$

and $V_R = \frac{V_0 R}{|Z|} = \boxed{\frac{V_0}{\sqrt{1 + (1/\omega RC)^2}}}$

Capacitor Voltage: $\tilde{v}_c = \tilde{i}\,\tilde{z}_c$

Using $\tilde{i} = \frac{V_0}{|Z|} e^{j(\omega t - \theta)}$ and $\tilde{z}_c = 1/j\omega C = \left(\frac{1}{\omega C}\right) e^{-j\pi/2}$ gives:

$$\tilde{v}_c = \frac{V_0}{\omega C |Z|} e^{j(\omega t - \theta - \pi/2)}$$

$\longrightarrow$ $v_c = V_c \cos(\omega t + \beta)$ with $\beta = -\theta - \pi/2 = \boxed{\arctan(1/\omega RC) - \pi/2}$

and $V_c = \frac{V_0}{\omega C |Z|} = \boxed{\frac{V_0}{\sqrt{1 + (\omega RC)^2}}}$

or

Using the voltage divider eq'n $\tilde{v}_c = \tilde{v} \frac{\tilde{z}_c}{R + \tilde{z}_c}$:

$$\tilde{v}_c = \frac{V_0 e^{j\omega t}}{j\omega C (R + 1/j\omega C)} = \frac{V_0 e^{j\omega t}}{(1 + j\omega RC)} = \frac{V_0 e^{j\omega t}}{\sqrt{1 + (\omega RC)^2}\, e^{j\phi}} = \frac{V_0 e^{j(\omega t - \phi)}}{\sqrt{1 + (\omega RC)^2}}$$

where $\tan\phi = \omega RC$

Then, $v_c = V_c \cos(\omega t + \beta)$ with $\boxed{\beta = -\phi = -\arctan(\omega RC)}$

and $\boxed{V_c = V_0 / \sqrt{1 + (\omega RC)^2}}$

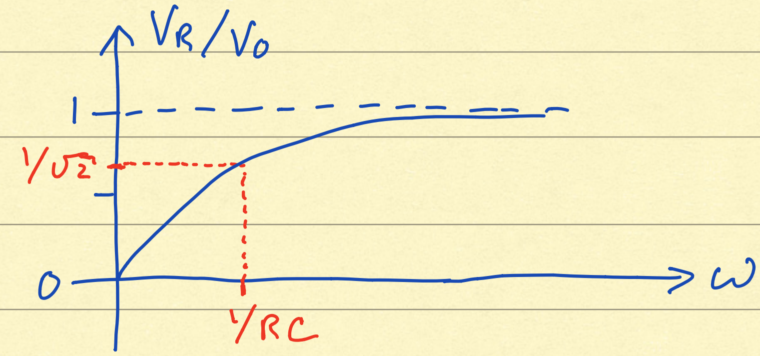
(Note: $\arctan(1/x) - \pi/2 = -\arctan x$)

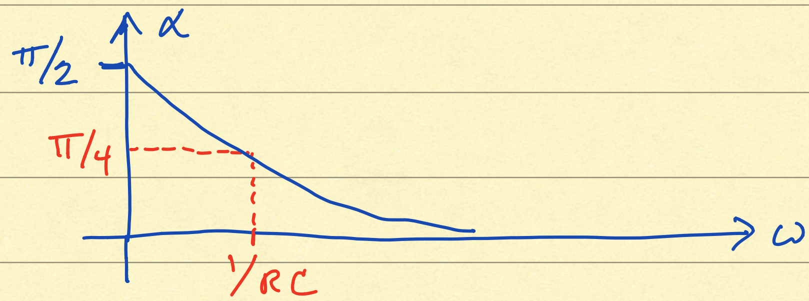
c) Organize your results in a table:

Element:	Amplitude	Phase relative to $v(t)$
Resistor	$V_R = V_0 / \sqrt{1 + (1/\omega RC)^2}$	$\alpha = \arctan(1/\omega RC)$
Capacitor	$V_C = V_0 / \sqrt{1 + (\omega RC)^2}$	$\beta = -\arctan(\omega RC)$

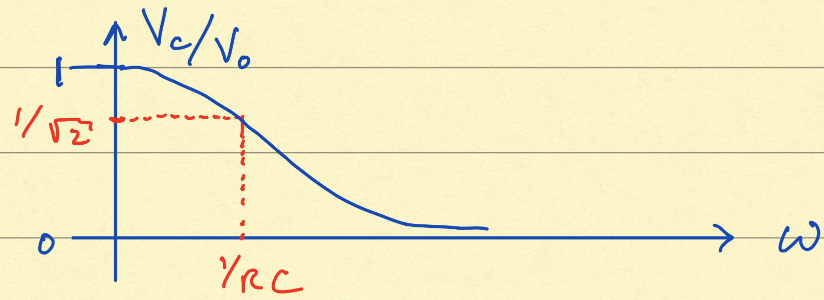
d) Sketch graphs of the amplitude and phase relations for the capacitor and resistor as functions of frequency.

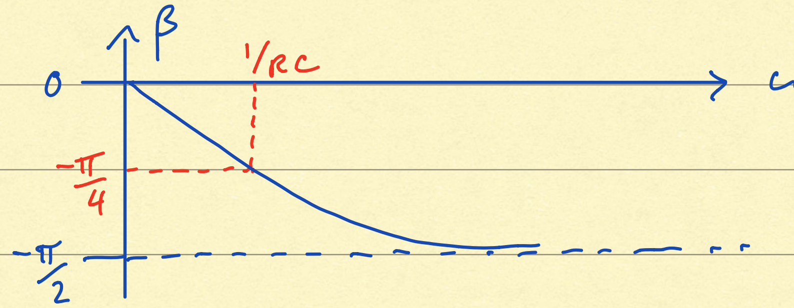
Resistor:

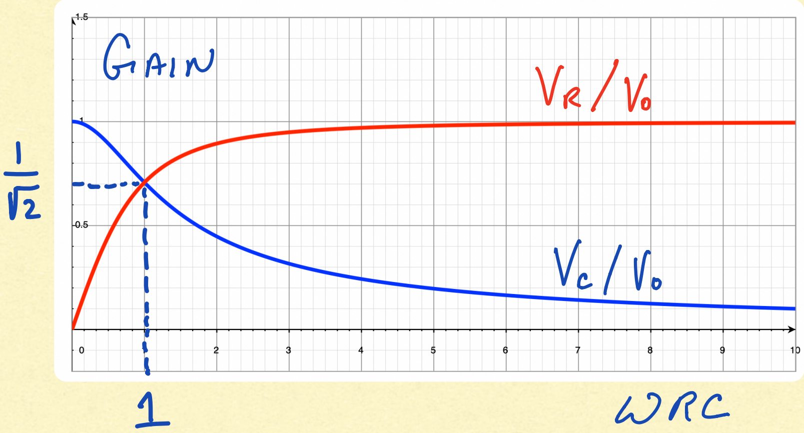


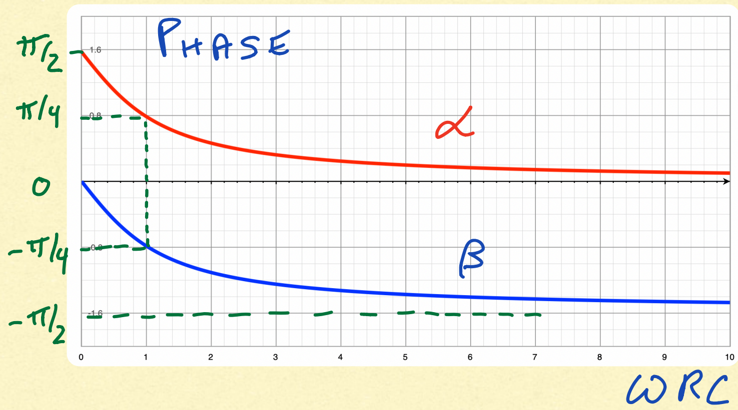


Capacitor

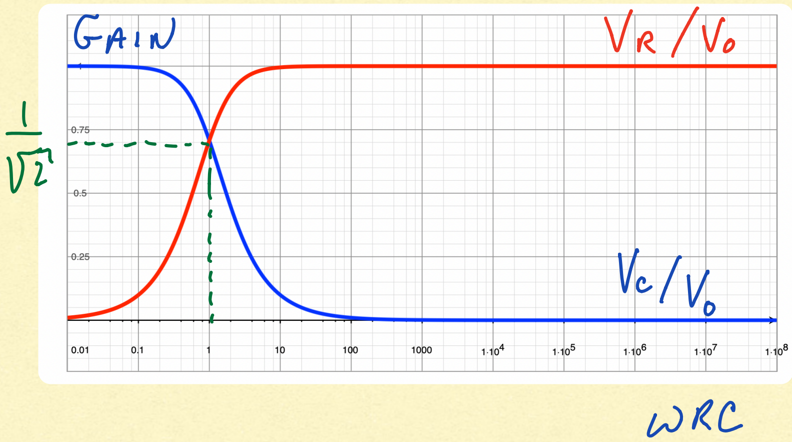


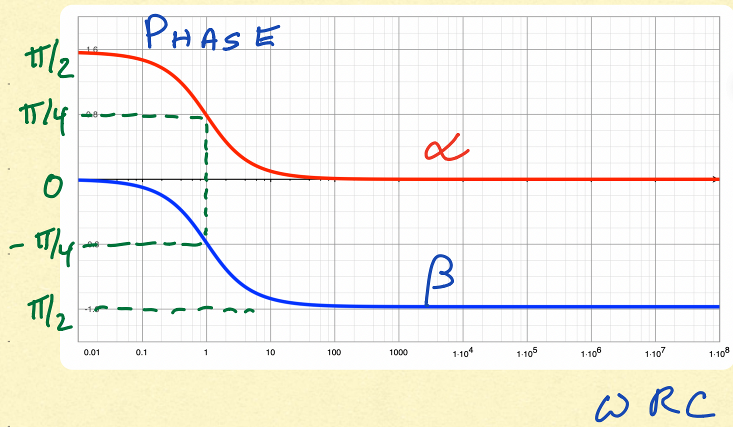



GAIN
V_R/V_0
V_C/V_0
$\frac{1}{\sqrt{2}}$
1
ωRC


PHASE
π/2
π/4
0
-π/4
-π/2
α
β
ωRC

Log Scales:


GAIN
V_R/V_0
V_C/V_0
$\frac{1}{\sqrt{2}}$
ωRC


PHASE
π/2
π/4
0
-π/4
π/2
α
β
ωRC