Phys2610 (2019) Prelab exercises for experiment 2

Pre-Lab exercises:

a) Find the time constant for nominal component values $R = 10\ k\Omega, \quad C = 0.03\ \mu F$. What approximate frequency setting f on the function generator would allow for 10 "time constants" of the RC response time to be observed before switching to the opposite polarity of the waveform?

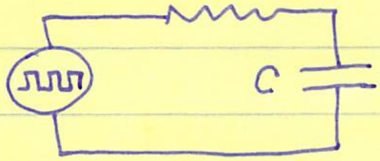
b) Find the potential difference across the capacitor and resistor as a function of time, for both transitions of the square wave, and check that your solutions satisfy Kirchhoff's loop theorem.

c) Draw a sketch of the expected waveforms for the EMF supplied by the function generator, the potential difference across the resistor, and the potential difference across the capacitor, for a frequency such that each half-cycle corresponds to 4 time constants of the RC circuit. Include at least one full cycle of the square waveform. Draw your diagrams to scale.

d) For the high to low transition, show that the waveform can be simply expressed as $v_C'(t) = v_C'(0)e^{-\frac{t}{RC}}$ where $t = 0$ represents the start of the cycle, and $v_C' = 0$ for $t \to \infty$. What is the value of $v_C'(0)$ in terms of V_0? A graph of $\ln\left(\frac{v_C'(t)}{v_C'(0)}\right)$ vs t should be a straight line with slope $(-1/RC)$. Note the prime here simply represents another variable, not a derivative.

e) Watch the introductory videos linked to the course web page for this experiment !!!

Phys 2610 (2019) Pre-lab exercises 2 solutions

a) 

For $R = 10\,k\Omega$, $C = 0.03\,\mu F$,
$\tau = RC = 3\times10^{-4}\,s = \boxed{0.30\,ms}$

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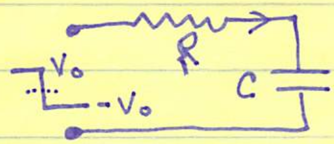
For $T = 20\tau$ (i.e. $T/2 = 10\tau$), $f = \frac{1}{T} = \boxed{167\,Hz}$

b) (i) 

$i = dq/dt$

loop rule (after transition): $-V_0 - iR - q/C = 0 \rightarrow \underline{\frac{dq}{dt} + \frac{q}{RC} = \frac{-V_0}{R}}$

Solution: $q = A e^{-t/RC} - V_0 C$

5 For $t = 0$, $q = V_0 C$, so $A = 2V_0 C$, giving

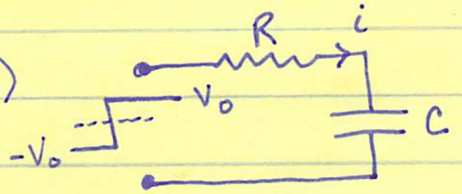
$$q = CV_0\left(2e^{-t/RC} - 1\right)$$, so that

$\boxed{v_C = \frac{q}{C} = V_0\left(2e^{-t/RC} - 1\right)}$ and $v_R = iR = \frac{dq}{dt}R$
$= R\left(\frac{2V_0C}{RC} e^{-t/RC}\right)$

or

$\boxed{v_R = -2V_0 e^{-t/RC}}$

$\therefore$ $v_R + v_C = -V_0$, satisfying K.L.

(ii) 

$i = dq/dt$

Here $V_0 - iR - q/C = 0 \Rightarrow \dfrac{dq}{dt} + \dfrac{q}{RC} = \dfrac{V_0}{R}$

Solution: $q = A e^{-t/RC} + V_0 V$

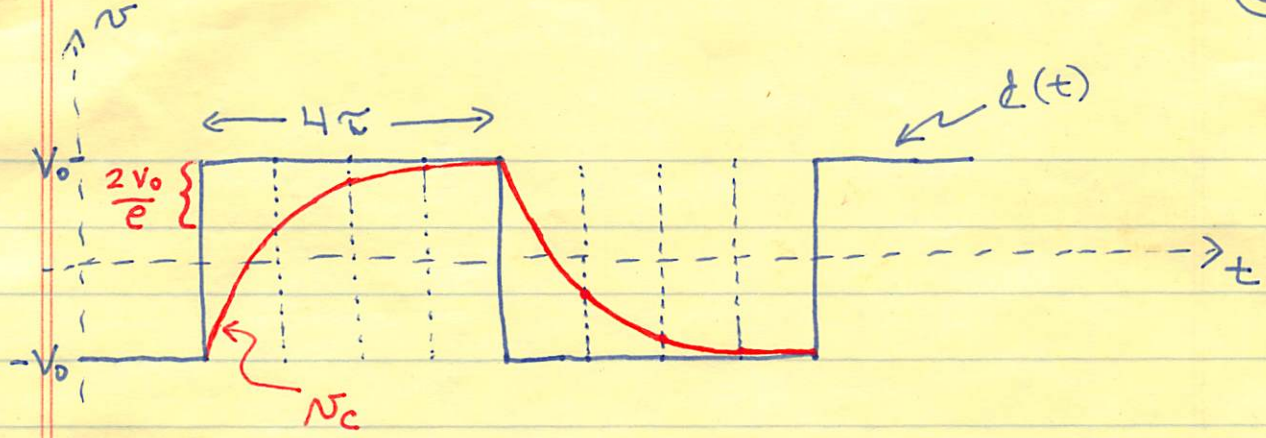
5 For $t = 0$, $q = -V_0 C$, so $A = -2V_0 C$, giving

$$q = CV_0\left(1 - 2e^{-t/RC}\right), \text{ so that}$$

$\boxed{v_C = \dfrac{q}{C} = V_0\left(1 - 2e^{-t/RC}\right)}$ and $\boxed{v_R = iR = 2V_0 e^{-t/RC}}$

$\therefore$ $v_C + v_R = +V_0$ satisfying K.L.

c)



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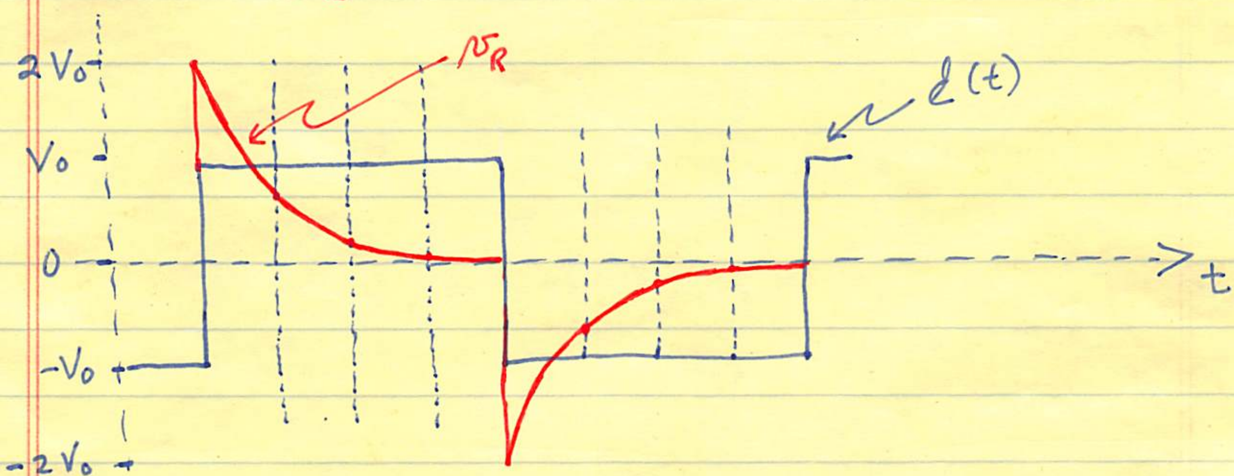

d) Consider $v' = v_C + V_0 = 2V_0 e^{-t/RC}$

Then $v'(0) = 2V_0$ and $v'(\infty) = 0$

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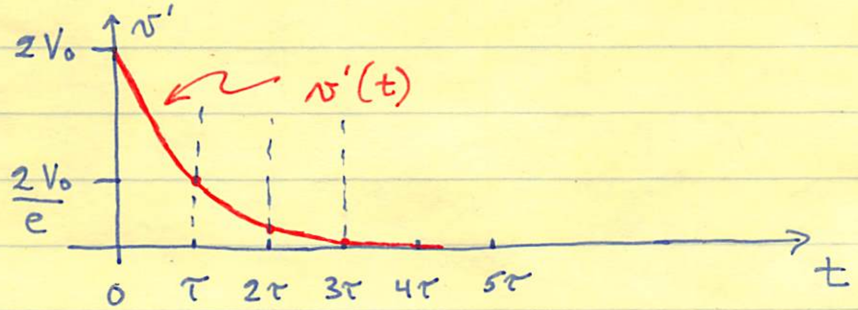
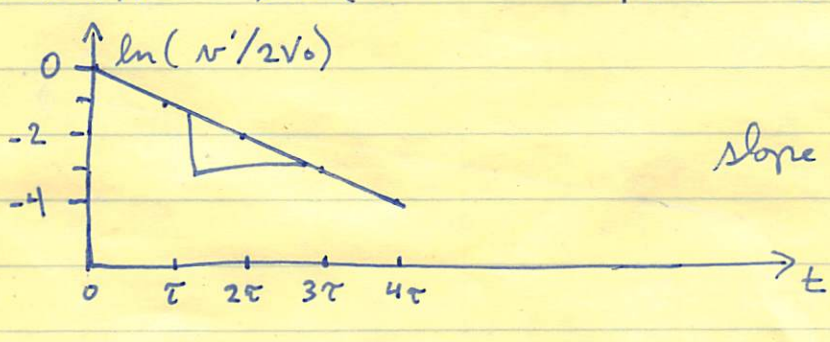


Also $\ln v' = \ln(2V_0) - t/RC \rightarrow \ln\left(\frac{v'}{2V_0}\right) = -t/RC$



slope = $-1/RC$

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