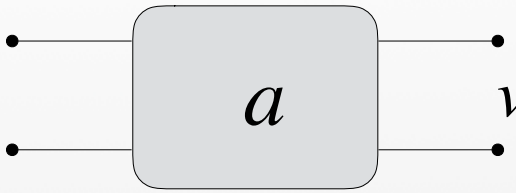
6) RC filters and gain in dB

$v_1 = V_1 e^{j\omega t}$



$v_2 = V_2 e^{j(\omega t+\phi)}$

Complex gain: $$a = \frac{v_2}{v_1} = \frac{V_2 e^{j(\omega t+\phi)}}{V_1 e^{j\omega t}} = \frac{V_2}{V_1} e^{j\phi}$$

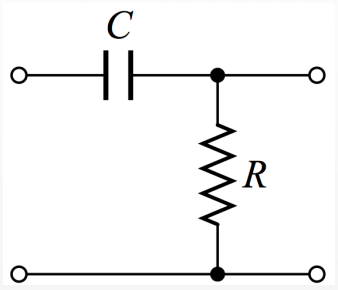
Magnitude: $$|a| = \left|\frac{v_2}{v_1}\right| = \frac{V_2}{V_1}$$

Phase: $$\phi$$

Gain in dB: $$G_{dB} = 20\log\left(\frac{V_2}{V_1}\right)$$

(a)High pass filter

$$v_1 = V_1 e^{j\omega t}$$



$$v_2 = V_2 e^{j(\omega t+\phi)}$$

$$|Z| = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

$$\frac{V_2}{V_1} = \frac{R}{|Z|} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \begin{cases} \approx \omega RC & \text{for small } \omega \\ = 1/\sqrt{2} & \text{for } \omega = 1/RC \\ \approx 1 & \text{for large } \omega \end{cases}$$

$G_{dB} = 20\log(RC) + 20\log(\omega)$

$G_{dB} = -3dB$

$G_{dB} = 0$

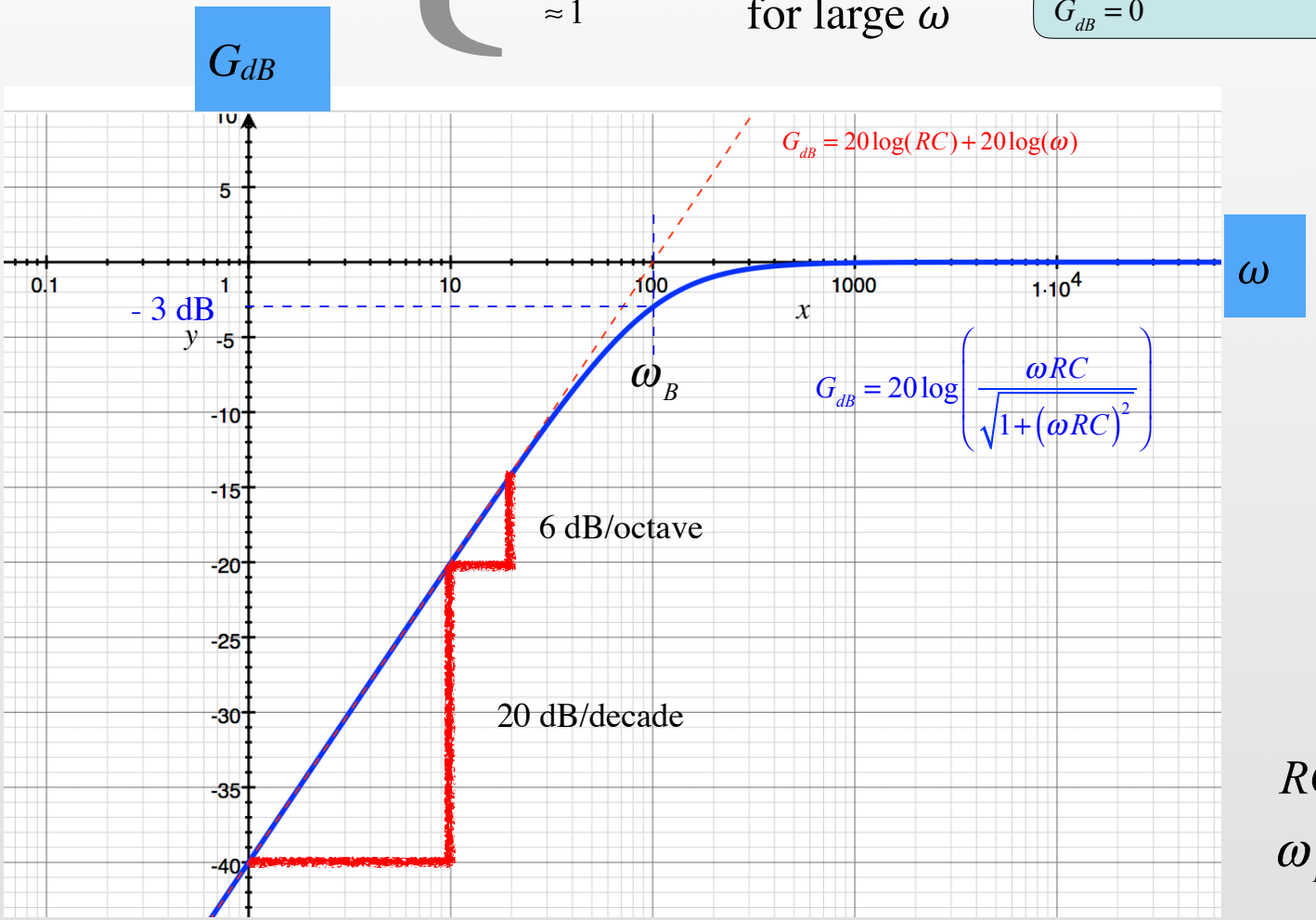
$$\frac{V_2}{V_1} = \frac{R}{|Z|} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$$

$$\begin{cases} \approx \omega RC & \text{for small } \omega \\ = 1/\sqrt{2} & \text{for } \omega = 1/RC \\ \approx 1 & \text{for large } \omega \end{cases}$$

$G_{dB} = 20\log(RC) + 20\log(\omega)$

$G_{dB} = -3dB$

$G_{dB} = 0$

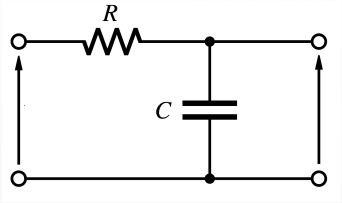


$RC = 0.01\text{s}$

$\omega_B = 100\text{rad/s}$

(b)Low pass filter

$v_1 = V_1 e^{j\omega t}$



$v_2 = V_2 e^{j(\omega t+\phi)}$

$$|Z| = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

$$\frac{V_2}{V_1} = \frac{1/\omega C}{|Z|} = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\begin{cases} \approx \frac{1}{\omega RC} & \text{for large } \omega \\ = 1/\sqrt{2} & \text{for } \omega = 1/RC \\ \approx 1 & \text{for small } \omega \end{cases}$$

$G_{dB} = -20\log(RC) - 20\log(\omega)$

$G_{dB} = -3dB$

$G_{dB} = 0$

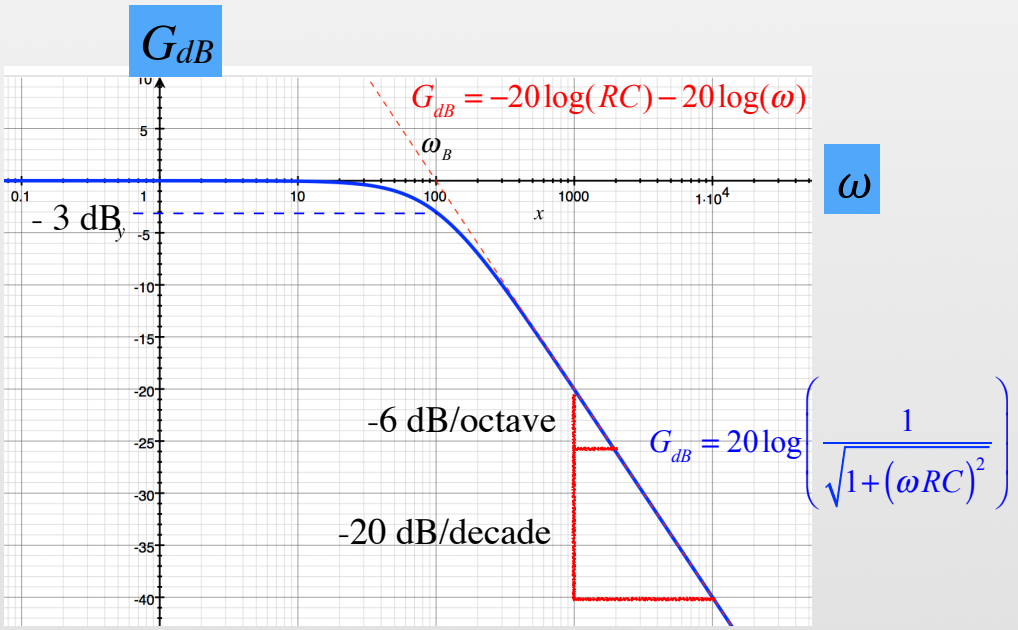
$$\frac{V_2}{V_1} = \frac{1/\omega C}{|Z|} = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\begin{cases} \approx \frac{1}{\omega RC} & \text{for large } \omega \\ = 1/\sqrt{2} & \text{for } \omega = 1/RC \\ \approx 1 & \text{for small } \omega \end{cases}$$

$G_{dB} = -20\log(RC) - 20\log(\omega)$

$G_{dB} = -3dB$

$G_{dB} = 0$



$RC = 0.01\text{s}$

$\omega_B = 100\text{rad/s}$