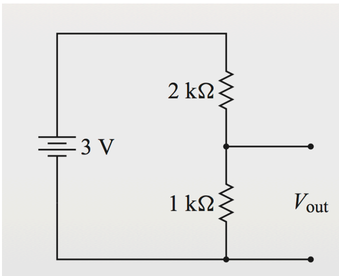
Phys2610 (2019) Assignment 1
Due 24 Jan 2019

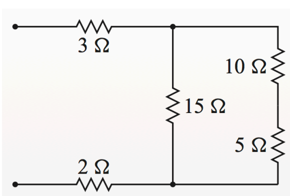
1. What is the resistance of a tungsten wire 0.3 mm in diameter and 0.1 m in length?

2. The output of the voltage divider shown is to be measured with voltmeters with input resistances of 10 kΩ, and 10 MΩ. What voltage will each indicate?

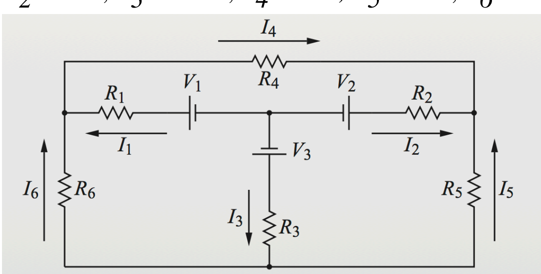


3. A real battery can be modeled as an ideal voltage source in series with a resistor (the internal resistance). An analog voltmeter measures the voltage of a worn-out 1.5 V flashlight battery with an internal resistance of 450 Ω as 1.2 V. What is the internal resistance of the analog meter?

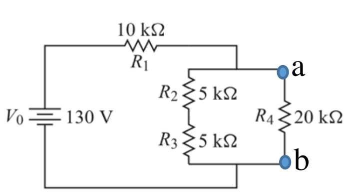
4. Find the current in each branch of the circuit below, if a 9 V battery is connected to the terminals.



5. Compute all the currents labeled in the circuit below, assuming the following values: V_1 = 10 V, V_2 = 6 V, V_3 = 12 V, R_1 = 4 Ω, R_2 = 2 Ω, R_3 = 10 Ω, R_4 = 5 Ω, R_5 = 7 Ω, R_6 = 3 Ω.



6. (a) Compute the current through the 20 kΩ resistor in the circuit shown below by reducing and expanding parallel and series combinations of resistors.
(b) Now find the Thevenin voltage, the Thevenin resistance, and the Norton current for the circuit with the terminals a and b, when the 20 kΩ resistor is removed.
(c) Show that, if the 20 kΩ resistor is connected to the Thevenin equivalent circuit, the current through the 20 kΩ resistor matches the value found in part (a). Do the same for the Norton equivalent circuit.

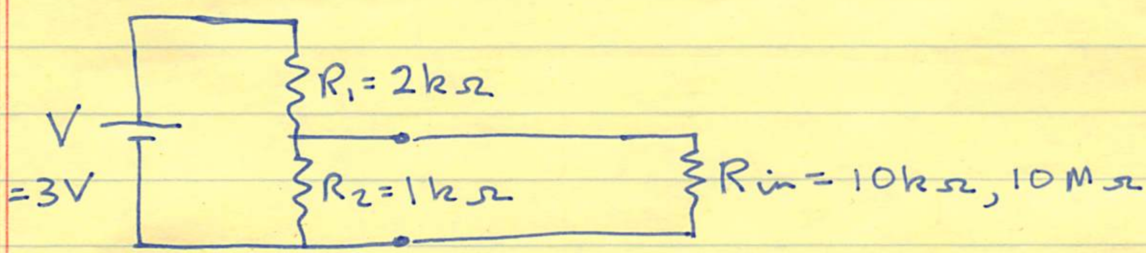


Phys2610 (2019) Assignment 1 solutions

① $R = \rho \frac{l}{A} = \underline{\underline{0.079\,\Omega}}$ using $\rho = 5.60\times 10^{-8}\ \Omega\,m$ for tungsten at room temperature

2 and $A = \pi r^2$ with $r = 0.15\ mm$.

②



$$V_{out} = \frac{V\,(R_2 /\!/ R_{in})}{R_1 + R_2 /\!/ R_{in}}$$

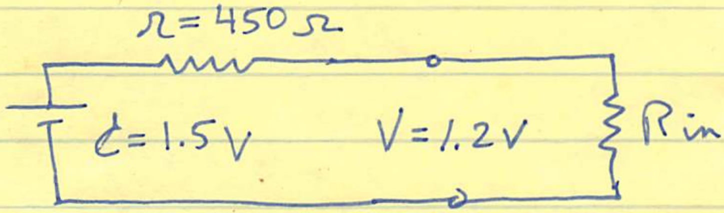
4 Here $R_2 /\!/ R_{in} = \frac{R_2 R_{in}}{R_2 + R_{in}} = 909\,\Omega,\ 999.9\,\Omega$

so

$$V_{out} = \underline{\underline{0.937}}\ V,\quad \underline{\underline{0.9999}}\ V$$

(2)

(3)

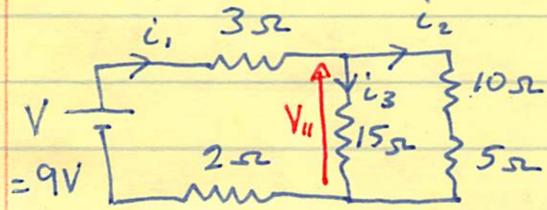


4 $$V = \frac{\mathcal{E} R_{in}}{R_{in} + r} \implies (R_{in} + r)V = \mathcal{E} R_{in}$$

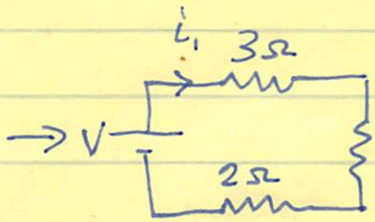
$$\implies R_{in}(V - \mathcal{E}) = -rV$$

$$\implies R_{in} = \frac{rV}{\mathcal{E} - V} = \underline{\underline{1.8\,k\Omega}}$$

(4)

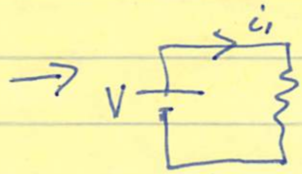


$\rightarrow$



$R_{\parallel} = (15\,\Omega) \parallel (10\,\Omega + 5\,\Omega)$
$= 7.5\,\Omega$

$\rightarrow$



$R = 2\,\Omega + R_{\parallel} + 3\,\Omega$
$= 12.5\,\Omega$

So

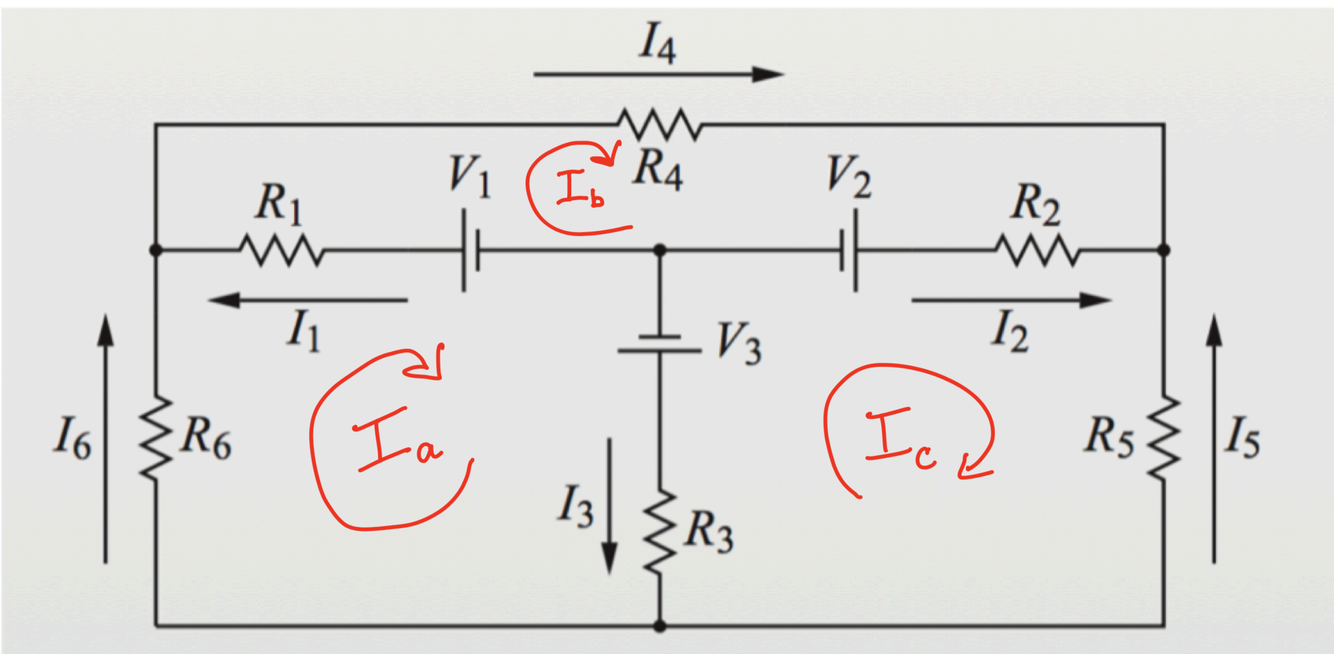
6 $$i_1 = \frac{V}{R} = 0.72\,A$$

Half of i_1 flows through each of the two equivalent $15\,\Omega$ branches, so $i_2 = i_3 = \frac{i_1}{2} = \underline{\underline{0.36\,A}}$

(Also $V_{\parallel} = i_1 R_{\parallel} = 5.4\,V \implies i_2 = i_3 = \frac{V_{\parallel}}{15\,\Omega} = 0.36\,A$)

page 2.5

(5) Compute all the currents labeled in the circuit below, assuming the following values: $V_1 = 10$ V, $V_2 = 6$ V, $V_3 = 12$ V, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 7\ \Omega$, $R_6 = 3\ \Omega$.



(see p. 2.5)

5 Applying the loop rule CW to the 3 inside loops, rearranging & substituting

a) $-I_a R_6 - (I_a - I_b)R_1 - V_1 + V_3 - (I_a - I_c)R_3 = 0$

$\rightarrow (-R_6 - R_1 - R_3) I_a + R_1 I_b + R_3 I_c = V_1 - V_3$

$\rightarrow (-17\Omega) I_a + (4\Omega) I_b + (10\Omega) I_c = -2\,V$ (1)

8 b) $-I_b R_4 - (I_b - I_c) R_2 - V_2 + V_1 - (I_b - I_a) R_1 = 0$

$\rightarrow (4\Omega) I_a + (-11\Omega) I_b + (2\Omega) I_c = -4\,V$ (2)

c) $-(I_c - I_a) R_3 - V_3 + V_2 - (I_c - I_b) R_2 - R_5 I_c = 0$

$\rightarrow (10\Omega) I_a + (2\Omega) I_b + (-19\Omega) I_c = 6\,V$ (3)

Eq'ns (1) to (3) can be written $\begin{pmatrix} -17 & 4 & 10 \\ 4 & -11 & 2 \\ 10 & 2 & -19 \end{pmatrix} \Omega \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ 6 \end{pmatrix} V$

$\mathbb{R}\, \mathbb{I} = \mathbb{V}$

Then $\mathbb{I} = \mathbb{R}^{-1} \mathbb{V} = \begin{pmatrix} 45 \\ 333 \\ -257 \end{pmatrix} mA$ using suitable software or Cramer's rule.

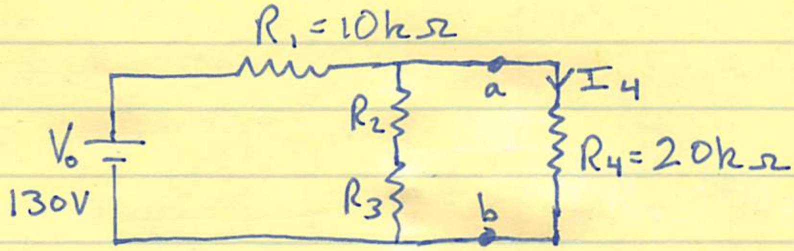
giving for each branch:

$I_1 = I_b - I_a = 288\ mA$ $\quad I_4 = I_b = 333\ mA$

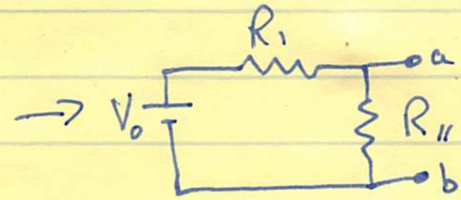
$I_2 = I_c - I_b = -590\ mA$ $\quad I_5 = -I_c = 257\ mA$

$I_3 = I_a - I_c = 302\ mA$ $\quad I_6 = I_a = 45\ mA$

(6) $R_2 = R_3 = 5\,k\Omega$



(a)

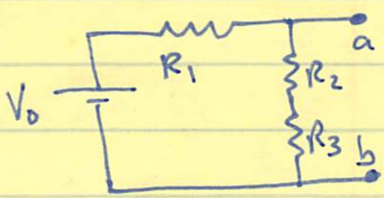


$\rightarrow$ $R_{\parallel} = R_4 \parallel (R_2+R_3) = 6.67\,k\Omega$

$$\Rightarrow V_{ab} = V_0 \frac{R_{\parallel}}{R_{\parallel}+R_1} = 52.0\,V$$

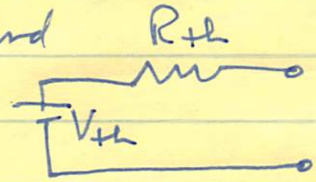
10 $\therefore$ current through R_4 is $I_4 = \dfrac{V_{ab}}{R_4} = \underline{\underline{2.60\,mA}}$

(b) With R_4 removed,



$$V_{ab} = V_{oc} = V_{th} = \frac{V_0\,(R_2+R_3)}{R_1+R_2+R_3} = \frac{V_0}{2} = \underline{\underline{65\,V}}$$

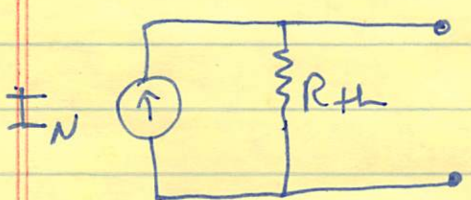
and



$R_{th} = R_1 \parallel (R_2+R_3) = \underline{\underline{5\,k\Omega}}$

(R_{ab} with V_0 shorted)

and



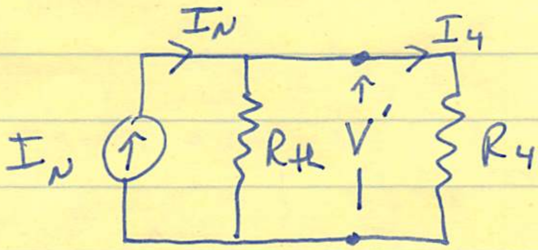
$I_N = \dfrac{V_{th}}{R_{th}} = \underline{\underline{13\,mA}}$

(short circuit current)

c) Thevenin circuit with R_4:

$$I_4 = \frac{V_{th}}{R_{th} + R_4} = \underline{\underline{2.6 \text{ mA}}}$$

Norton circuit with R_4:



$I_4 = \dfrac{V'}{R_4}$ where $V' = I_N \; R_{th} \parallel R_4$

so $I_4 = \dfrac{I_N \; R_{th}}{R_4 + R_{th}} = \underline{\underline{2.6 \text{ mA}}}$

34