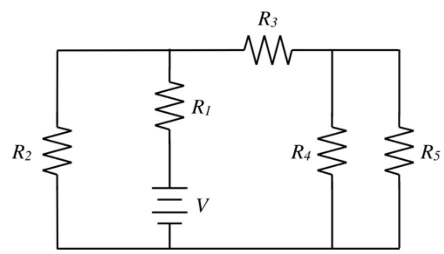
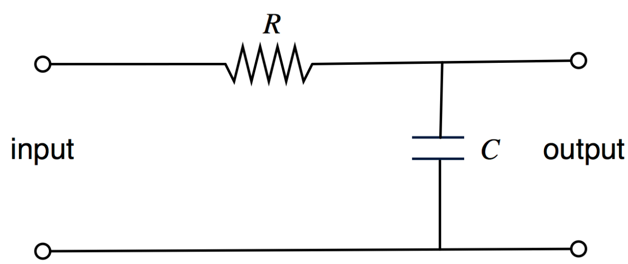
All questions have equal value, except number 5, which has double weight.

Corrected version

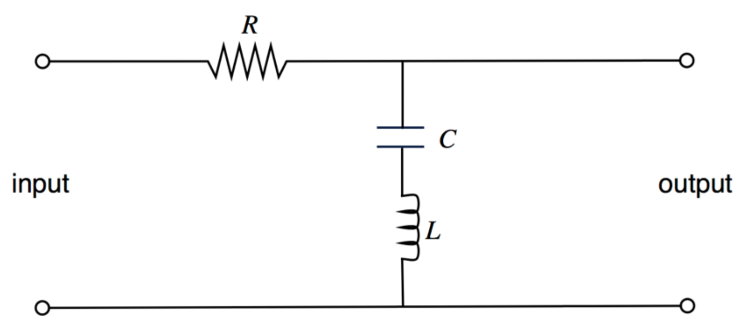
1. For the following circuit find the total current supplied by the battery, and the current through R_4, if $V = 10$ V, $R_1 = 2\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 5\ \Omega$, and $R_5 = 10\ \Omega$.



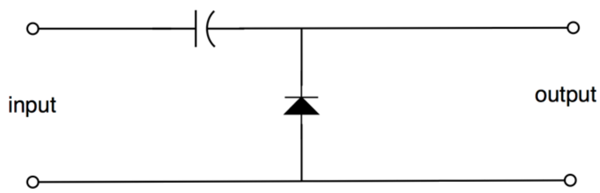
2. In the RC circuit shown below, with $R = 15$ kΩ and $C = 1\ \mu$F, determine the charge on the capacitor 15 ms after the input switches from -5 V to +5 V. What is the limiting output voltage after a long time, and how long will it take to reach 99% of that value? (Assume the input was at -5 V for a long time before it switched.)



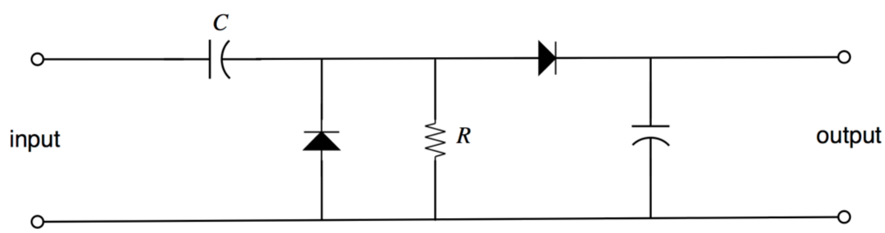
3. For the following circuit, give an expression for the output if the input is given by $v_{in} = V_{in}\cos(\omega t)$. What is the ratio of the output amplitude to the input amplitude at zero frequency (dc), at high frequency, and at resonance frequency?



4. (a) Sketch the output waveform for the following circuit if the input is sinusoidal. Assume the turn-on voltage for the diode is zero.

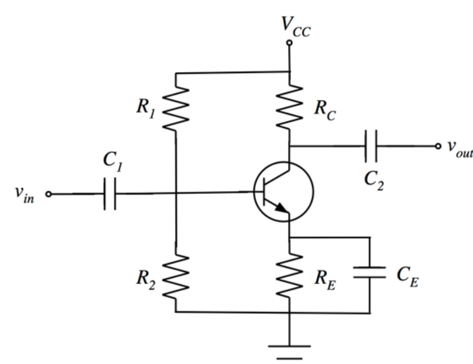


(b) What is the output of the following circuit if the input is sinusoidal with a peak voltage of 2.0 V? Assume the RC time constant is much longer than the period of the input, and both diodes turn on at zero volts.

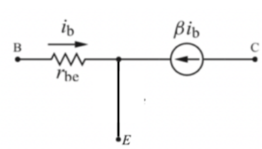


Double value

5. (a) For common emitter amplifier shown below, with $V_{CC} = 20$ V, $R_1 = 35$ kΩ, $R_2 = 2.8$ kΩ, $R_C = 2.25$ kΩ, $R_E = 250$ Ω, and $\beta = 100$, what are V_{CE} and I_C at the operating point?



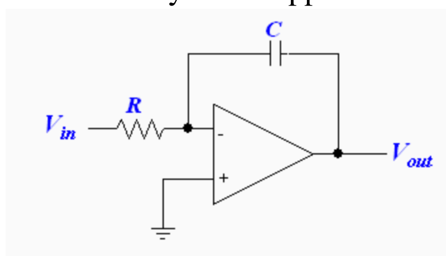
(b) Using the circuit shown for the transistor with $r_{be} = 2000$ Ω, draw an ac-equivalent circuit for the amplifier, and calculate the midband gain, the input impedance, and the output impedance.



(c) What value for C_1 would result in 3 dB decrease in the gain at 20 Hz from its value at midband, not considering attenuation from other parts of the circuit.

(d) If C_E were removed, and the output taken from the emitter, what would the voltage gain be?

6. Show that the output of the circuit below is approximately proportional to the integral of the input, and give the condition for the validity of the approximation.

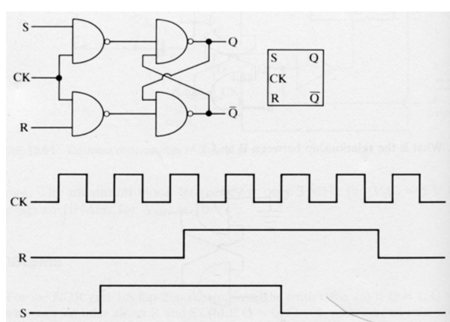


Draw the circuit for a passive integrator, and give the condition for its validity.

7. (a) Draw the schematic diagram for an inverting amplifier with a gain of -75 and in input impedance of 5 kΩ, using an op-amp.

(b) Draw a circuit diagram for a simple inverter (NOT gate) using one npn transistor and resistors. Practical gates add additional stages for more speed and to reduce the output impedance. Explain what limits the switching speed and output impedance of your single transistor gate.

8. (a) For the clocked RS flip-flop shown, with $Q = 0, \bar{Q} = 1$, sketch Q for the CK, R, and S inputs shown. If R is held at 0, sketch Q for the CK and S inputs shown



(b) Use a truth table to prove $\overline{A+B} = \bar{A} \cdot \bar{B}$

9. (a) A half-adder takes two inputs (A, B) and provides two outputs $(A \oplus B, A \cdot B)$. Show how to implement a half adder using only NAND gates.

(b) A full adder takes three inputs (A, B, C) and provides two outputs $(A \oplus B \oplus C, A \cdot B + A \oplus B \cdot C)$. Show how to implement a full adder using two half-adders and NAND gates. Can you make a full adder with 9 NAND gates?

The End

PHYS 2610: Final Exam Formula Sheet 2017

Current: $i = \frac{dq}{dt} = \int \mathbf{J} \cdot \overrightarrow{da}$

Steady state: $\frac{di}{dt} = 0; \oint \mathbf{J} \cdot \overrightarrow{da}$

Ohm's law: $\mathbf{J} = \sigma\mathbf{E} = \frac{\mathbf{E}}{\rho} \Rightarrow v = iR$ with $R = \rho\ell/A$

Current density: $\mathbf{J} = ne\vec{v}_d$

Gauss's law: $\oint \mathbf{E} \cdot \overrightarrow{da} = q_{net}/\varepsilon_0$

Electric potential and potential energy: $V = U/q;\ dU = qdV$

Potential difference and emf: $\int_a^b \mathbf{E} \cdot \overrightarrow{dl} = -(V_b - V_a);\ \oint \mathbf{E} \cdot \overrightarrow{dl} = 0$

Power: $P = vi$

Capacitor: $q = CV,\ U = q^2/(2C)$

Solution to $\frac{dy}{dx} + ax = b$ has the form $y = Ae^{-ax} + b/a$

Faraday's law: $\mathcal{E}_{ind} = \int_a^b \mathbf{E} \cdot \overrightarrow{dl} = -\frac{d}{dt}\int \mathbf{B} \cdot \overrightarrow{da} = -L\frac{di}{dt}$

Inductor: $\mathcal{E} = L\frac{di}{dt}$

Magnetic field of ideal solenoid: $B = \mu_0 nI$

Euler's formula: $e^{j\theta} = \cos\theta + j\sin\theta$

Complex impedance: $Z = R + jX = |Z|e^{j\phi}$; $\tilde{v} = Z\tilde{\imath}$; $v = \mathrm{Re}(\tilde{v}) = V cos\omega t$

Capacitive impedance: $Z_C = -jX_C = \frac{1}{j\omega C}$

Inductive impedance: $Z_L = jX_L = j\omega L$

Series impedance: $Z = \sum Z_i$

Parallel impedance: $\frac{1}{Z} = \sum \frac{1}{Z_i}$

Voltage gain: $a = \frac{v_{out}}{v_{in}}$

Gain in dB: $G_{dB} = 20\log\left|\frac{v_2}{v_1}\right|$

Q Factor: $Q = \omega_0 L/R$

Schockley diode equation: $I = I_s(e^{eV/\eta kT}$-1); η is the ideality factor ~ 2 for Si

Bipolar transistor current gains: $\alpha = \frac{I_C}{I_E}$; $\beta = \frac{I_C}{I_B}$

DeMorgan's theorems: $\overline{A+B} = \bar{A}\cdot\bar{B}$; $\overline{A\cdot B} = \bar{A}+\bar{B}$; $A\cdot B = \overline{\bar{A}+\bar{B}}$; $A+B = \overline{\bar{A}\cdot\bar{B}}$

Half adder: $S = A\oplus B$; $C = A\cdot B$

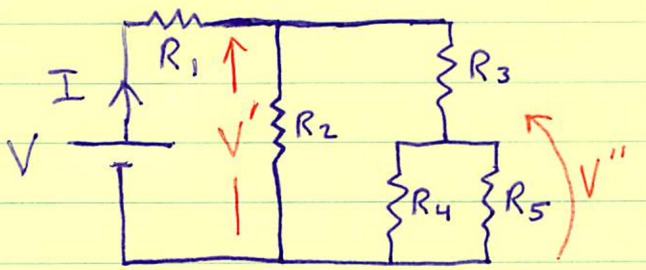
Full adder: $S_n = A_n \oplus B_n \oplus C_{n-1}$; $C_n = A_n\cdot B_n + C_{n-1}\cdot(A_n\oplus B_n)$

Ones' complement: complement each bit

Two's complement: one's complement plus 1

Phys 2610 (2018)   Final Exam Solutions

① equivalent circuit:



$R_1 = 2\,\Omega$ $V = 10\,V$
$R_2 = 5\,\Omega$
$R_3 = 2\,\Omega$
$R_4 = 5\,\Omega$
$R_5 = 10\,\Omega$

Equivalent resistance: $R = R_1 + R_2 // (R_3 + R_4 // R_5)$

$R_4 // R_5 = (5)(10)/(5+10)\,\Omega = 3.33\,\Omega$

$R_3 + R_4 // R_5 = 2\,\Omega + 3.33\,\Omega = 5.33\,\Omega$

$R_2 // (R_3 + R_4 // R_5) = (5)(5.33)/(5+5.33)\,\Omega = 2.58\,\Omega$

$R = 2\,\Omega + 2.58\,\Omega = 4.58\,\Omega$

Current supplied by battery: $I = \frac{V}{R} = \frac{10\,V}{4.58\,\Omega} = \boxed{2.18\,A}$

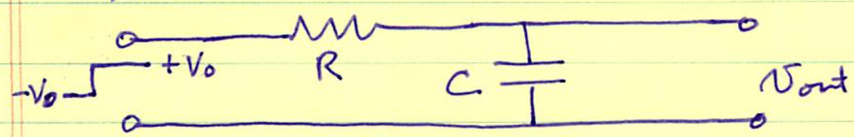
Then $V' = V - IR_1 = 10\,V - (2.18\,A)(2\,\Omega) = 5.64\,V$

and $V'' = \frac{V'(R_4 // R_5)}{R_3 + R_4 // R_5} = \frac{5.64\,V(3.33\,\Omega)}{5.33\,\Omega} = 3.52\,V$

so $I_4 = V''/R_4 = 3.52\,V/5\,\Omega = \boxed{0.705\,A}$

$V_0 = 5V$

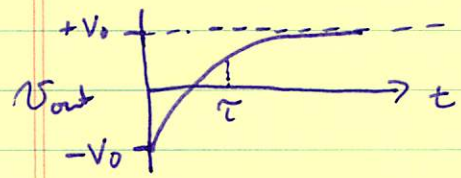
2



$R = 15k\Omega$, $C = 1\mu F$

$\Rightarrow \tau = RC = 15\,ms$

For const. v_{in}, after a long time, $v_{out} = v_{in}$, so v_{out} starts at $-V_0$ and approaches $+V_0$ exponentially:



$\Rightarrow \quad v_{out} = 2V_0(1-e^{-t/\tau}) - V_0$

(by inspection)

or $\quad \boxed{v_{out} = V_0(1-2e^{-t/\tau})}$ *

(i) For $t = 15\,ms\,(=\tau)$, $v_{out} = V_0(1-e^{-1}) = 0.264\,V_0$

$\rightarrow q = v_{out}C = 0.264(5V)(1\mu F) = \boxed{1.32\mu C}$

(ii) For $t \to \infty$, $\boxed{v_0 \to V_0 = 5V}$

(iii) For $v_0 = 0.99V_0$, $1-2e^{-t/\tau} = 0.99 \rightarrow 2e^{-t/\tau} = 0.01$

$\rightarrow -t/\tau = \ln(.005) \rightarrow t = -\tau\ln(.005) = \boxed{79.5\,ms}$

(To complete 99% of the $-V_0 \rightarrow +V_0$ transition, $1-e^{-t/\tau} = 0.99$
$\Rightarrow t = -\tau\ln(.01) = 69.1\,ms$)

---

* From KVL, $V_0 - iR - q/C = 0 \Rightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{V_0}{R}$

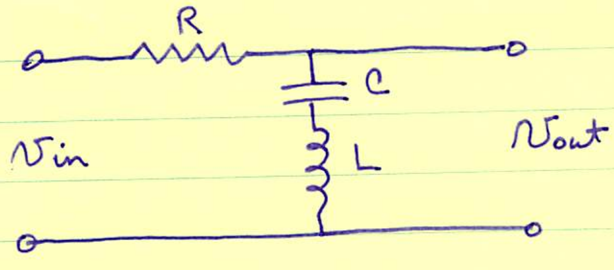
$\Rightarrow q = Ae^{-t/RC} + V_0C$

For $t=0$, $q = -V_0C \Rightarrow A = -2V_0C$, so

$q = V_0C - 2V_0Ce^{-t/\tau} = V_0C(1-2e^{-t/\tau})$

or $\quad \boxed{v_{out} = V_0(1-2e^{-t/\tau})}$

3



Voltage divider: $v_{out} = \dfrac{v_{in} Z}{R+Z}$ where $Z = \dfrac{1}{j\omega C} + j\omega L$

$= j(\omega L - 1/\omega C)$

The ratio of amplitudes is then

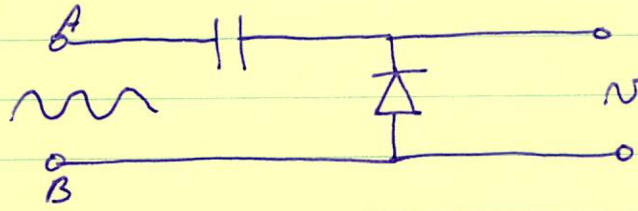
$$\frac{V_{out}}{V_{in}} = \frac{|v_{out}|}{|v_{in}|} = \frac{|Z|}{|R+Z|} = \frac{(\omega L - 1/\omega C)}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

For $\omega = 0$, $\dfrac{V_{out}}{V_{in}} \rightarrow \dfrac{1/\omega C}{1/\omega C} \rightarrow \boxed{1}$ (C presents ∞ impedance)

For $\omega \rightarrow \infty$, $\dfrac{V_{out}}{V_{in}} \rightarrow \dfrac{\omega L}{\omega L} \rightarrow \boxed{1}$ (L presents ∞ impedance)

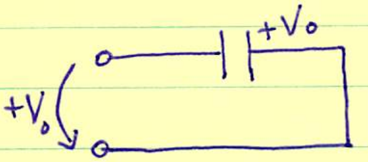
For $\omega = 1/\sqrt{LC}$, $\dfrac{V_{out}}{V_{in}} = \boxed{0}$ (LC is short cct)

(4) (a)

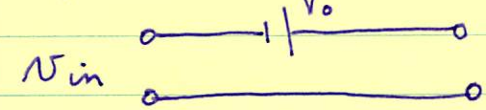


$v_{in} = V_0 \cos\omega t$

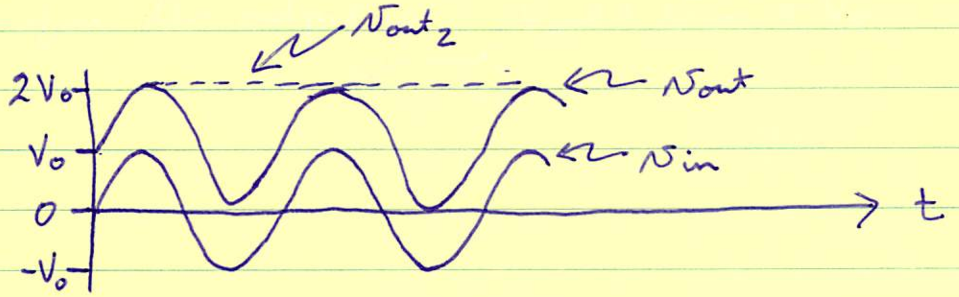
When B first goes positive w.r.t. A the diode closes and the capacitor charges to the peak voltage.



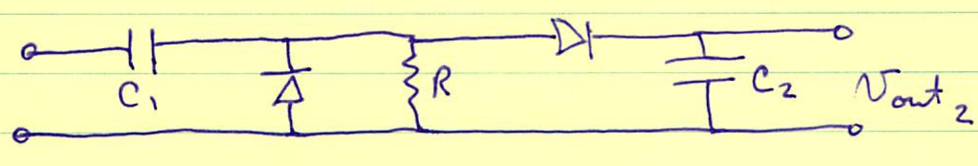
When B decreases, the diode opens and the capacitor has no path to discharge, so it maintains the voltage V_0, acting like a battery. Subsequently, the diode only opens when v_{in} reaches $-V_0$ (or slightly before if the capacitor leaks a small amount of charge).



$\Longrightarrow \quad v_{out} = v_{in} + V_0$



(b)



The second diode is a closed switch until C_2 charges to the maximum output of the first stage, which is $2V_0$ from part (a) (assuming C_1 does not discharge appreciably through R). C_2 has no path to discharge and so holds the voltage $2V_0 = 4V$.

5 (a) $V_B = \dfrac{V_{CC} R_2}{R_1 + R_2} = 1.48\,V$ assuming I_B is small

$\Rightarrow V_E = V_B - V_{BE} = V_B - 0.6V = 0.88\,V$

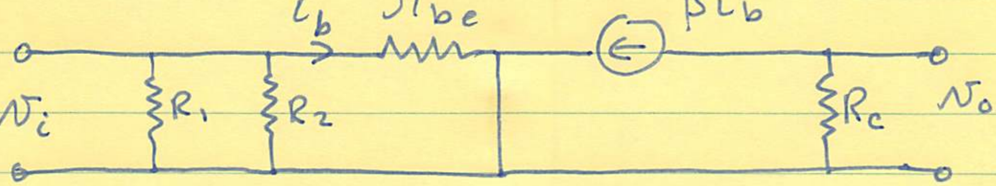
$\Rightarrow I_E = I_C = V_E / R_E = \boxed{3.52\,A}$

but

$I_C = \dfrac{(V_{CC} - V_{CE})}{R_C + R_E} \Longrightarrow V_{CE} = -I_C(R_C + R_E) + V_{CC}$

$= \boxed{11.2V}$

(b)



$r_{be} = 1000\,\Omega$

(i) gain $a_v = \dfrac{v_o}{v_i} = \dfrac{-\beta i_b R_c}{i_b r_{be}} = -\dfrac{\beta R_c}{r_{be}} = -112$

(ii) input impedance: $r_{in} = R_1 // R_2 // r_{be}$

$= \left(\dfrac{1}{2.8k\Omega} + \dfrac{1}{35k\Omega} + \dfrac{1}{2k\Omega}\right)^{-1} = \boxed{1.1k\Omega}$

(iii) output impedance:

$r_{out} = \dfrac{v_{out}(\text{open})}{i_{out}(\text{short})} = \dfrac{\beta i_b R_c}{\beta i_b} = R_C = \boxed{2.25k\Omega}$

(c) C_1 forms a high pass filter with r_{in}.

For 3dB attenuation at 20 Hz due to this filter:

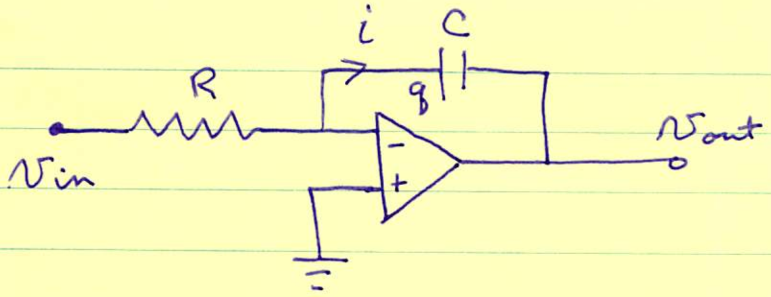
$$\omega_{3dB} = \frac{1}{r_{in} C_1} \Rightarrow C_1 = \frac{1}{2\pi f_{3dB}\, r_{in}} = \boxed{7.2\mu F}$$

(d) The gain for an emitter follower is $\boxed{\text{unity}}$, since

$V_E = V_B - V_{BE}$ and V_{BE} is ~ constant

$\rightarrow v_e = v_b$ or $v_{out} = v_{in}$

6



Since $v_- = v_+$, $v_- = 0$ (virtual ground)

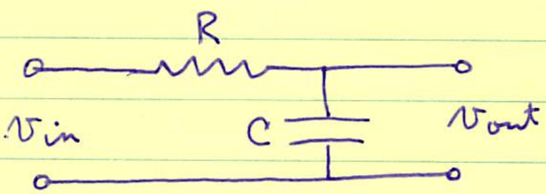
Then $v_{out} = -q/C = -\frac{1}{C}\int i\,dt$

But $v_{in} = iR$ so $\boxed{v_{out} = \frac{-1}{RC}\int v_{in}\,dt}$

This is valid for $|a_0| \gg |a|$ or $a_0 \gg \frac{1/\omega C}{R}$

or $\boxed{\omega RC \gg 1/a_0}$

A passive integrator is



Here $v_{out} = q/C = \frac{1}{C}\int i\,dt$

and $v_{in} - iR = v_{out} \Rightarrow i = \frac{1}{R}(v_{in} - v_{out})$.

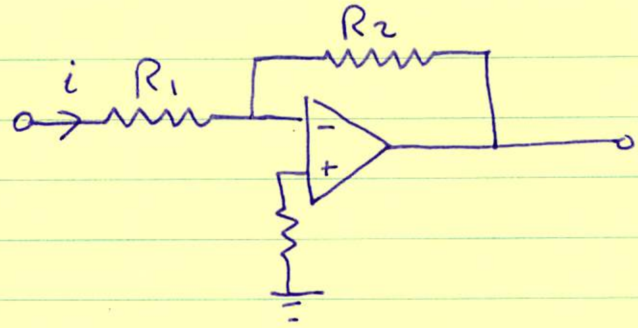
Then for $v_{out} \ll v_{in}$,

$$i = \frac{1}{RC}\int v_{in}\,dt$$

The condition $v_{out} \ll v_{in} \Rightarrow 1/\omega C \ll R$

$$\Rightarrow \boxed{\omega RC \gg 1}$$

7 (a)

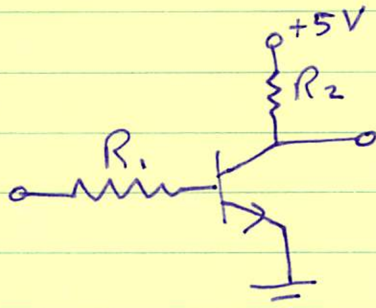


Input impedance: $R_{in} = \frac{v_{in}}{i} = R_1$ since $v_- = v_+ = 0$

$\rightarrow \boxed{R_1 = 5\,k\Omega}$

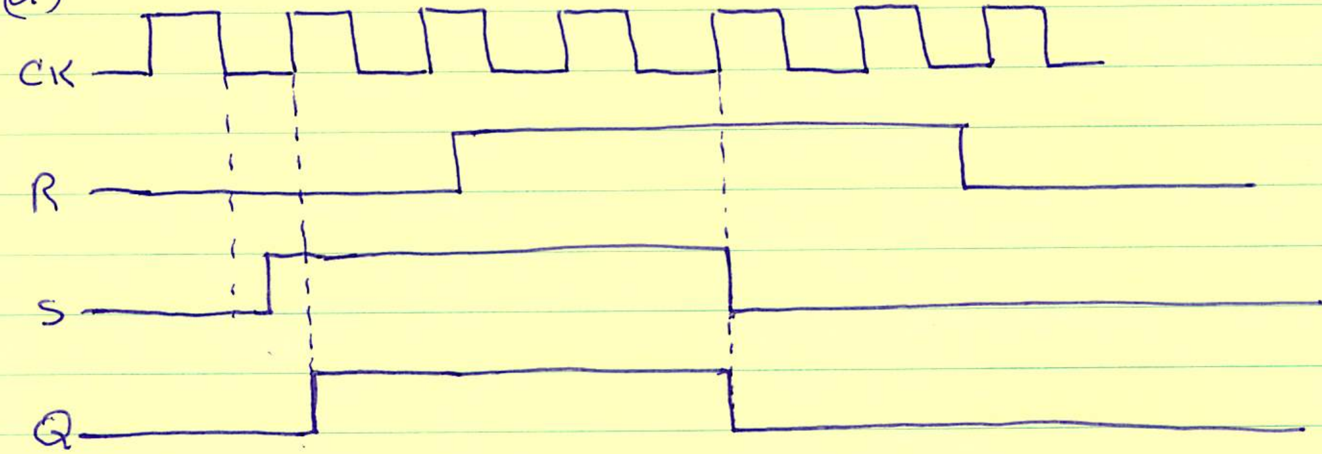
Gain: $a = \frac{-R_2}{R_1} \rightarrow R_2 = -aR_1 = -(-75)(5\,k\Omega) = \boxed{375\,k\Omega}$

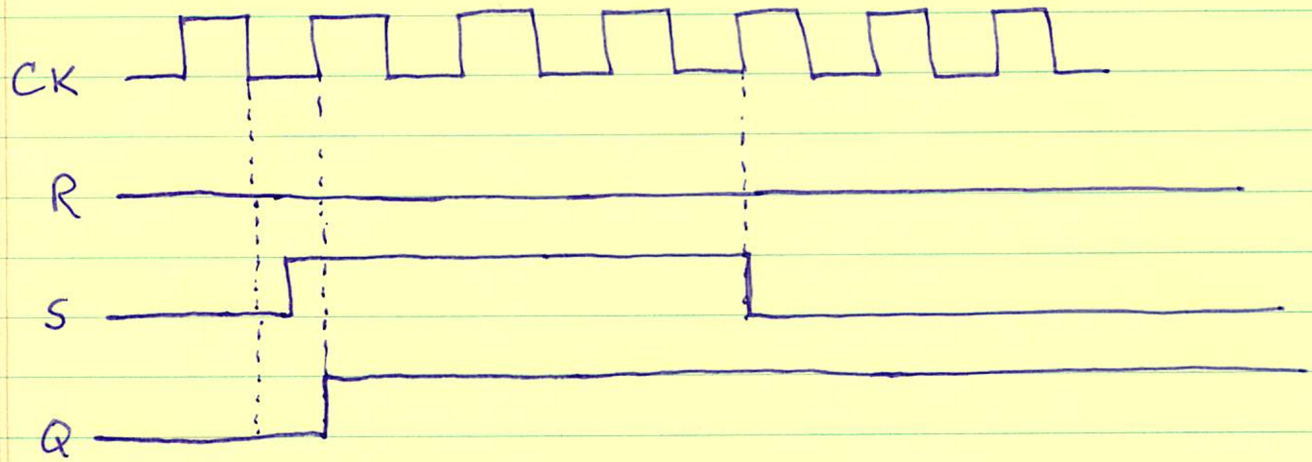
(b) Inverter:



R2 is needed to limit the current when the transistor is on, but it limits the speed because the effective capacitance of the transistor must charge through R2 when it turns off. R2 is also the output impedance which limits the load that it can drive.

8 (a)





(b)

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

$\therefore \overline{A+B} = \overline{A}\cdot\overline{B}$

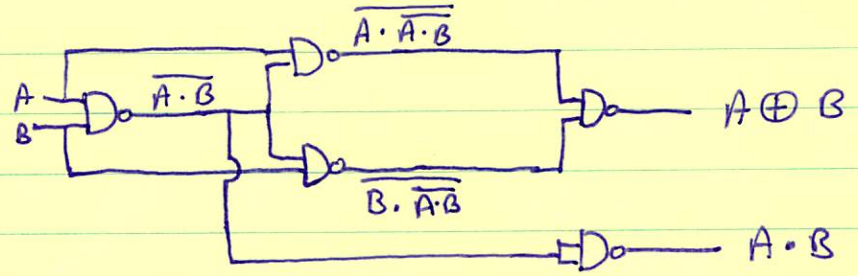
9 (a) $A \oplus B = (A+B) \cdot \overline{A \cdot B}$

$= A \cdot \overline{A \cdot B} + B \cdot \overline{A \cdot B}$ distributive prop.

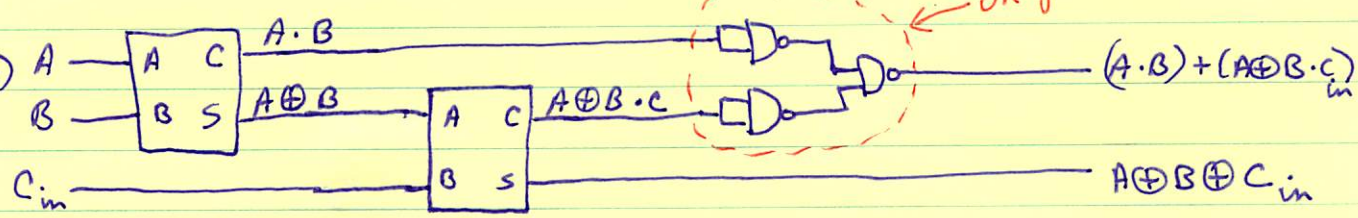
$= \overline{\overline{A \cdot \overline{A \cdot B} + B \cdot \overline{A \cdot B}}}$ $\overline{\overline{A}} = A$

$= \overline{\overline{A \cdot \overline{A \cdot B}} \cdot \overline{B \cdot \overline{A \cdot B}}}$ De Morgan

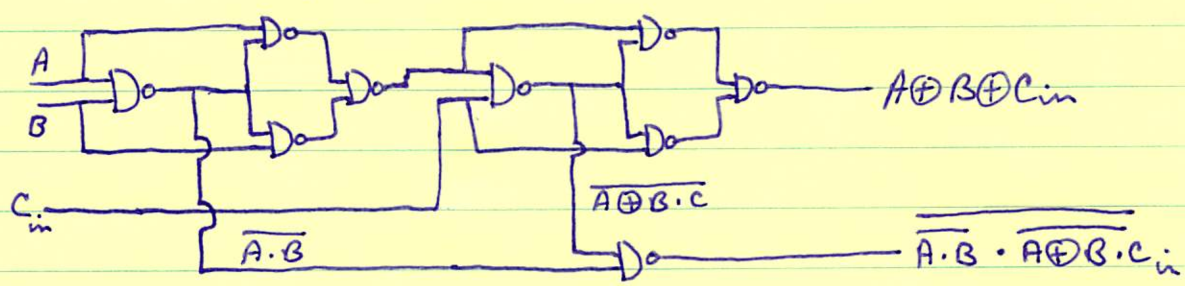
Then



(b)



Adder with 9 gates



$= A \cdot B + A \oplus B \cdot C_{in}$