UNIVERSITY OF MANITOBA

March 4, 2019 | MIDTERM EXAM

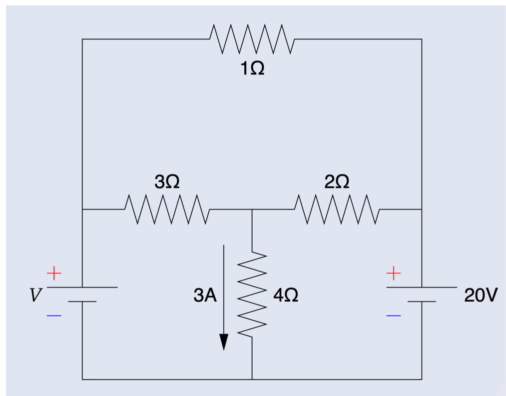
PAGE NO.: 1

DEPARTMENT & COURSE NO.: PHYS 2610 | TIME: 1.5 hours

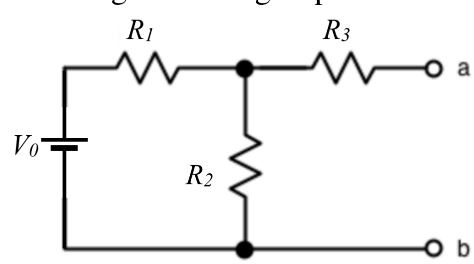
EXAMINATION: Circuit Theory and Introductory Electronics | EXAMINER: W Ens

All questions have equal value.

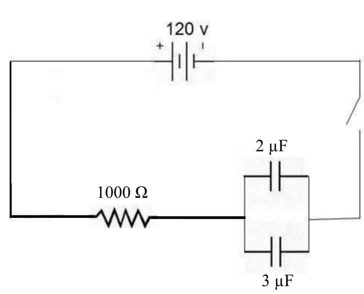
1. For the circuit shown below, with the current through the 4 Ω resistor equal to 3 A (as indicated), find (a) the current through each of the other resistors, (b) the voltage of the battery on the left, and (c) the power delivered by the 20 V battery.



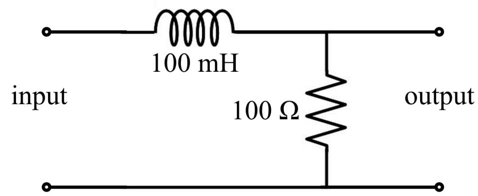
2. Using $R_1 = 60\ \Omega$, $R_2 = 150\ \Omega$, $R_3 = 90\ \Omega$, and $V_0 = 10$ V, for the circuit shown below, find the values for Thevenin and Norton equivalent circuits with respect to the terminals a and b. What load resistance connected to the output terminals would give the largest power transfer?



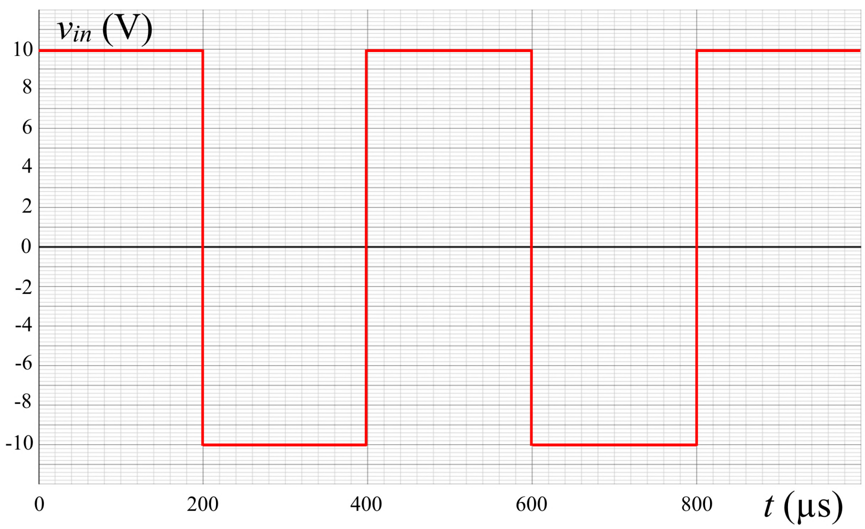
3. For the RC circuit below, determine the current delivered by the battery 2 ms after the switch closes. Find the charge on each of the capacitors after equilibrium is reached with the switch closed.



4. For the RL circuit below, sketch the output waveform for each of the two input waveforms shown.

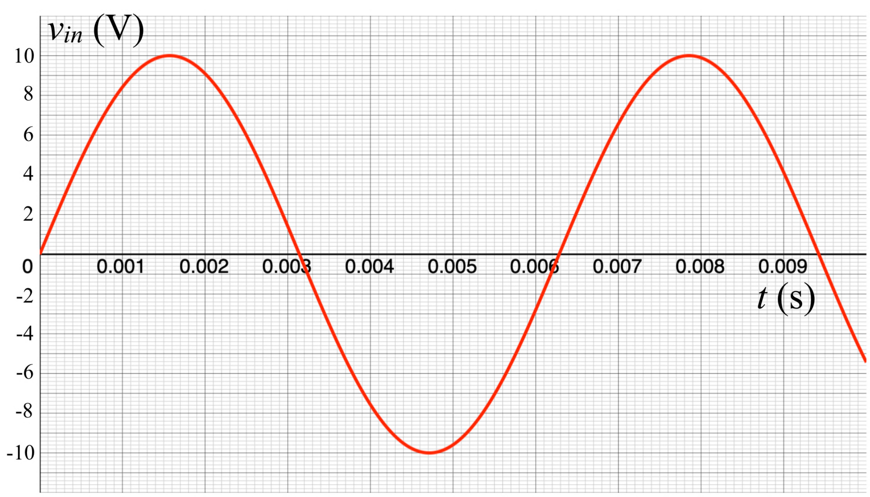


(a) A square wave with an amplitude of 10 V and a period of 400 μs.



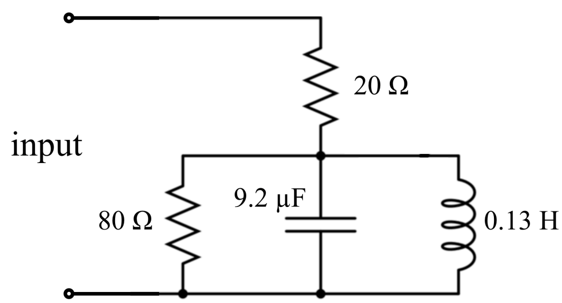
(b) A sine wave with an amplitude of 10 V and an angular frequency of 1000 rad/s:

$$v_{in} = 10V \sin\left(1000\ s^{-1} t\right)$$



Make your sketches to scale directly on the diagrams above, and don't forget to hand in this question sheet.

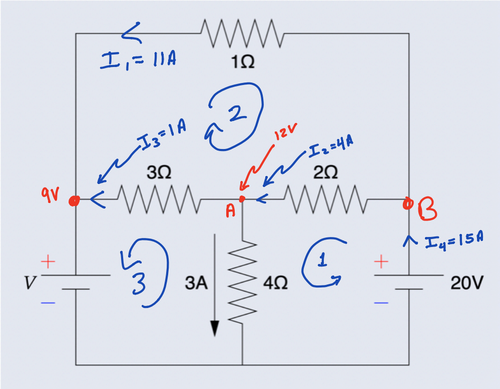
5. Find the magnitude and phase of the impedance across the input for the circuit below. If a sinusoidal input is connected with 160 V amplitude and 250 Hz frequency, find the voltage signal across the 20-Ω resistor, including the phase with respect to the input.



6. A 10-Ω resistor, 10-mH inductor, and 10-μF capacitor are connected in series with a 10-kHz sinusoidal voltage source. The rms current through the circuit is 0.20 A. Find the rms voltage drop across each of the 3 elements. What would the rms current be at resonance, if the input rms voltage were the same?

Phys 2610 (2019) Midterm solutions

1. For the circuit shown below, with the current through the 4 Ω resistor equal to 3 A (as indicated), find (a) the current through each of the other resistors, (b) the voltage of the battery on the left, and (c) the power delivered by the 20 V battery.



(a)

Loop 1 : $20V - I_2(2\Omega) - (3A)(4\Omega) = 0 \Rightarrow I_2 = \frac{20V - 12V}{2\Omega} \Rightarrow \boxed{I_2 = 4A}$

Jctn A : $I_2 = I_3 + 3A \Rightarrow I_3 = I_2 - 3A = 1A \Rightarrow \boxed{I_3 = 1A}$

Loop 2 : $-I_3\, 3\Omega + I_1\, 1\Omega - I_2\, 2\Omega = 0 \Rightarrow I_1 = \frac{I_3\, 3\Omega + I_2\, 2\Omega}{1\Omega} = \frac{3V + 8V}{1\Omega} = 11A$

$\Rightarrow \boxed{I_1 = 11A}$

(b)

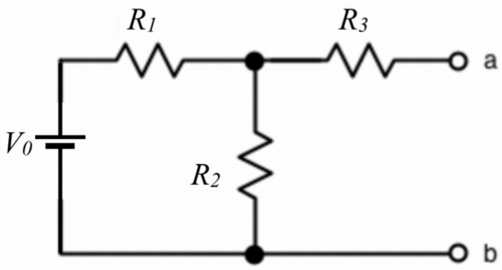
Loop 3 : $(3A)(4\Omega) - I_3(3\Omega) - V = 0 \Rightarrow V = 12V - (1A)(3\Omega) = 12V - 3V = 9V$

$\Rightarrow \boxed{V = 9V}$

(c)

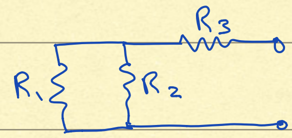
Jctn B: $I_4 = I_2 + I_1 = 15A \rightarrow P = VI = (20V)(15A) \Rightarrow \boxed{P = 300W}$

2. Using $R_1 = 60\ \Omega$, $R_2 = 150\ \Omega$, $R_3 = 90\ \Omega$, and $V_0 = 10$ V, for the circuit shown below, find the values for Thevenin and Norton equivalent circuits with respect to the terminals a and b. What load resistance connected to the output terminals would give the largest power transfer?



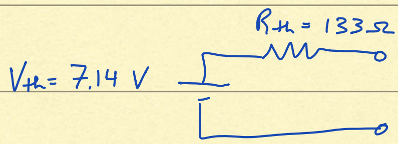
Thevenin: V_{th} = open cct voltage = $V_0 \dfrac{R_2}{R_1+R_2} = (10V)\left(\dfrac{150}{150+60}\right) = \boxed{7.14V}$

R_{th} = resistance with V_0 shorted.



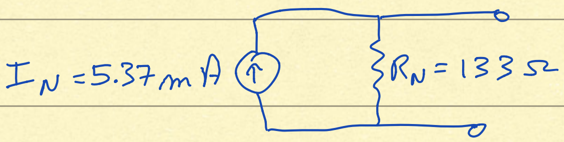
$= R_3 + R_1 // R_2 = 90\Omega + 150\Omega // 60\Omega$

$= 90\Omega + \dfrac{(150)(60)}{(150+60)}\Omega = 133\Omega$ $\boxed{R_{th} = 133\Omega}$



Norton: $\boxed{R_N = R_{th} = 133\Omega}$

I_N = closed cct current = $\dfrac{V_{th}}{R_{th}} = \dfrac{7.14}{133}$ A = 53.7 mA $\boxed{I_N = 53.7\ mA}$



Max power transfer: $\boxed{R_L = R_{th} = 133\Omega}$

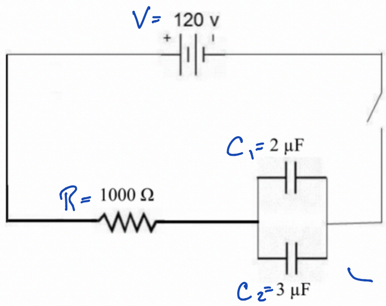
3. For the RC circuit below, determine the current delivered by the battery 2 ms after the switch closes. Find the charge on each of the capacitors after equilibrium is reached with the switch closed.

$V - iR - q/C = 0 \implies \frac{di}{dt}R + \frac{i}{C} = 0$

$\implies \frac{di}{dt} + \frac{i}{R} = 0 \implies i = Ae^{-t/RC}$

For $t=0$, $i = \frac{V}{R} \implies A = \frac{V}{R} \implies \boxed{i = \frac{V}{R} e^{-t/RC}}$

or $i = 120\text{mA}\, e^{-t/5\text{ms}}$



$C = 2\mu F + 3\mu F = 5\mu F$

or by inspection $i = \frac{V}{R} e^{-t/\tau}$, where $\tau = RC = 1000\Omega\ 5\mu F = 5\text{ ms}$

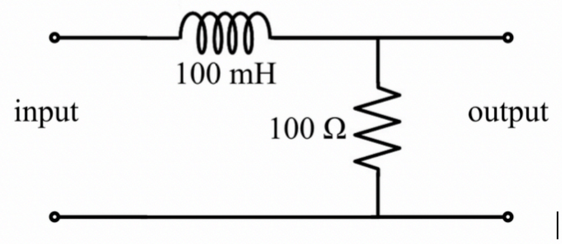
At $t = 2\text{ms}$, $i = \frac{V}{R} e^{-2/5} = 120\text{ mA}\ e^{-2/5} = \boxed{80.4\text{ mA}}$

At $t \to \infty$, $i \to 0$. $\therefore V_C = V_{C_1} = V_{C_2} = V = 120\text{ V}$

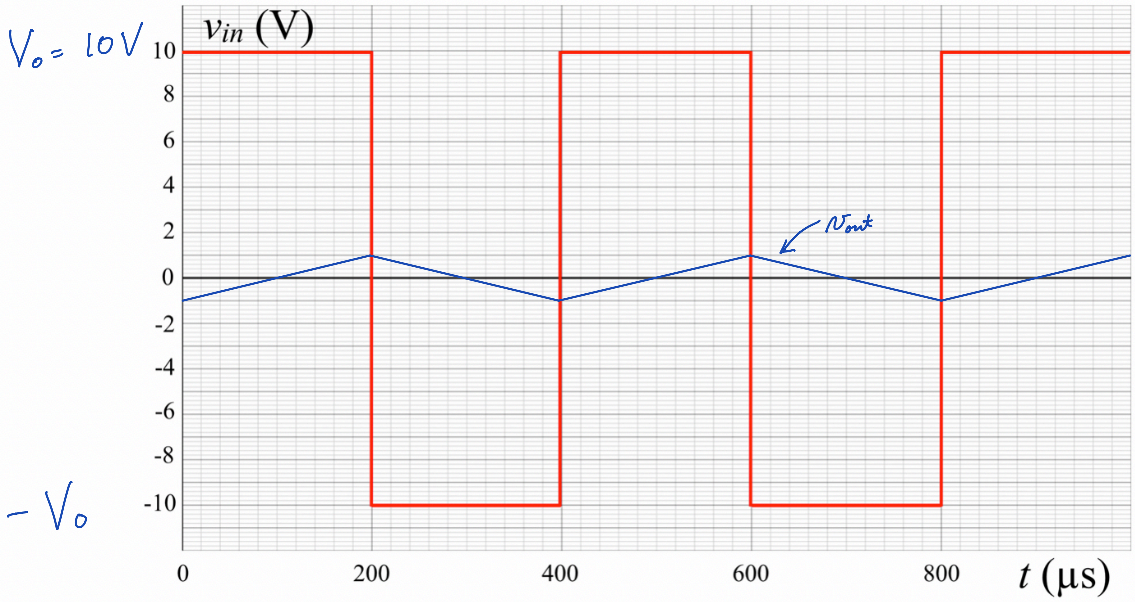
$\implies q_1 = V_{C_1} C_1 = 120\text{V}\ (2\mu F) = \boxed{240\ \mu C}$

$q_2 = V_{C_2} C_2 = 120\text{V}\ (3\mu F) = \boxed{360\ \mu C}$

4. For the RL circuit below, sketch the output waveform for each of the two input waveforms shown.



(a) A square wave with an amplitude of 10 V and a period of 400 μs.



(0.2τ)

For a step voltage from zero to $\pm V_o$, as $t \to \infty$, $v_{out} \to \pm V_o$:

$$v_{out} = \pm V_o (1 - e^{-t/\tau}) \quad \text{with} \quad \tau = L/R \qquad [1]$$

Here $\tau = \frac{L}{R} = 1ms$, so $t_{max} = 200\mu s << \tau$.

For $t << \tau$, $|v_{out}| << V_o$ (so the step is always from near zero to $\pm V_o$)

Then $v_{out} \cong \pm V_o (1 - (1 - t/\tau)) = \pm V_o t/\tau$ [2]

This is a straight line with slope $\pm V_o/\tau = \frac{\pm 10V}{1ms} = \frac{\pm 2V}{200\mu s}$

After many cycles v_{out} is symmetric about zero, so the result is a triangular wave for $-1V \to +1V \to -1V$ ... [3]

Notes for Question 4 b

[1] This result can be obtained by inspection, or from

$$\pm V_0 - L\frac{di}{dt} - iR = 0 \longrightarrow \frac{di}{dt} + \frac{iR}{L} = \pm\frac{V_0}{L} \longrightarrow i = A_1 e^{-t/\tau} \pm \frac{V_0}{R}$$

$$\longrightarrow v_{out} = iR = A e^{-t/\tau} \pm V_0 \quad *$$

If $v_{out} = 0$ at $t = 0$, $0 = A \pm V_0 \rightarrow A = \mp V_0$ so $v_{out} = \pm V_0 (1 - e^{-t/\tau})$

Since v_{out} is not exactly zero when the input steps to $\pm V_0$ (at $t = 0$), better boundary conditions on $*$ are:

For a symmetric output, $v_{out}(t=0) = -v_{out}(t = 0.2\tau)$

$$\longrightarrow A \pm V_0 = -(0.82A \pm V_0)$$

$$\longrightarrow A(1 + .82) = \mp 2V_0 \longrightarrow A = \mp 1.1\, V_0$$

Then $v_{out} = \pm V_0 (1 - 1.1 e^{-t/\tau})$

and $v_{out}(t=0) = -v_{out}(t = 0.2\tau) = \pm V_0 (1 - 1.1) = \pm V_0(-.1) = \mp 1V$

again giving triangular wave: $-1V \rightarrow +1V \rightarrow -1V \cdots$

[2] This result is also obtained by recognizing that the circuit integrates if $v_{out} \ll v_{in}$: $v_{in} - L\frac{di}{dt} = v_{out} \Longrightarrow i = \frac{1}{L}\int (v_{in} - v_{out})\, dt \cong \frac{1}{L}\int v_{in}\, dt$

(for $v_{out} \ll v_{in}$)

$$\Longrightarrow v_{out} = iR \cong \frac{R}{L}\int v_{in}\, dt = \frac{1}{\tau}\int \pm V_0\, dt = \frac{1}{\tau}(\pm V_0 t + \text{const.})$$

$$= \pm V_0\left(\frac{t}{\tau}\right) + \text{const.}$$ (Here, to give a symmetric result, const $= -1V$)

[3] If a symmetric output is not assumed, and $v_{out}(t=0)=0$ is taken for first half-cycle, then $v_{out} = V_o(1-e^{-t/\tau})$ applies and $v_{out}(t=200\mu s) = 2V$.

For the second half-cycle, $v_{out}(t=0) = 2V$, so * becomes

$$2V = A - V_o \Rightarrow A = 12V \quad \text{so} \quad v_{out} = 12V e^{-t/\tau} - V_o$$

$$\text{At } t = 200\mu s, \quad v_{out}(200\mu s) = 12V e^{-.2} - 10V = -0.18V$$

For the third half-cycle, $v_{out}(t=0) = -0.18V$, so * becomes

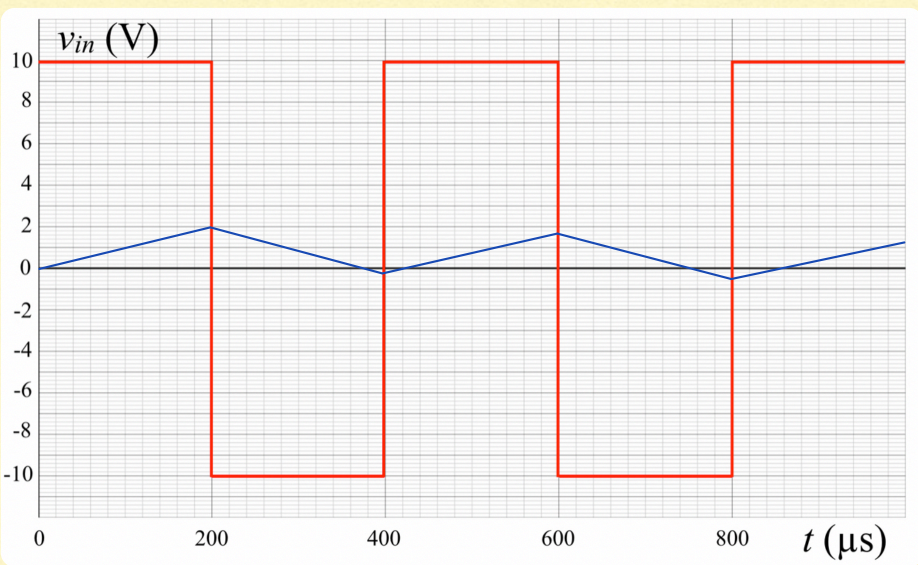
$$-0.18V = A + V_o \Rightarrow A = -10.18V \Rightarrow v_{out} = -10.18V e^{-t/\tau} + V_o$$

$$\text{At } t = 200\mu s, \quad v_{out}(200\mu s) = -10.18V e^{-.2} + 10V = 1.67V$$

For the fourth, $v_{out}(200\mu s) = -0.45V$

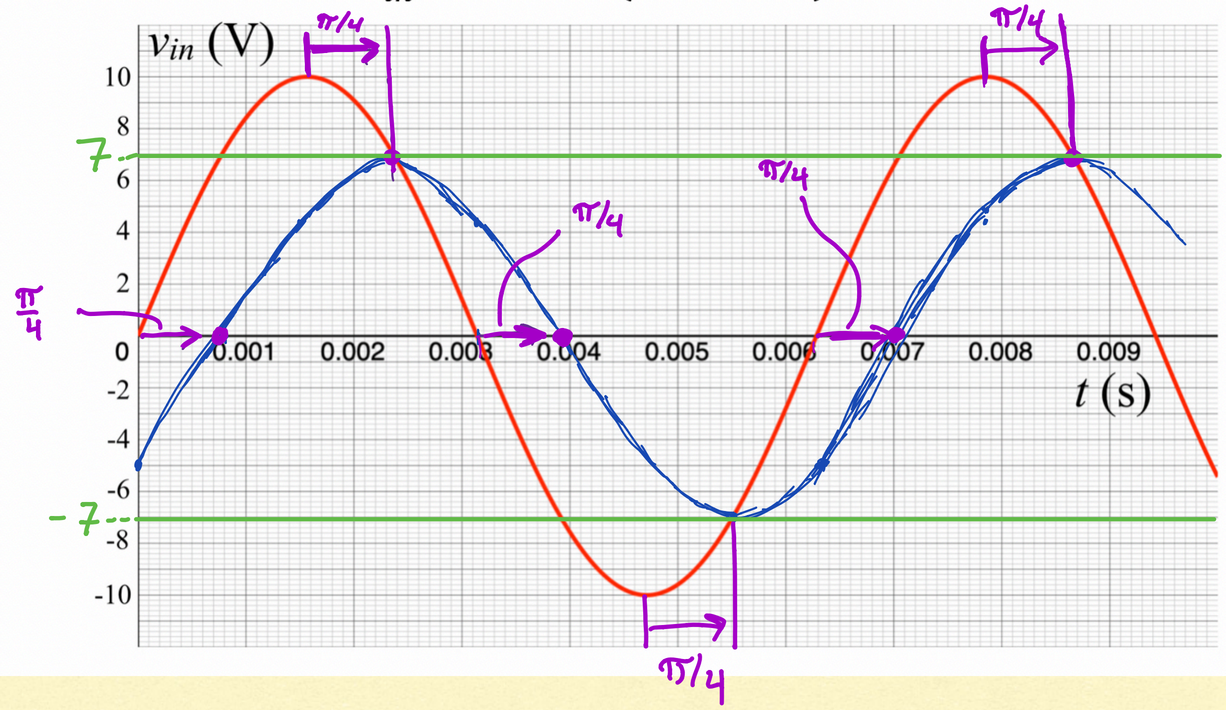
fifth, $v_{out}(200\mu s) = 1.3V$

(The results converge to $\pm 1V$ after about 20 cycles.)



(b) A sine wave with an amplitude of 10 V and an angular frequency of 1000 rad/s:

$$v_{in} = 10V \sin(1000\, s^{-1} t)$$



Here $\omega = 1000 \text{ rad/s} = \frac{1}{\tau} = \frac{R}{L} = \omega_B \longrightarrow$ 3dB attenuation, phase shift $= -\pi/4$

$\rightarrow v_{out} = \frac{V_{in}}{\sqrt{2}} \sin(\omega t - \pi/4) = (7V) \sin((1000 s^{-1}) t - \pi/4)$ [1]

Neg. phase shift corresponds to a shift to the right $\therefore$ peaks + zero crossings shift $\pi/4$ to the right $\rightarrow \Delta t \cong .0008$ s (Note: for $t=0$, $v_o = 7V \sin(-\pi/4) = -5V$)

[1] For sinusoidal input v_{in}:

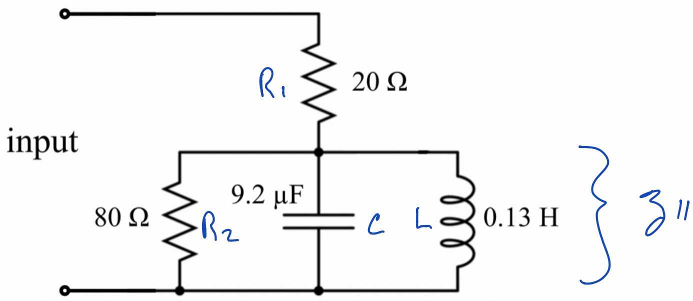
$v_{out} = iR = \frac{v_{in}}{z} R$ with $z = R + j\omega L = \sqrt{R^2 + (\omega L)^2}\, e^{j\theta}$ and $\tan\theta = \frac{\omega L}{R}$

$\rightarrow v_{out} = \frac{v_{in} R}{\sqrt{R^2 + (\omega L)^2}\, e^{j\theta}} = \frac{v_{in}}{\sqrt{R^2 + (\omega L)^2}} e^{-j\theta}$ (phase shift is $-\theta$)

For $\omega = 1000$ rad/s, $\frac{\omega L}{R} = \frac{(1000)(.1)}{100} = 1$ (i.e. $\omega = \omega_B = R/L$)

Then $\frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{1}{\sqrt{2}}$ and $\tan\theta = 1 \Rightarrow \theta = \pi/4 \Rightarrow$ phase shift $= -\pi/4$

5. Find the magnitude and phase of the impedance across the input for the circuit below. If a sinusoidal input is connected with 160 V amplitude and 250 Hz frequency, find the voltage signal across the 20-Ω resistor, including the phase with respect to the input.



Component impedance at 250 Hz [1]

$$Z_C = \frac{1}{j\omega C} = -j\,\frac{1}{2\pi 250\,(9.2\times10^{-6})}\,\Omega = -j\,69.2\,\Omega$$

$$Z_L = j\omega C = j\,(2\pi 250)(0.13)\,\Omega = j\,204\,\Omega$$

Then,

$$Z_{\parallel} = \left(\frac{1}{R_2} + \frac{1}{Z_C} + \frac{1}{Z_L}\right)^{-1} = \left(\frac{1}{80} + \frac{1}{(-j69.2)} + \frac{1}{(j204)}\right)^{-1}\Omega = \frac{1}{\left(\frac{1}{80} + j(.00954)\right)}\,\Omega\,\frac{\left(\frac{1}{80} - j(.00954)\right)}{\left(\frac{1}{80} - j(.00954)\right)}$$

$$= \frac{1/80 - j(.00954)}{(1/80)^2 + (.00954)^2}\,\Omega = 4044\left(1/80 - j(.00954)\right)\Omega = 50 - j\,39\,\Omega$$

Total impedance: $Z_{total} = R_1 + Z_{\parallel} = (70 - j\,39)\,\Omega = |Z|\,e^{j\theta}$

where $|Z| = \sqrt{70^2 + 39^2}\,\Omega = \boxed{80\,\Omega}$

and $\tan\theta = \frac{-39}{70} \Rightarrow \boxed{\theta = -.51\text{ rad} = -29^\circ}$

Voltage across 20 Ω resistor: $v_{R_1} = i R_1$ where $i = \frac{v_{in}}{Z_{total}} = \frac{V_{in}\,e^{j\omega t}}{|Z|\,e^{j\theta}} = \frac{V_{in}}{|Z|}\,e^{j(\omega t - \theta)}$

$$\Rightarrow v_{R_1} = \frac{V_{in} R_1}{|Z|}\,e^{j(\omega t - \theta)} = \frac{(160)(20)}{80}\,V\,e^{j(\omega t + 0.5\text{ rad})}$$

$$= 40V\,e^{j(\omega t + 0.5\text{ rad})}$$

$\boxed{\Longrightarrow \text{amplitude: } 40V,\ \text{phase} = 0.5\text{ rad} = 29^\circ}$

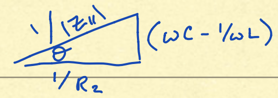
Note for question 5

[1] Full credit for the numerical solution, but algebraic solution is given below:

$$Z_{\parallel} = \left(\frac{1}{R_2} + j\omega C + \frac{1}{j\omega L}\right)^{-1} = \frac{1}{\frac{1}{R_2} + j(\omega C - 1/\omega L)} = \frac{1}{\sqrt{(\frac{1}{R_2})^2 + (\omega C - 1/\omega L)^2}\; e^{j\theta}}$$

where $\tan\theta = R_2(\omega C - 1/\omega L)$

$$= |Z_{\parallel}|\, e^{-j\theta} \quad \text{with} \quad |Z_{\parallel}| = \frac{1}{\sqrt{(1/R_2)^2 + (\omega C - 1/\omega L)^2}}$$



Then

$$Z = R_1 + Z_{\parallel} = \left(R_1 + |Z_{\parallel}|\cos\theta\right) - j\left(|Z_{\parallel}|\sin\theta\right)$$

where $\cos\theta = \frac{|Z_{\parallel}|}{R_2}$ & $\sin\theta = |Z_{\parallel}|(\omega C - 1/\omega L)$

$$= \left(R_1 + \frac{|Z_{\parallel}|^2}{R_2}\right) - j\left(|Z_{\parallel}|^2(\omega C - 1/\omega L)\right)$$

or, explicitly (without simplification):

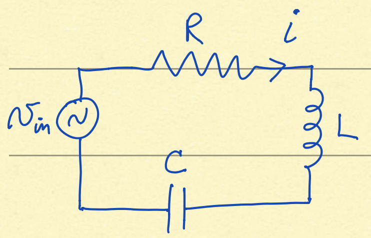
$$Z = \left[R_1 + \frac{1}{R_2\left((1/R_2)^2 + (\omega C - 1/\omega L)^2\right)}\right] - j\,\frac{(\omega L - 1/\omega C)}{\left((1/R_2)^2 + (\omega L - 1/\omega C)^2\right)}$$

Using $R_1 = 20\,\Omega$, $R_2 = 80\,\Omega$, $C = 9.2\,\mu F$, $L = 0.13\,H$, $f = 250\,Hz$ $\longrightarrow \omega = 2\pi f = 1571\, s^{-1} \longrightarrow (\omega C - 1/\omega L) = 9.56\times10^{-3}\,\Omega^{-1}$

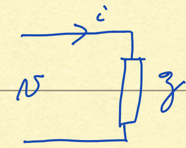
$\longrightarrow \left(\frac{1}{R}\right)^2 + (\omega C - 1/\omega L)^2 = 2.48\times10^{-4}\,\Omega^{-2}$

$$Z = \left(20 + \frac{1}{80(2.48\times10^{-4})}\right) - j\,\frac{9.56\times10^{-3}}{2.48\times10^{-4}}\,\Omega = (20 + 50) - j39\,\Omega = 70 - j39\,\Omega$$

6. A 10-Ω resistor, 10-mH inductor, and 10-μF capacitor are connected in series with a 10-kHz sinusoidal voltage source. The rms current through the circuit is 0.20 A. Find the rms voltage drop across each of the 3 elements. What would the rms current be at resonance, if the input rms voltage were the same?



Voltage across arbitrary impedance z: $v = i z$



$$\Rightarrow V_p e^{j\omega t} = I_p e^{j(\omega t + \phi)} |Z| e^{j\theta} \quad \Rightarrow V_p = I_p |Z| \quad \Rightarrow V_{rms} = I_{rms} |Z|$$

Resistor: $V_{R rms} = I_{rms} R = (0.2A)(10\Omega) \Rightarrow \boxed{V_{R rms} = 2V}$

Capacitor: $V_{C rms} = I_{rms} X_C = \dfrac{I_{rms}}{\omega C} = \dfrac{0.2A}{2\pi(10kHz)(10\mu F)} \Rightarrow \boxed{V_{C rms} = 0.32V}$

Inductor $V_{L rms} = I_{rms} X_L = I_{rms}\,\omega L = (0.2A)2\pi(10kHz)(10mH) \Rightarrow \boxed{V_{L rms} = 126V}$

rms current at resonance (same voltage)

First, find voltage from current at 10kHz:

$V_{rms} = I_{rms}|Z|$. At 10kHz, $I_{rms} = 0.2A$, and

$$|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{10^2 + \left[2\pi 10^4 (10^{-2}) - 1/(2\pi 10^4 10^{-6})\right]^2} = 627\Omega$$

$$\Rightarrow V_{rms} = (0.2A)(627\Omega) = 125V$$

Then find current at resonance

At resonance, $\omega L - 1/\omega C = 0 \Rightarrow |Z| = R$, so $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{125V}{10\Omega} = \boxed{12.5A}$