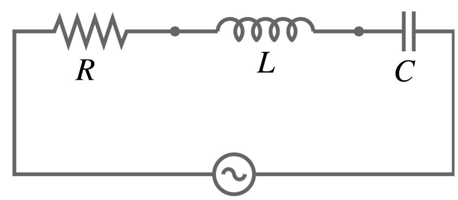
Experiment 4: LRC Series Circuit

Goal: To characterize the ac frequency response of an *LRC* series circuit.

Summary: The function generator will be used to apply a sinusoidal signal, and the resonance peak will be examined by measuring the voltage across the resistor. The amplitude and phase of the capacitor voltage will be measured in detail and compared with theory.



$v = V_0 \cos(\omega t)$

The nominal component values for this circuit will be $L = 25$ mH, $C = 2.2$ nF, with $R = 100$, 1000, and 2000 Ω.

The intrinsic resistance of the inductor, r_L, and the capacitance of the oscilloscope probe will be considered in this experiment.

Pre-Lab exercises:

Use the nominal component values given above for the following exercises.

a) Calculate the circuit resonant angular frequency ω_0 (in rad/s), and the resonant temporal frequency f_0 (in Hz) for which the current is maximum. Find the value of Q for each resistor value.

b) The complex impedance of the circuit is given by $z = |Z|e^{j\theta}$, where

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \quad \text{and} \quad \tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}.$$

What are the values of the phase at low, resonant and high frequencies?

c) The voltage across the resistor is $v_R = iR = \frac{v}{z}R$, so the real signal is

$$v_R = V_R \cos(\omega t + \alpha), \quad \text{where} \quad V_R = V_0 \frac{R}{|Z|}, \quad \text{and} \quad \alpha = -\theta.$$

What are the expected amplitude and phase at resonance?

d) The voltage across the capacitor is $v_C = iZ_C = \frac{v}{z} z_C$. Using the expression for z from (b) and $z_C = \frac{1}{j\omega C} = \frac{1}{\omega C e^{j\pi/2}}$, we can write

$$v_C = V_C \cos(\omega t + \beta), \quad \text{where } V_C = V_0 \frac{1}{\omega C|Z|} = \frac{V_0}{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}} \text{ , and } \beta = -\left(\theta + \frac{\pi}{2}\right)$$

(i) What is the ratio of amplitudes V_C / V_0 at resonance for each value of R? How can this be greater than 1?

(ii) Determine the low, resonant, and high frequency values of β. Sketch a graph of β versus ω from low to high frequency, marking the resonance condition clearly.

e) Defining "capacitor resonance" to be the condition for which the amplitude ratio V_C / V_0 is a maximum, give the expression for the value of ω at this resonance. What is the value of this resonance, and what is its value in the $R = 0$ limit? Calculate the amplitude ratio at this capacitor resonance for the 3 resistor values.

Measurements and Analysis:

Select components with nominal values: $R = 100, 1000, 2000\ \Omega$; $L = 25$ mH; $C = 2.2$ nF.

Measure and record the actual component values, and also measure the inductor's resistance (r_L), and the capacitance of the oscilloscope probe (C_P). Then calculate the effective values for the circuit resistance ($R + r_L$), and the effective capacitance when the probe is connected across the capacitor ($C + C_P$). (Note that the input resistance of the oscilloscope (1 MΩ) can be neglected, and the probe capacitance can be neglected when it is connected across the resistor.)

Calculate the value of the resonant angular frequency ($\omega_0 = \frac{1}{\sqrt{LC}}$), and the corresponding temporal frequency in Hz using the effective component values with and without the oscilloscope probe connected across the capacitor.

Assemble your components into a series circuit, and then measure the output across the resistor and capacitor as the frequency is changed. (Note that the resistor and capacitor will have to be swapped for the two measurements to ensure the scope and signal generator share a ground.)

1. Resistor: Determine the resonant frequency by maximizing the resistor voltage, and compare with the predicted value. What is the observed amplitude ratio at this frequency? Is it consistent with the prediction of question (c) above? You can use any of the 3 resistors for this measurement, but the resonance will be sharper for the 100 Ω resistor.

2. Capacitor:

(a) With the frequency set to the resonance determined in part 1, measure the amplitude of the signal across the capacitor using each of the 3 resistors. Compare it to the expected amplitude.

(b) Using the 2 kΩ resistor, adjust the frequency to maximize the capacitor voltage and compare the frequency and amplitude with expectations for this condition.

(c) Using the 100 Ω resistor, and for a wide range of frequencies from about 150 Hz to 150 kHz, measure the amplitude and phase response of the capacitor waveform, as follows:

- Trigger on the function generator signal (channel 1), and set the amplitude of the function generator to be a few volts. Scan through a wide frequency range and verify that the amplitude does not change with frequency. Note the peak-to-peak value of the function generator signal.

- Set the frequency for each measurement to be a convenient value such that one cycle starts and ends on a major division mark of the screen. Full screen is 10 divisions!

- Read the peak-to-peak waveform values, using the position controls to move the trace around so that it is convenient to read.

- When measuring the phase shift, adjust the horizontal scale (time base) to enhance the spacing between the two signals. Adjust the vertical scales for good resolution.

You may use the digital measuring tools of the oscilloscope to obtain your measurements of peak-to-peak wave form values and the time difference between zero crossings as needed for the phase difference test. Verify that you have the setup working correctly by making one detailed comparison between the digital measurements and a visual reading of the 'scope trace using the calibration factors (Volts/div) and the (sec/div) for the vertical + horizontal scales, respectively.

Plot the amplitude ratio as a function of frequency (symbols) and superpose the theoretical prediction as a solid line. Do the same for the phase shift β.