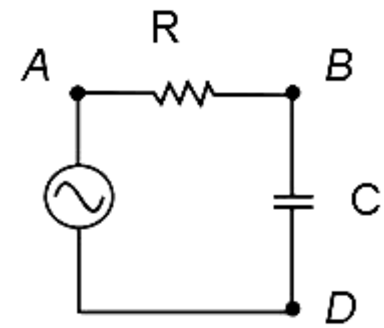
Experiment 3: RC Circuit AC Response



Goal: To characterize the ac response of an *RC* series circuit.

Circuit: The function generator will be used to apply a sinusoidal *emf* with **amplitude** V_0 at angular frequency $\omega = 2\pi f$ across the *RC* components shown in the diagram. The amplitude and phase of the capacitor waveform, relative to the function generator, will be measured and compared with theory.

Pre-Lab exercises:

a) Working with the complex waveforms $\tilde{v}(t) = V_0 e^{j\omega t}$, $v(t) = Re(\tilde{v})$, use the complex impedance formalism to show that the current in the circuit is given by:

$i(t) = \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos(\omega t - \theta)$, where $\tan\theta = -1/\omega RC$.

b) Based on the solution to part a), find expressions for the potential differences across the resistor, $v_R(t) = V_R \cos(\omega t + \alpha)$, and capacitor, $v_C(t) = V_C \cos(\omega t + \beta)$, showing the phases (α and β) with respect to the input voltage. The capacitor values will be examined in this experiment.

c) Organize your results in a table:

Element:	Amplitude	Phase relative to $v(t)$
Resistor	$V_R=$	$\alpha =$
Capacitor	$V_C=$	$\beta =$

d) Sketch graphs of the amplitude and phase relations for the capacitor and resistor as functions of frequency.

Equipment:

Function generator, Digital oscilloscope and associated circuit probes, breadboard, resistor and capacitor, Digital multimeter and capacitance meter

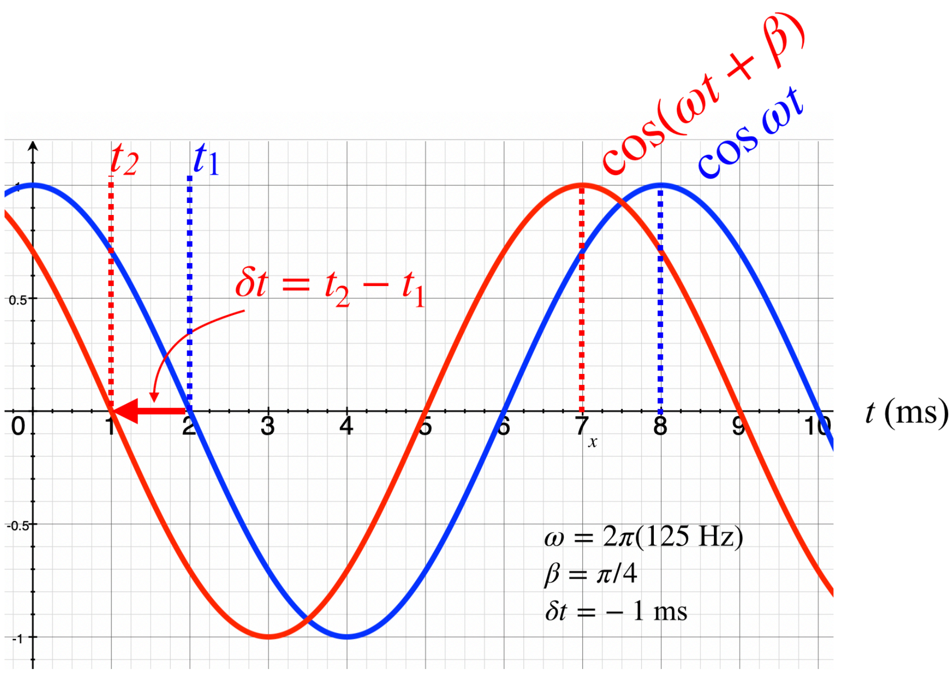
Preparation and Practice:

- Select components as follows: $R = 10$ kΩ, $C = 0.033$ µF;

  measure and record the actual component values.
- Assemble the circuit using the breadboard, function generator, and leads. Set the function generator to output a sine wave voltage signal with a few volts amplitude; display this on channel 1 of the oscilloscope, and use it for the trigger.

- Next, attach the second probe to channel 2, and use it to observe the voltage across the capacitor in your circuit, being careful to establish only one ground point in the circuit.
- For the amplitude study, it is convenient to use the "Measure" function available with the digital scope. Figure out how to display both channels simultaneously and have the "measure" function display the peak-to-peak voltages of both channels on the screen.
- To measure the phase shift between two signals, use the time between the zero crossings, because that can be identified easily even if the amplitudes are different. As illustrated in the figure below, a positive phase shift β results in the waveform shifted to the left (lower time). Since $\cos \omega t_1 = \cos(\omega t_2 + \beta)$, then $\omega t_1 = \omega t_2 + \beta$ (for $\beta < \pi/2$), so, taking δt as negative for a shift to lower time (and positive otherwise), $\delta t = t_2 - t_1$, gives

$$\beta = \omega(t_1 - t_2) = -\omega\, \delta t$$



- To test the method, set a relatively high frequency (e.g. ~ 10 kHz) on the function generator and observe the two signals:
    - Set the voltage gains relatively high so that the traces have a steep slope at their zero crossings. Expand the horizontal scale so that the traces are separated by several divisions at the center of the screen.
    - Make sure the waveforms are symmetric about the middle horizontal line. One way to be sure of this is to zero (ground) the channel and position the line at the center, and then use ac coupling for the input.
    - Read the time difference by eye in units of "divisions" and note the calibration time per division that is displayed on the screen; compare this to the time difference obtained using the "measure" function on the scope.

- Run through a quick frequency scan noting the amplitude and phase response of the capacitor signal to make sure the qualitative behavior is consistent with expectations, before proceeding to record the data. You may want to have an instructor check it.

Measurements and analysis -- Capacitor waveform:

1. Amplitude study:

For a wide range of frequencies from about 100 Hz to 20 kHz, measure the amplitude response of the capacitor waveform, recording the peak-to-peak voltage values for both channel 1 and channel 2.

Experimental uncertainty will be dominated by the peak-to-peak voltage readings. Estimate this uncertainty based on the noise in the measurements. A convenient format to record the data is shown below:

f (Hz)	V_0 (peak-peak) (V)	V_C (peak-peak) (V)

Analysis: Use the lab computer to calculate and plot V_C/V_0 as a function of frequency and superpose the prediction of your prelab exercise with *measured* component values as a solid curve. Compare the results using linear and log scales for frequency. Are the results consistent within the uncertainties of the voltage measurements?

2. Phase study:

For a wide range of frequencies from about 100 Hz to 20 kHz, measure the phase response of the capacitor waveform, as described above, i.e. by measuring the time difference between zero crossings in the center of the screen for the two waveforms. Use the vertical and horizontal gains to your advantage to obtain accurate readings. The frequency can be read directly from the scope screen.

Experimental uncertainty here will be dominated by the measurement of the time difference. Estimate this uncertainty based on your ability to read this value.

A convenient format for your data table is:

f (Hz)	δt (s)	Phase difference (rad) $\beta = (-)\omega\,\delta t = (-)2\pi f\,\delta t$

Analysis:

Use the lab computer to calculate and plot the capacitor phase $\beta = -2\pi f \delta t$ as a function of frequency and superpose the prediction of your prelab exercise b) with measured component values as a solid curve.

Compare the results using linear and log scales for frequency, taking account of the estimated uncertainty.