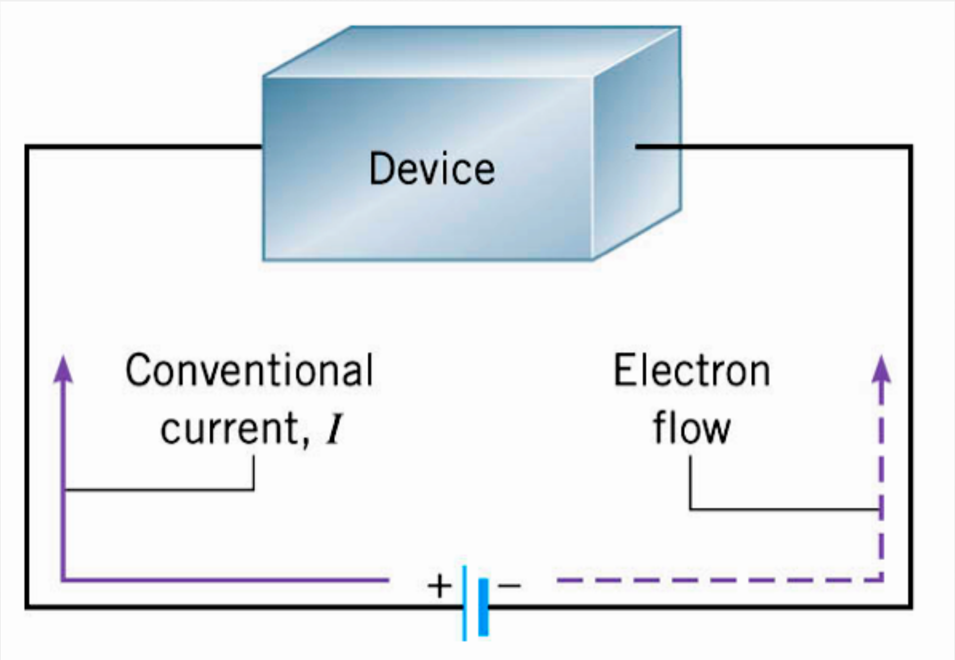
Chapter One
Basic Concepts

Current, Voltage, Resistance, Ohm's Law

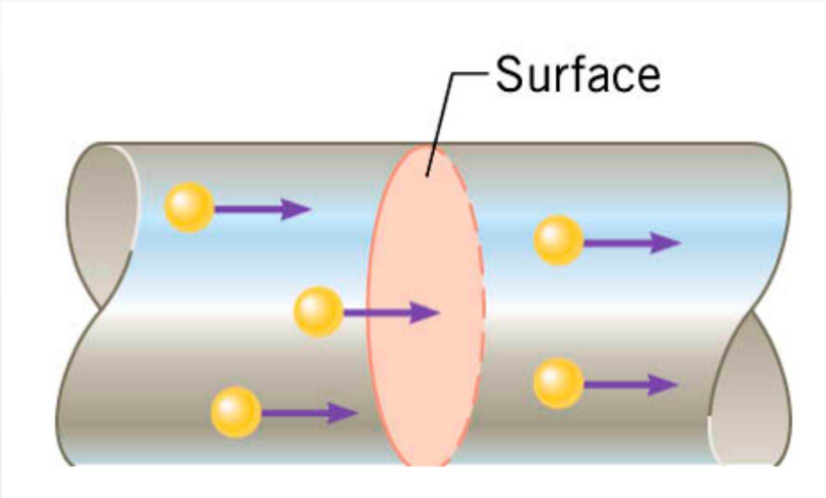
1) Electric current and 2) Voltage

Potential difference and charge flow

Battery produces potential difference causing flow of charge in conductor



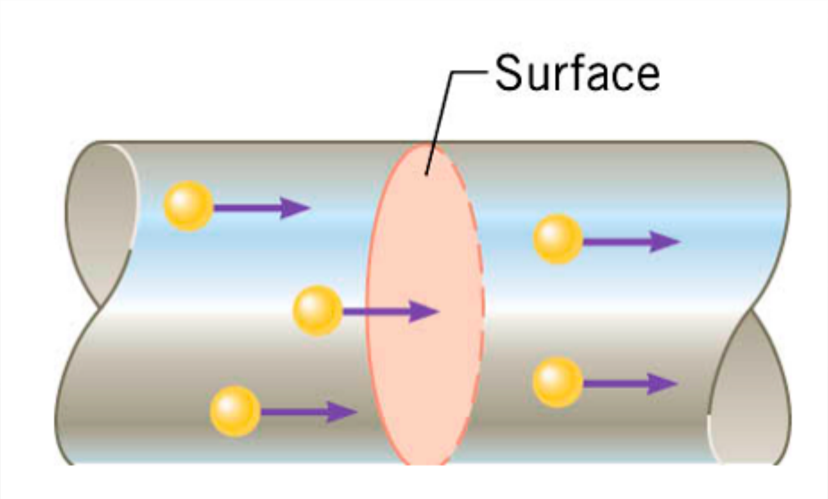
Current: $I = dq/dt$



dq is charge that passes the surface in time *dt*

Units: C/s = ampere = A

- Drift velocity: average velocity of electrons

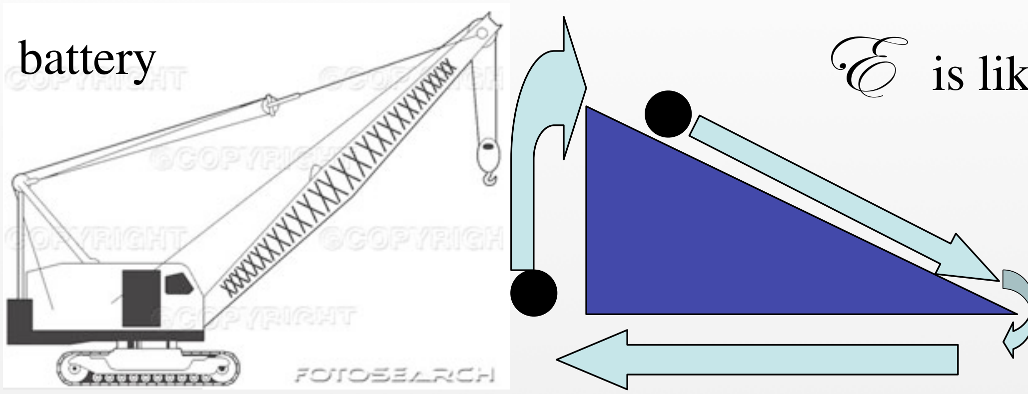


~ mm/s

- Signal velocity: speed of electric field

= speed of light in the material ~10^8 m/s

Electromotive force, emf

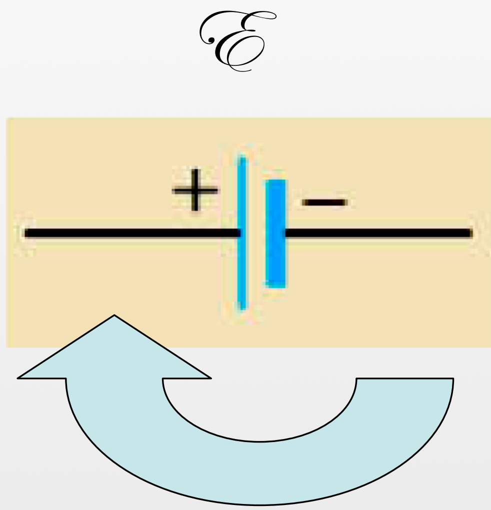


$\mathcal{E}$ is like gh

gravitational analogy for a circuit

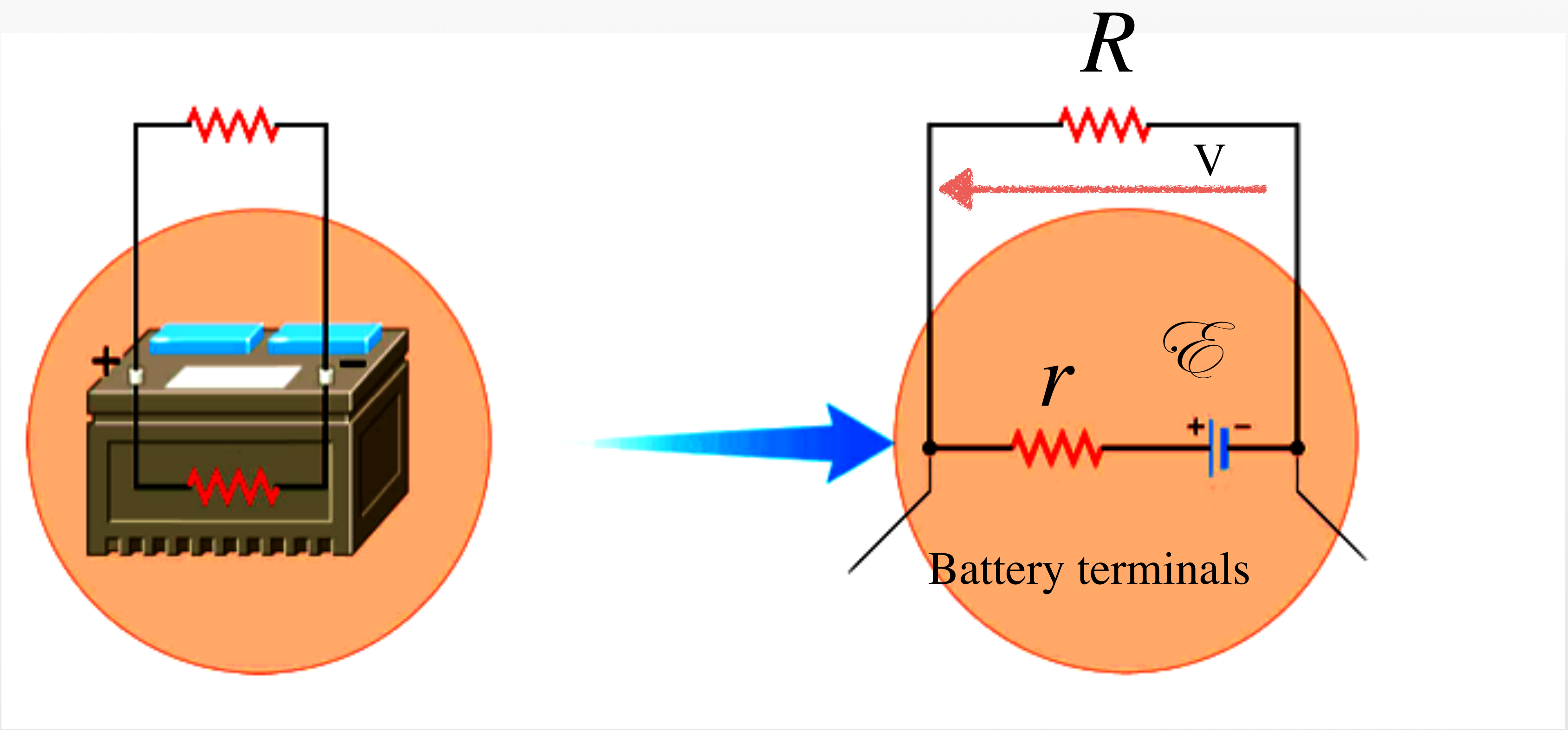
- *emf* = electromotive force = maximum potential difference produced by a device
- Symbol: $\mathcal{E}$
- *emf* is not a force, but it causes current to flow

• Symbol for a perfect seat of *emf*



$$V = \mathcal{E}$$

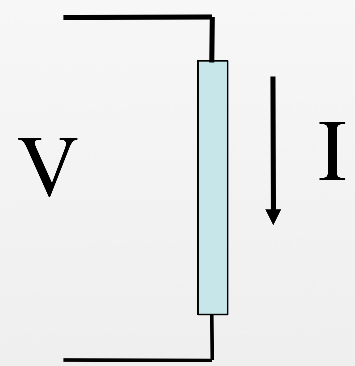
- Real battery



$V < \mathcal{E}$ **in general**

3) Power

Power dissipated in a device



- Energy lost or gained by dq is $dU=dqV$
- Power:

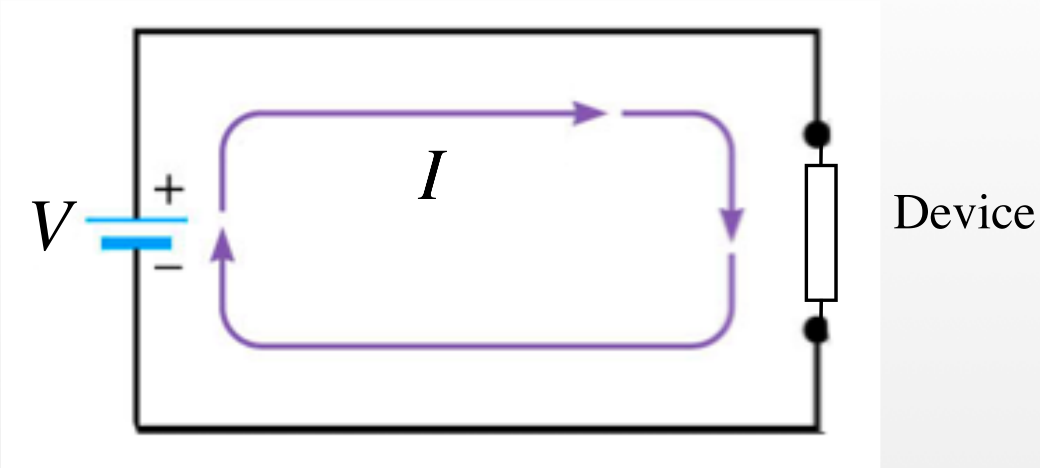
$$P = \frac{dU}{dt} = \frac{dqV}{dt}$$

$$P = VI$$

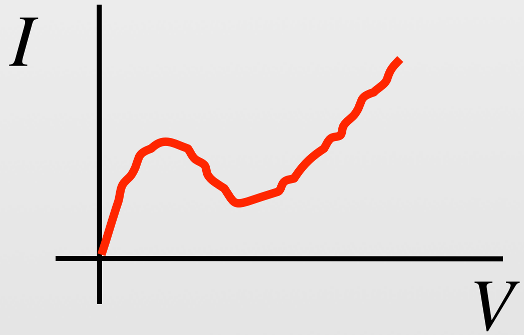
Units: (C/s)(J/C) = J/s = W

Consumed energy = $P\,t$: [kW h] = (1000 W) (3600 s) = 3.6 MJ

4) Resistance

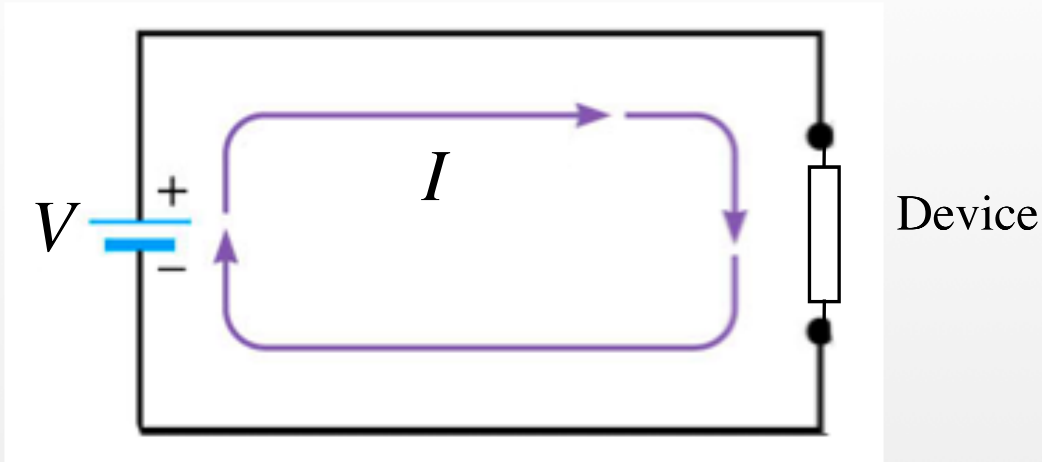


- Current depends on voltage and on the device

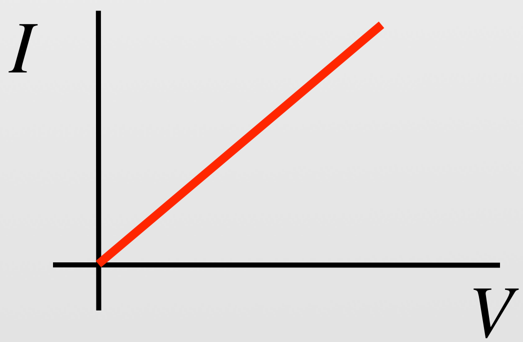


- called the I-V characteristics of the device

a) Ohm's law



- For some devices (conductors), I is proportional to V:

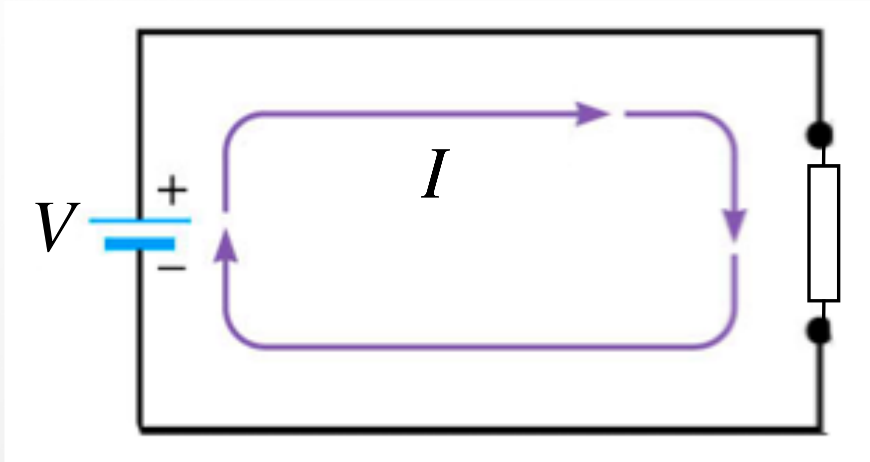


$$V = IR$$

- R = *Resistance* = proportionality constant = V/I

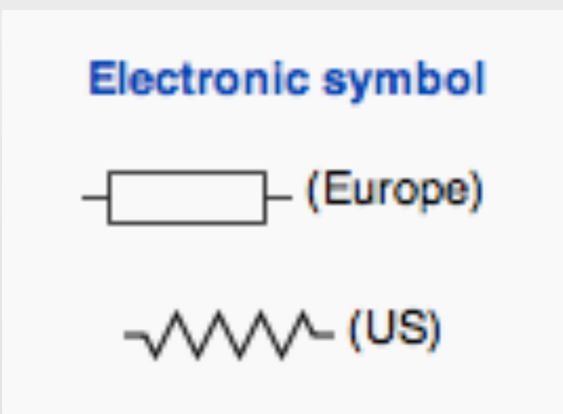
- R = *Resistance* = proportionality constant = V/I

Units: volt/ampere = ohm = Ω

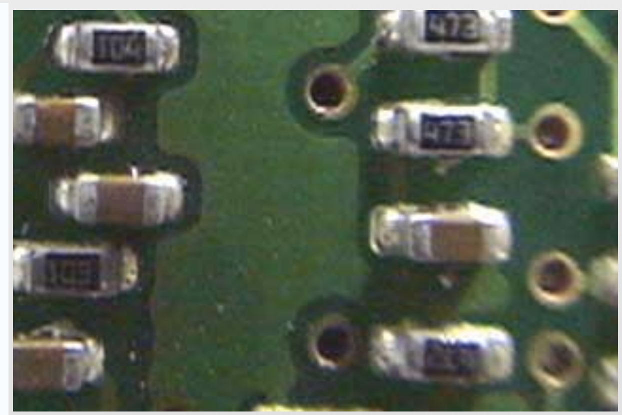


- Ohmic material obeys Ohm's Law: *R is constant*
- *R* is a property of the *device*

- images:

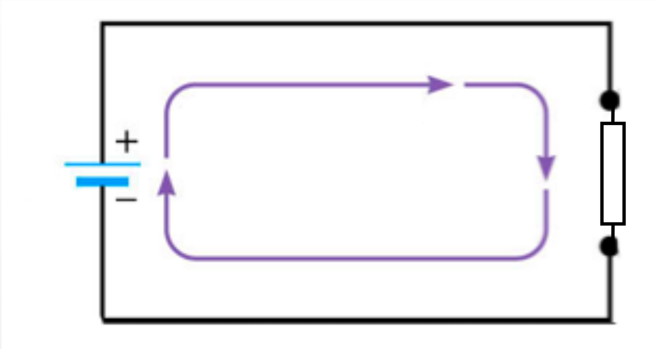


surface mount

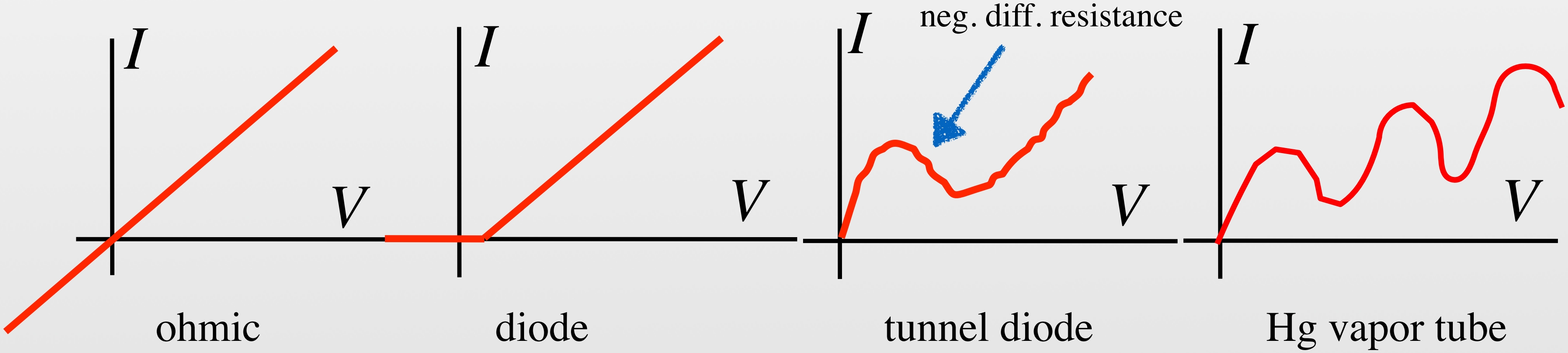


Ring color			Significant figures	Multiplier		Tolerance		Temperature coefficient	
Name	Code	RAL				Percent	Letter	ppm/K	Letter
None	–	–	–	–		±20%	M	–	
Pink	PK	3015	–	$\times 10^{-3}$[5]	×0.001	–		–	
Silver	SR	–	–	$\times 10^{-2}$	×0.01	±10%	K	–	
Gold	GD	–	–	$\times 10^{-1}$	×0.1	±5%	J	–	
Black	BK	9005	0	$\times 10^{0}$	×1	–		250	U
Brown	BN	8003	1	$\times 10^{1}$	×10	±1%	F	100	S
Red	RD	3000	2	$\times 10^{2}$	×100	±2%	G	50	R
Orange	OG	2003	3	$\times 10^{3}$	×1000	–		15	P
Yellow	YE	1021	4	$\times 10^{4}$	×10 000	(±5%[nb 1][6])	–	25	Q
Green	GN	6018	5	$\times 10^{5}$	×100 000	±0.5%	D	20	Z[nb 2]
Blue	BU	5015	6	$\times 10^{6}$	×1 000 000	±0.25%	C	10	Z[nb 2]
Violet	VT	4005	7	$\times 10^{7}$	×10 000 000	±0.1%	B	5	M
Gray	GY	7000	8	$\times 10^{8}$	×100 000 000	±0.05% (±10%[nb 1][6])	A	1	K
White	WH	1013	9	$\times 10^{9}$	×1 000 000 000	–		–	

- *non-ohmic resistance?*



- Current depends on voltage and on the device



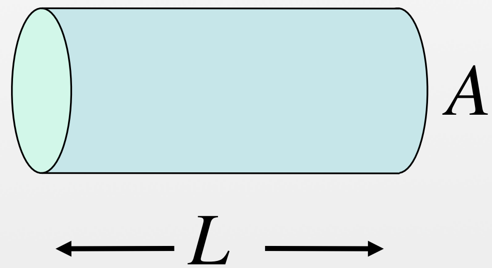
- dc resistance: $R = \dfrac{V}{I}$
- differential (or ac) resistance: $r = \dfrac{dV}{dI}$

(can be negative)

What’s the resistance of a 100 W light bulb if $I = 0.83$ A?

b) Resistivity

- Property of material; zero for superconductors
- For cylindrical conductor:



- R is proportional to L
- R is proportional to $1/A$
- R is proportional to L / A
- Define *resistivity* ρ as the proportionality constant

$$R = \rho \frac{L}{A}$$

For a cylinder, $\rho = R\dfrac{A}{L}$

Using $R = \dfrac{V}{i}$ gives $\rho = \dfrac{V}{i}\dfrac{A}{L} = \dfrac{V/L}{i/A}$

but $E = V/L$ and $J = i/A$ so

$$\rho = \frac{E}{J}$$

units: $\dfrac{\text{V/m}}{\text{A/m}^2} = \dfrac{\text{V}}{\text{A}}\text{m} = \Omega\text{m}$

$$R = \frac{V}{I}$$

Material	Resistivity (Ω·m) at 20 °C
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminium	2.82×10^{-8}
Calcium	3.36×10^{-8}
Tungsten	5.60×10^{-8}
Zinc	5.90×10^{-8}
Nickel	6.99×10^{-8}
Iron	1.0×10^{-7}
Tin	1.09×10^{-7}
Platinum	1.06×10^{-7}
Lead	2.2×10^{-7}
Manganin	4.82×10^{-7}
Constantan	4.9×10^{-7}
Mercury	9.8×10^{-7}
Nichrome[6]	1.10×10^{-6}
Carbon[7]	3.5×10^{-5}
Germanium[7]	4.6×10^{-1}
Silicon[7]	6.40×10^{2}
Glass	10^{10} to 10^{14}
Hard rubber	approx. 10^{13}
Sulfur	10^{15}
Paraffin	10^{17}
Quartz (fused)	7.5×10^{17}
PET	10^{20}
Teflon	10^{22} to 10^{24}

Resistance of copper wire.

20 gauge: $A = 5.2 \times 10^{-7}\ m^2$, $L = 5\ m$

-

c) Conductivity

$$\sigma = \frac{1}{\rho} \qquad (\Omega \mathrm{m})^{-1}$$

Conductance: $$G = \frac{1}{R} = \frac{I}{V} \qquad (\Omega^{-1}) \quad \text{(also ℧)}$$

Siemens (or mho)

d) Temperature dependence

- Resistivity is approx linear with temperature: $\rho = a + bT$

Define $\rho_0 =$ resistivity at $T = T_0$

$$\rho_0 = a + bT_0 \quad \rightarrow a = \rho_0 - bT_0$$

$$\rho = \rho_0 + b(T - T_0)$$

$$\rho / \rho_0 = 1 + \alpha(T - T_0)$$ $\alpha =$ coefficient of resistivity (C^{o-1})

$$\rho = \rho_0(1 + \alpha(T - T_0)) \quad \Rightarrow R = R_0(1 + \alpha(T - T_0))$$

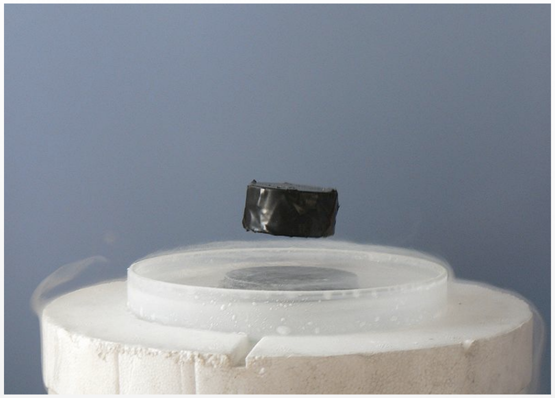
For metals, $\alpha > 0$ (resistance *increases* with temp)

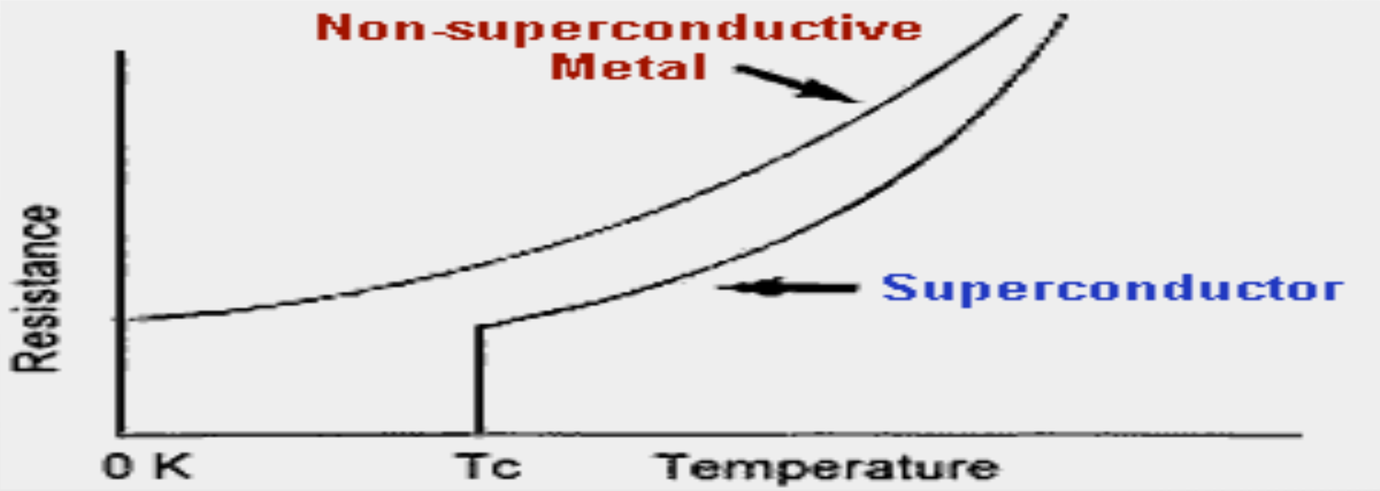
For semiconductors, $\alpha < 0$ (resistance *decreases*)

Material	Resistivity (Ω·m) at 20 °C	Temperature coefficient* [K^{-1}]
Silver	1.59×10^{-8}	0.0038
Copper	1.72×10^{-8}	0.0039
Gold	2.44×10^{-8}	0.0034
Aluminium	2.82×10^{-8}	0.0039
Calcium	3.36×10^{-8}	?
Tungsten	5.60×10^{-8}	0.0045
Zinc	5.90×10^{-8}	0.0037
Nickel	6.99×10^{-8}	?
Iron	1.0×10^{-7}	0.005
Tin	1.09×10^{-7}	0.0045
Platinum	1.06×10^{-7}	0.00392
Lead	2.2×10^{-7}	0.0039
Manganin	4.82×10^{-7}	0.000002
Constantan	4.9×10^{-7}	0.000 008
Mercury	9.8×10^{-7}	0.0009
Nichrome[6]	1.10×10^{-6}	0.0004
Carbon[7]	3.5×10^{-5}	−0.0005
Germanium[7]	4.6×10^{-1}	−0.048
Silicon[7]	6.40×10^{2}	−0.075
Glass	10^{10} to 10^{14}	?
Hard rubber	approx. 10^{13}	?
Sulfur	10^{15}	?
Paraffin	10^{17}	?
Quartz (fused)	7.5×10^{17}	?
PET	10^{20}	?
Teflon	10^{22} to 10^{24}	?

Superconductors

- Below critical temp T_c, ρ –> *0*
  - Current flows in loop indefinitely
  - Quantum transitions not possible





T_c typically < 10 K, but can be > ~ 77 K (high Tc ceramics), the BP of LN2 (record is 138 K)

Applications: MRI, MagLev trains

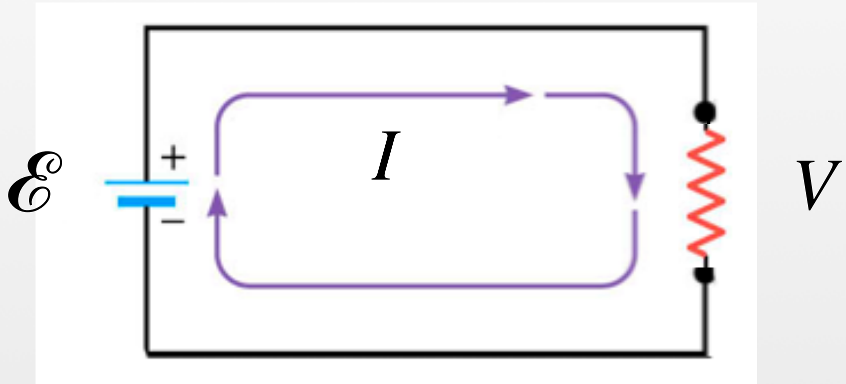
Confirmed critical temperatures

Critical temperature (T_c), crystal structure and lattice constants of some high-T_c superconductors

Formula	Notation	T_c (K)	No. of Cu-O planes in unit cell	Crystal structure
$YBa_2Cu_3O_7$	123	92	2	Orthorhombic
$Bi_2Sr_2CuO_6$	Bi-2201	20	1	Tetragonal
$Bi_2Sr_2CaCu_2O_8$	Bi-2212	85	2	Tetragonal
$Bi_2Sr_2Ca_2Cu_3O_6$	Bi-2223	110	3	Tetragonal
$Tl_2Ba_2CuO_6$	Tl-2201	80	1	Tetragonal
$Tl_2Ba_2CaCu_2O_8$	Tl-2212	108	2	Tetragonal
$Tl_2Ba_2Ca_2Cu_3O_{10}$	Tl-2223	125	3	Tetragonal
$TlBa_2Ca_3Cu_4O_{11}$	Tl-1234	122	4	Tetragonal
$HgBa_2CuO_4$	Hg-1201	94	1	Tetragonal
$HgBa_2CaCu_2O_6$	Hg-1212	128	2	Tetragonal
$HgBa_2Ca_2Cu_3O_8$	Hg-1223	134	3	Tetragonal

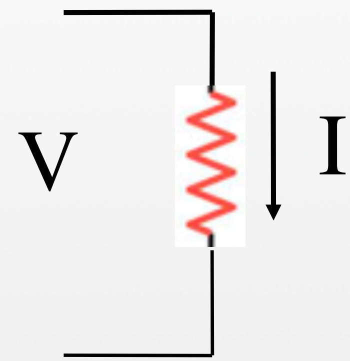
5) Resistor Circuits

a) Simple circuit



Ohm's Law: $V = IR$ or $\mathcal{E} = IR$

Power dissipated in resistors

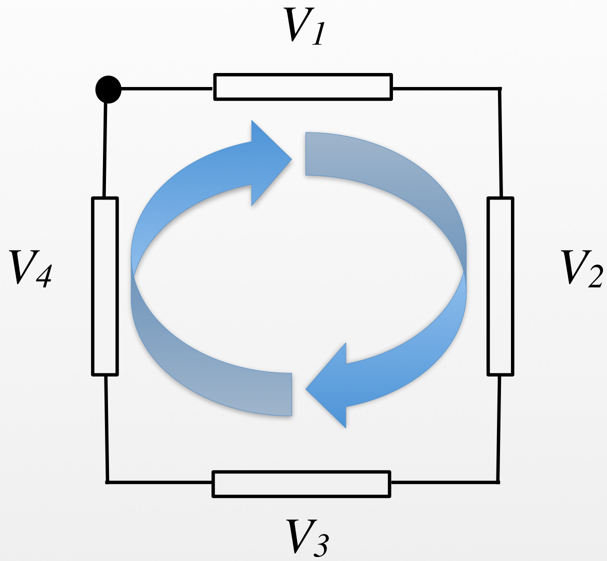


$$V = IR$$

$$P = VI = (IR)I \Rightarrow \quad P = I^2 R$$

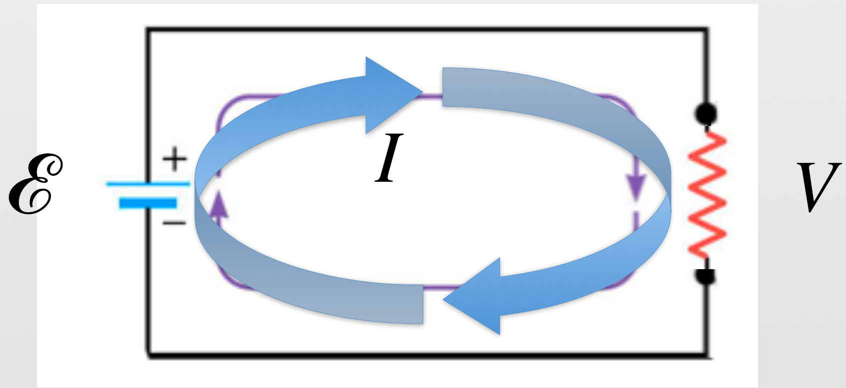
$$P = VI = V\frac{V}{R} \Rightarrow \quad P = \frac{V^2}{R}$$

b) Kirchhoff's loop rule



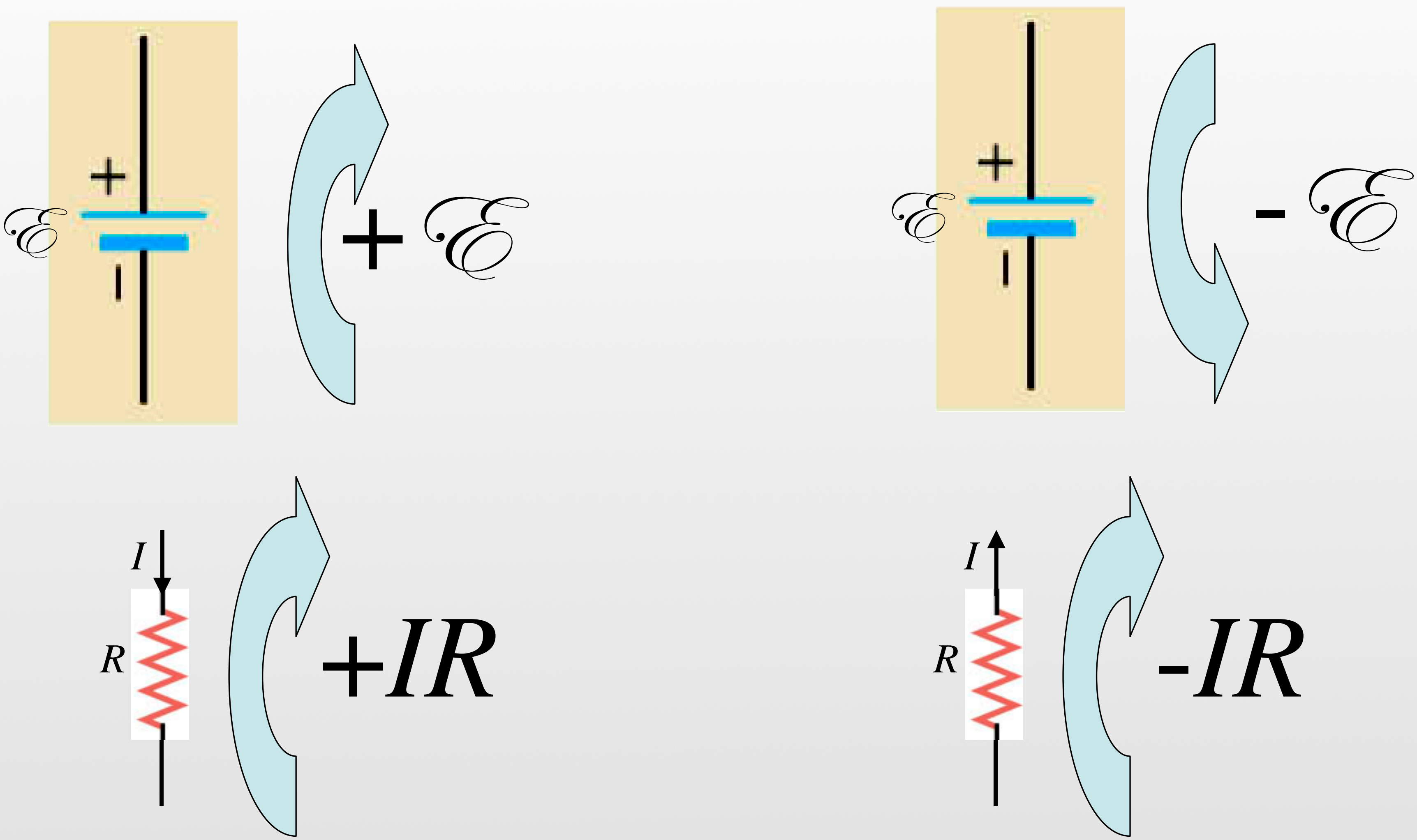
$$V_1 + V_2 + V_3 + V_4 = 0$$

$$\sum_i V_i = 0$$



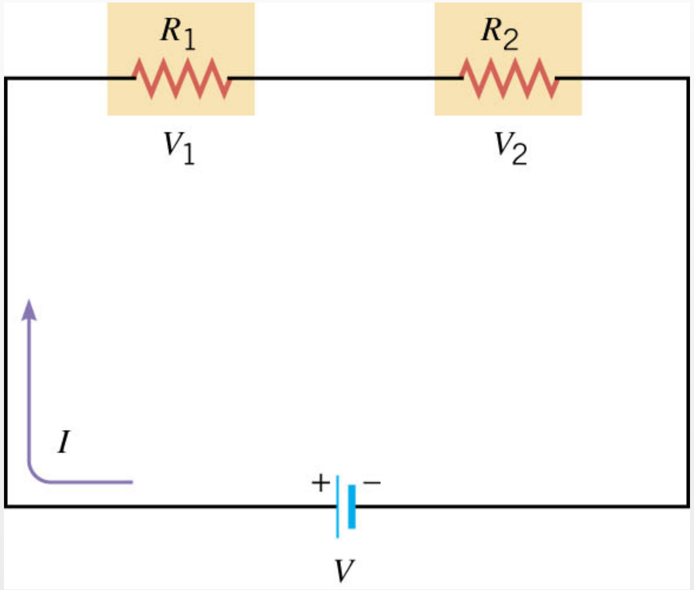
$$\mathcal{E} - IR = 0$$

Polarities:



c) Resistors in series

Same current



Loop rule:

$$V - IR_1 - IR_2 = 0$$

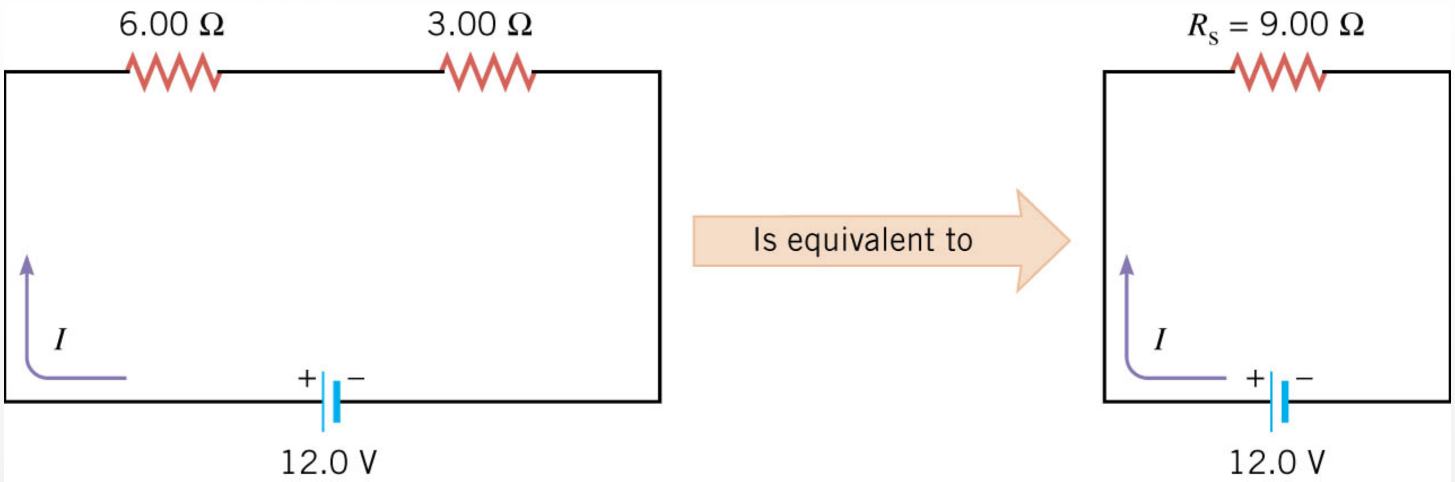
So, $V = IR_1 + IR_2$

$$= I(R_1 + R_2)$$

Or, $V = IR_S$

if

$$R_S = R_1 + R_2$$



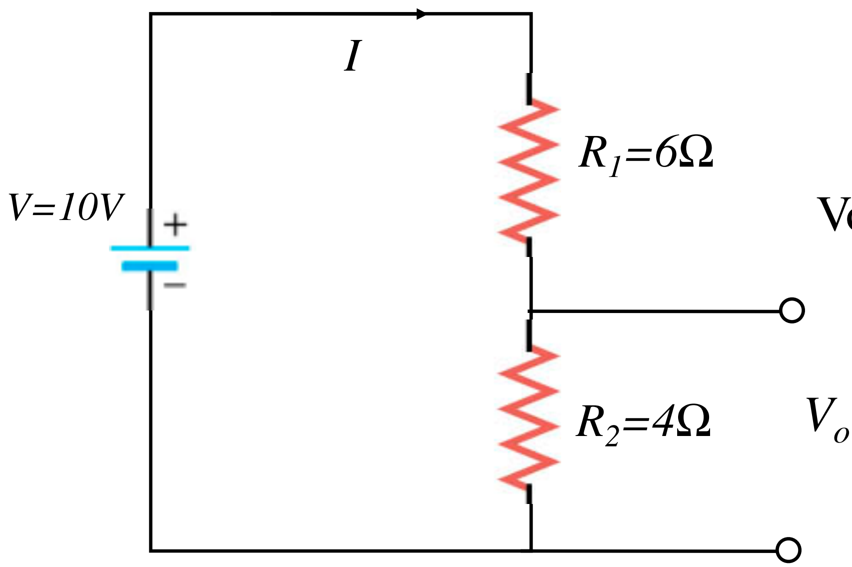
Find the current and the power through each resistor.

In general, for series resistors,

$$R_S = R_1 + R_2 + R_3 + \cdots$$

$$R_S = \sum_i R_i$$

Voltage divider



Current is the same in both resistors

$$I = \frac{V}{R_S} = \frac{V}{R_1 + R_2} \quad = 1\text{A}$$

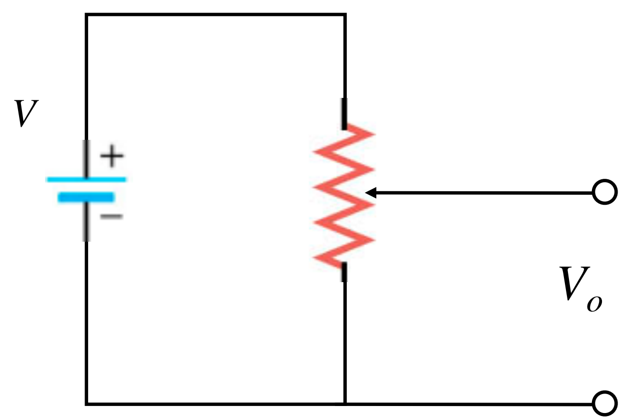
Voltages divide in proportion to R

$$V_1 = IR_1 = 6V$$

$$V_2 = IR_2 = 4V$$

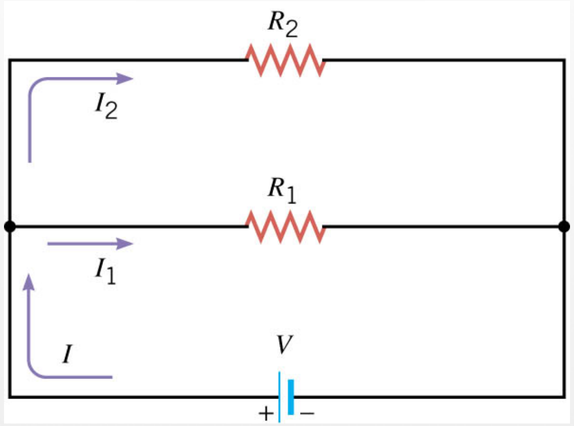
Output Voltage:

$$V_o = IR_2 = \frac{V}{R_1 + R_2} R_2 = V\left(\frac{R_2}{R_1 + R_2}\right)$$



d) Resistors in parallel

Same voltage



Kirchhoff's junction rule

$$I = I_1 + I_2$$

Ohm's Law (loop rule)

$$V = I_1 R_1 \quad \text{and} \quad V = I_2 R_2$$

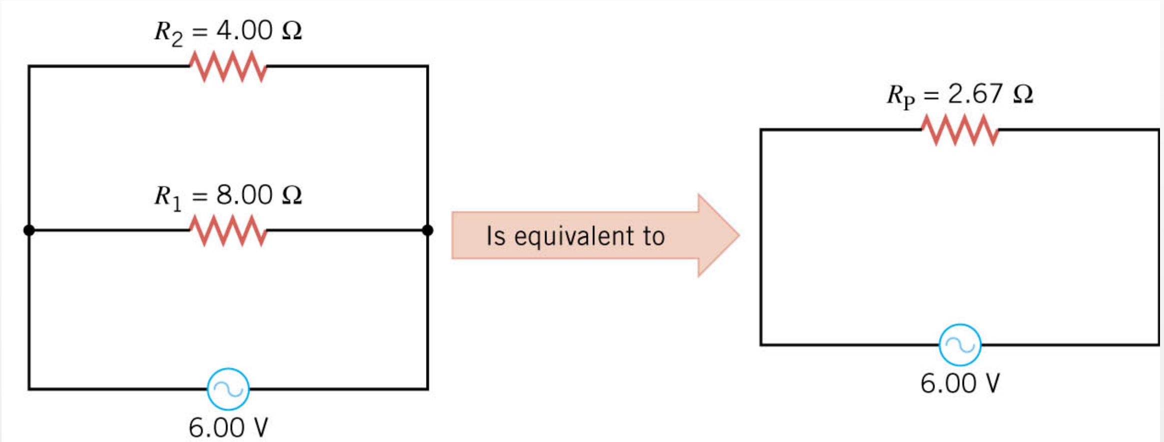
So, $$I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V\frac{1}{R_P}$$

Or, $V = IR_P$ if

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = R_1 // R_2$$



- Equivalent resistance is smaller than both R_1 and R_2
- Conductance adds

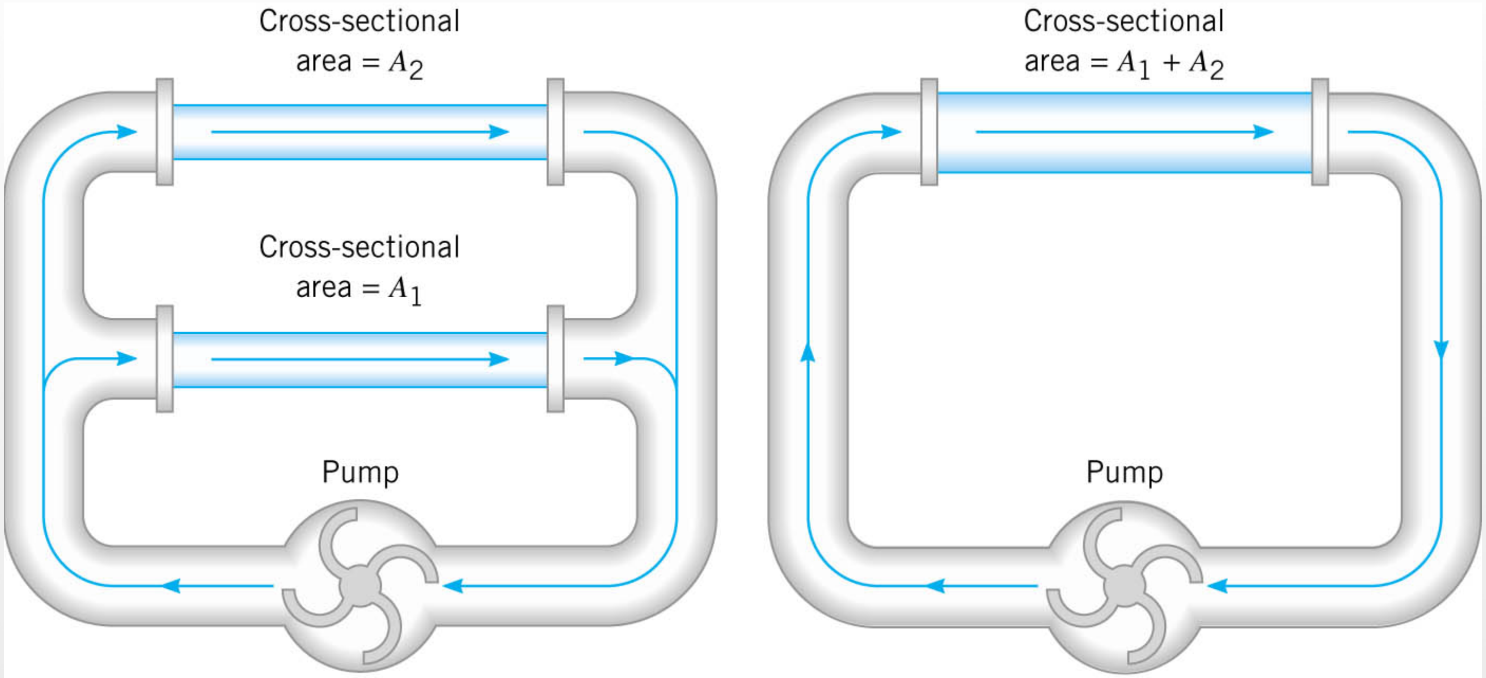
Find currents in each branch, power dissipated by each resistor, and the total power.

In general, for parallel resistors,

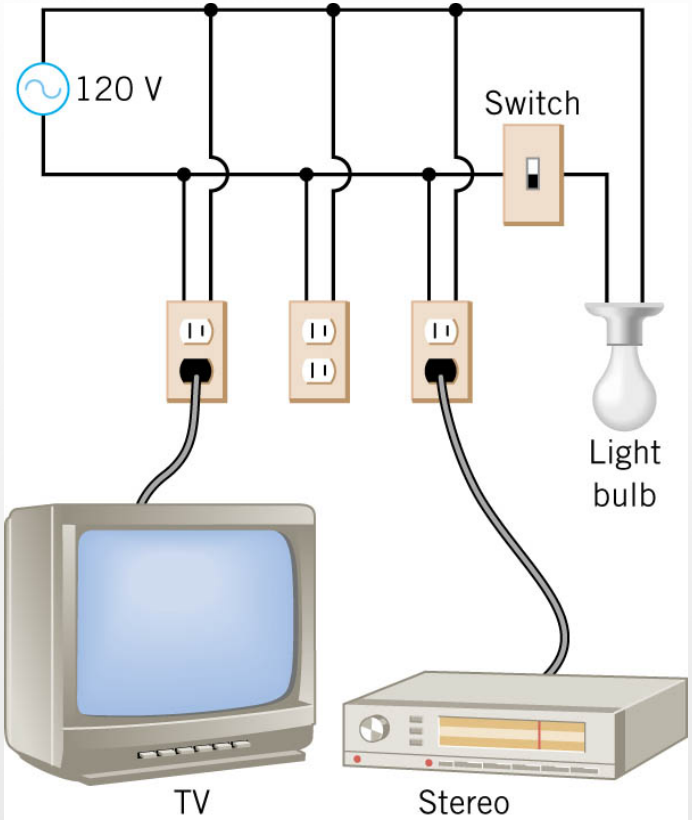
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

or

$$\frac{1}{R_P} = \sum_i \frac{1}{R_i}$$



conductance adds



parallel connections in the home

Special cases

i) *Equal resistance*

$$R_P = R // R = \frac{R^2}{2R} = \frac{R}{2}$$

ii) *Very unequal resistors* (e.g. 1Ω and 1 MΩ)

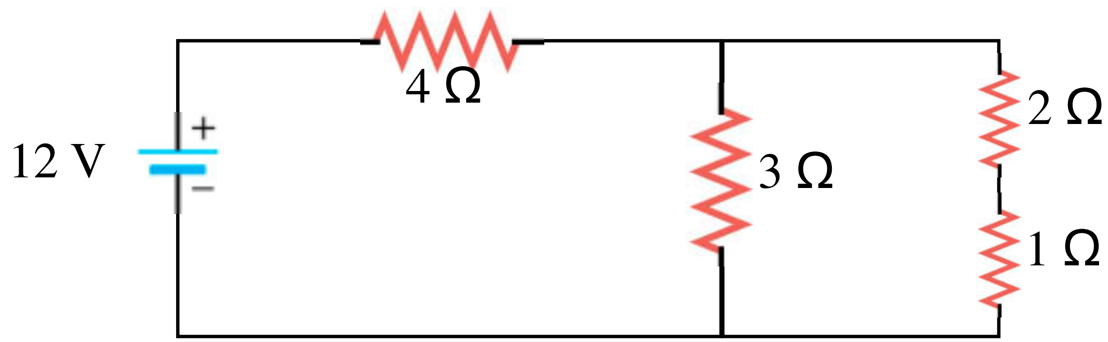
$$R_P = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{(1)(10^6)\Omega}{1 + 10^6} \cong 1\Omega$$

If $R_2 >> R_1$, then $R_1 + R_2 \cong R_2$

so $R_P \cong \dfrac{R_1 R_2}{R_2} = R_1$

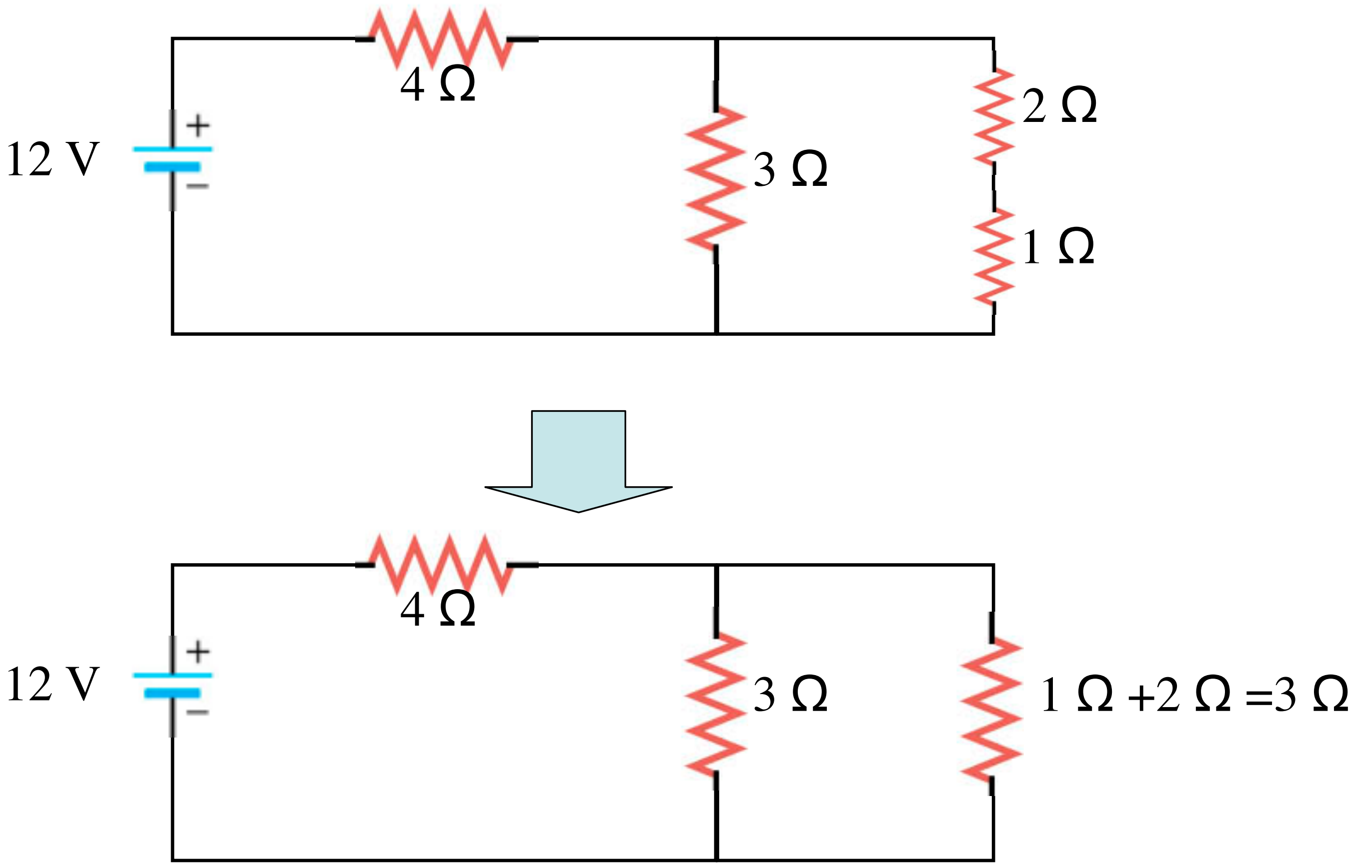
R_P = the smaller value

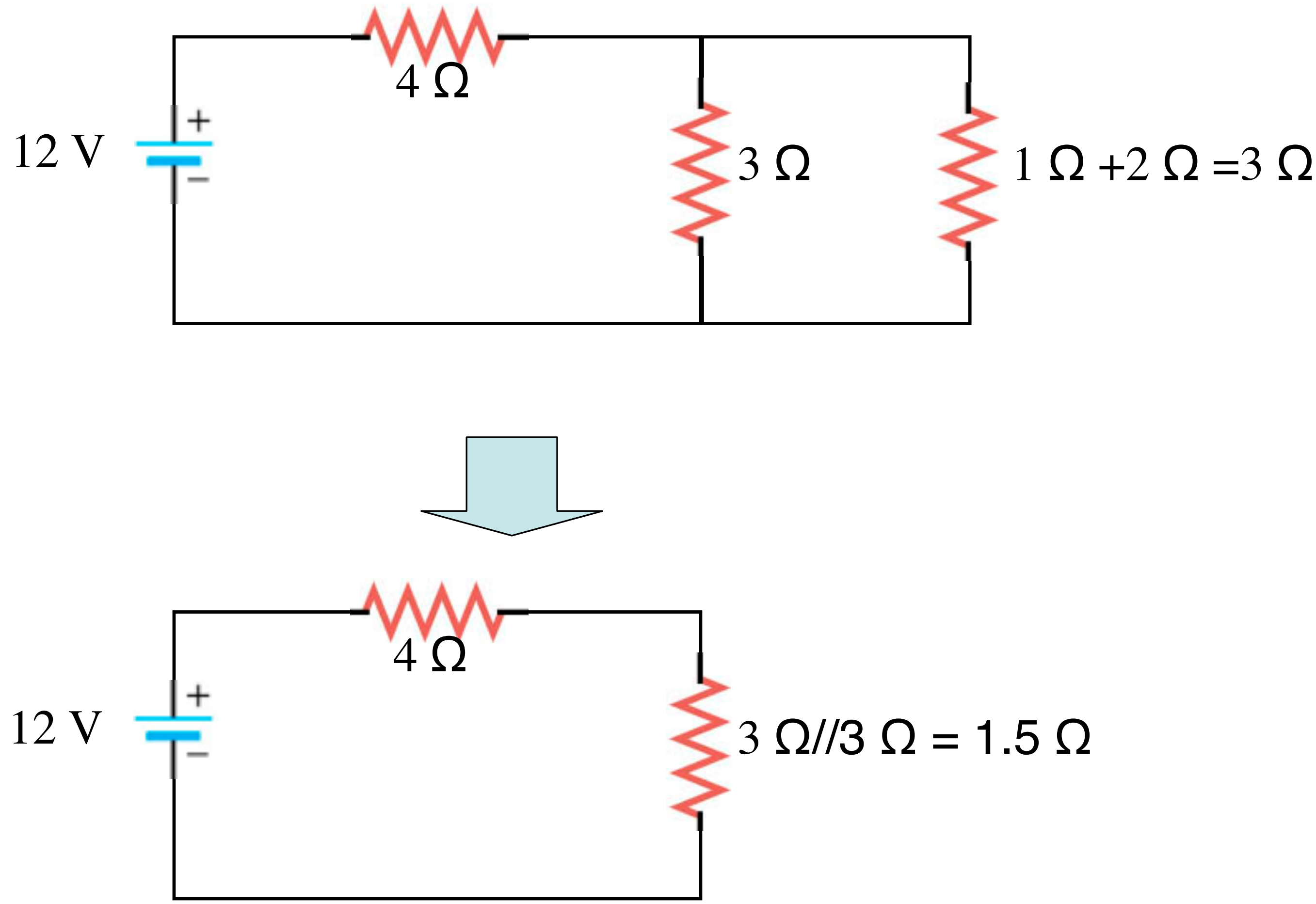
A series and parallel example

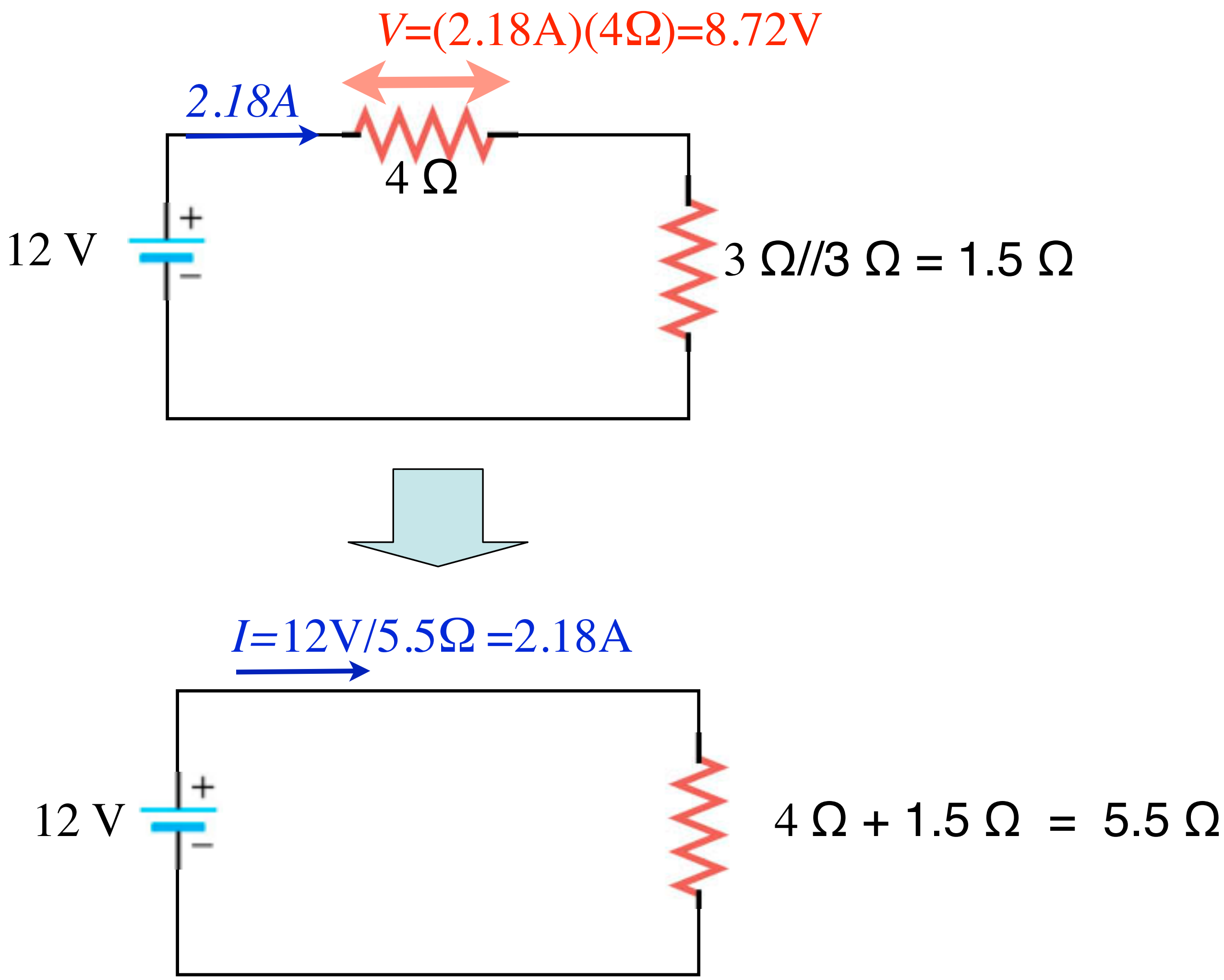


Find voltage across and current through each resistor

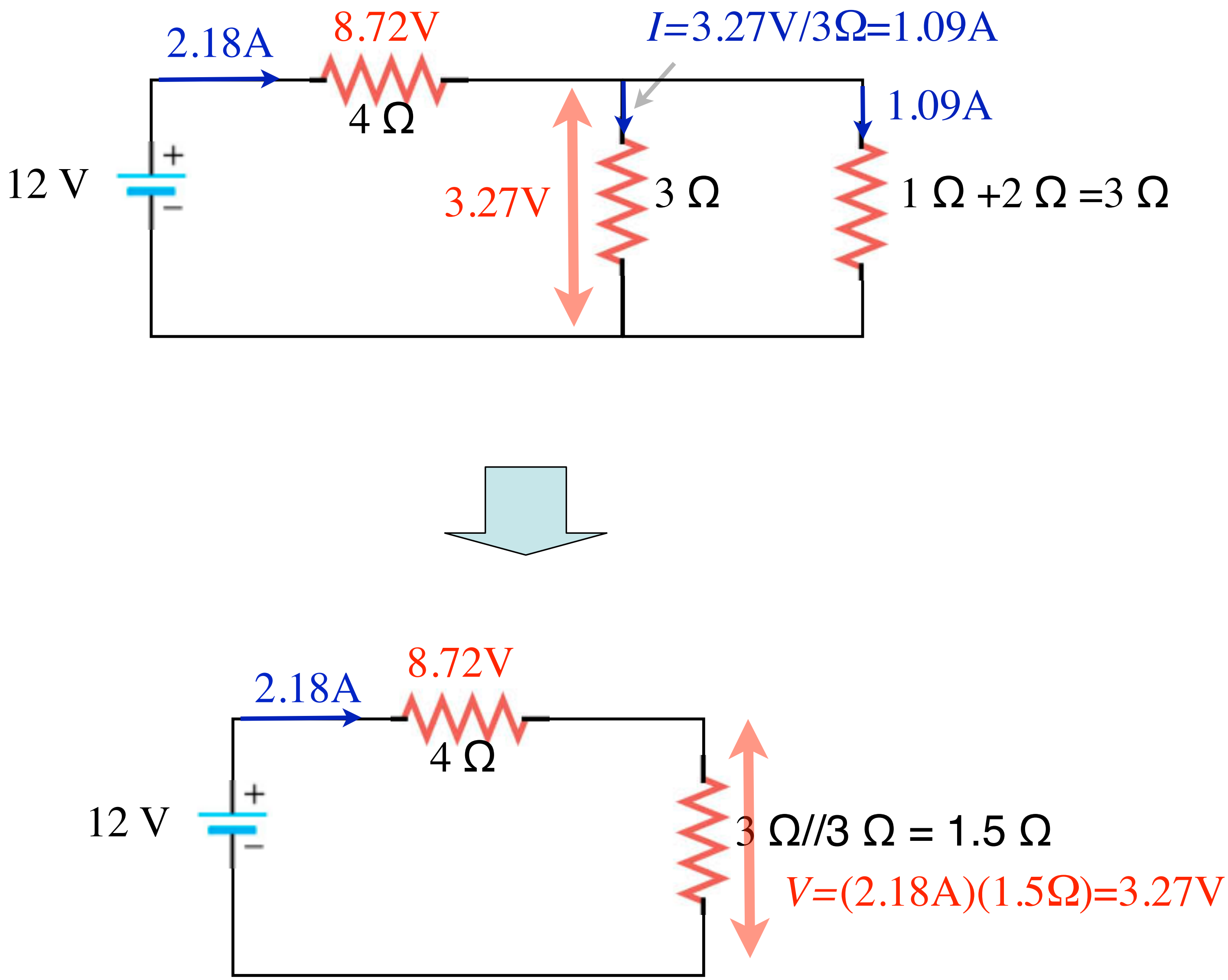
a) Reduce stepwise using series and parallel segments

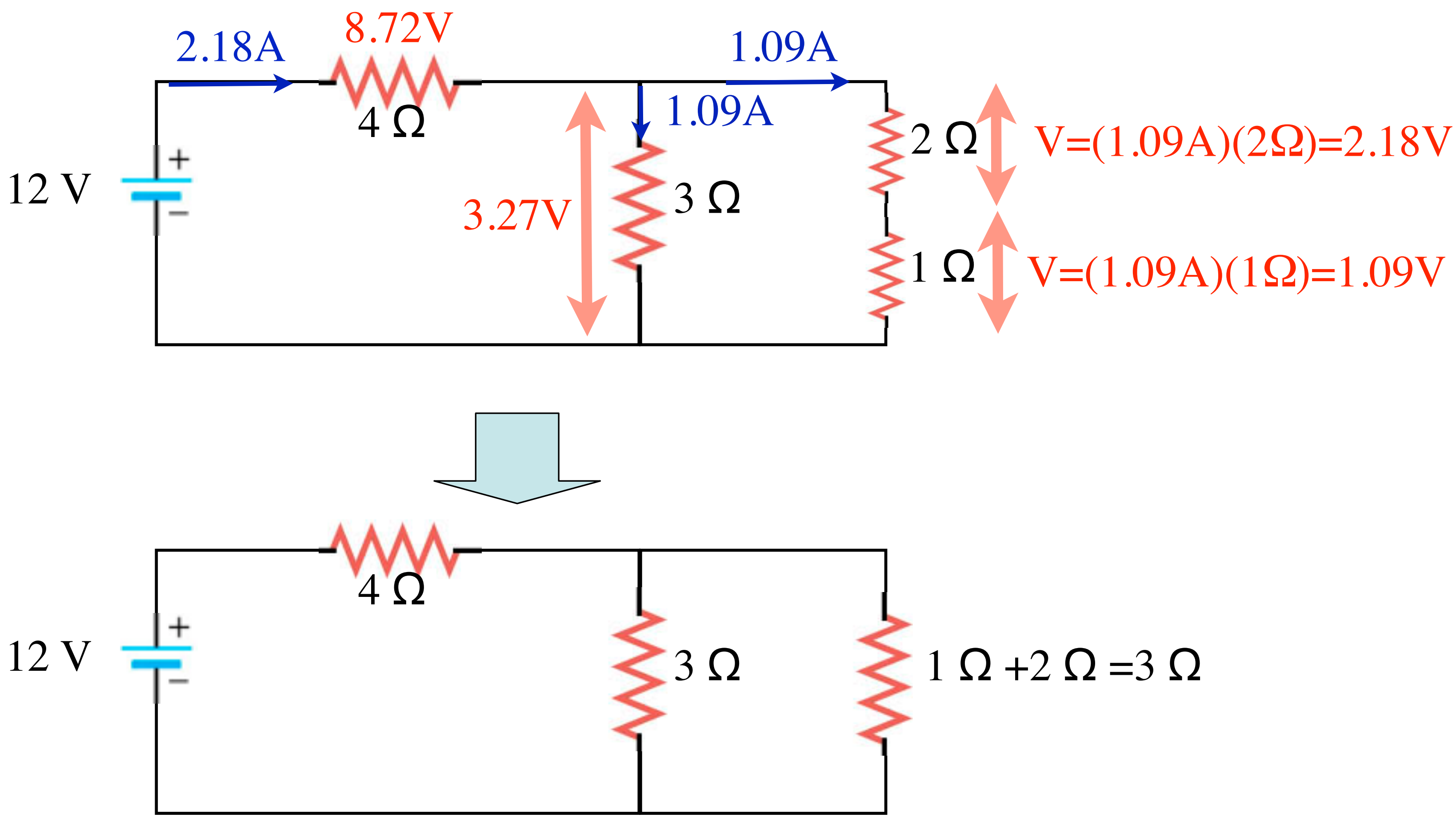


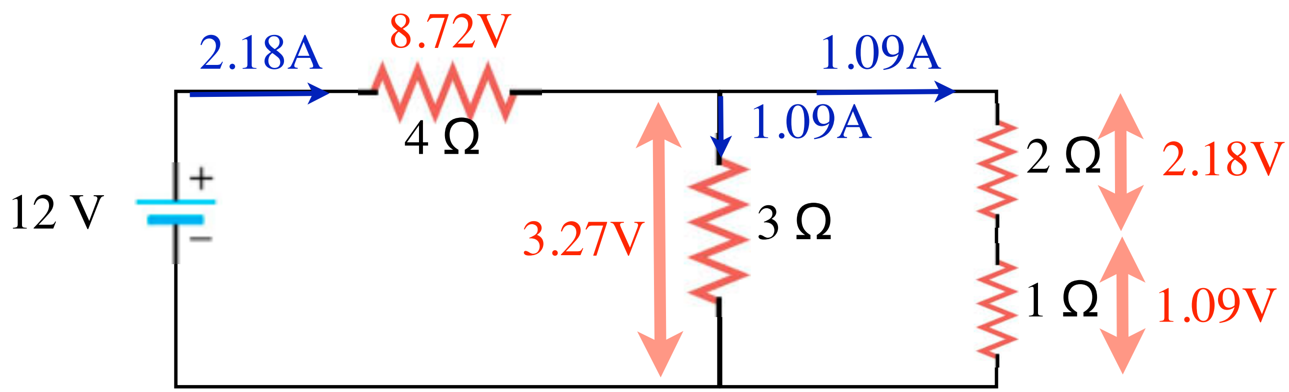

4 Ω
12 V
3 Ω
1 Ω +2 Ω =3 Ω
4 Ω
12 V
3 Ω//3 Ω = 1.5 Ω



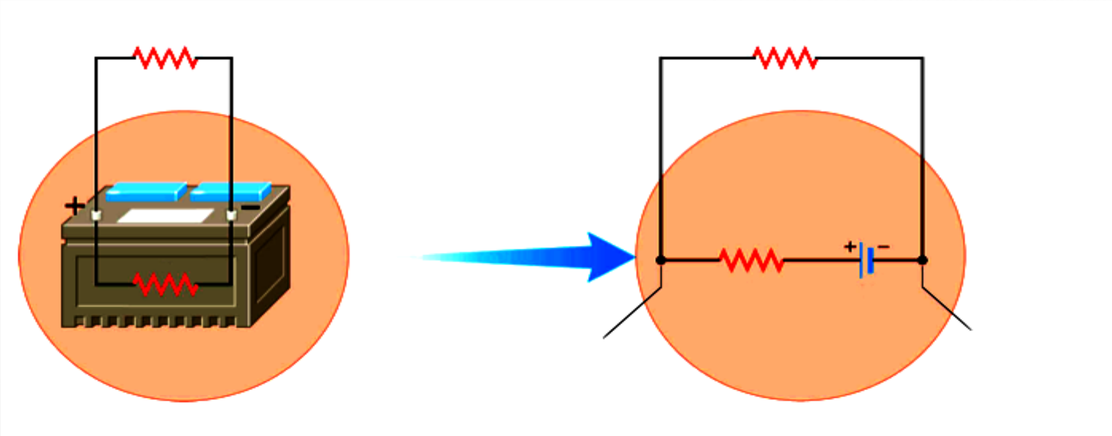
V=(2.18A)(4Ω)=8.72V
2.18A
4 Ω
12 V
+
−
3 Ω//3 Ω = 1.5 Ω
I=12V/5.5Ω =2.18A
12 V
+
−
4 Ω + 1.5 Ω = 5.5 Ω

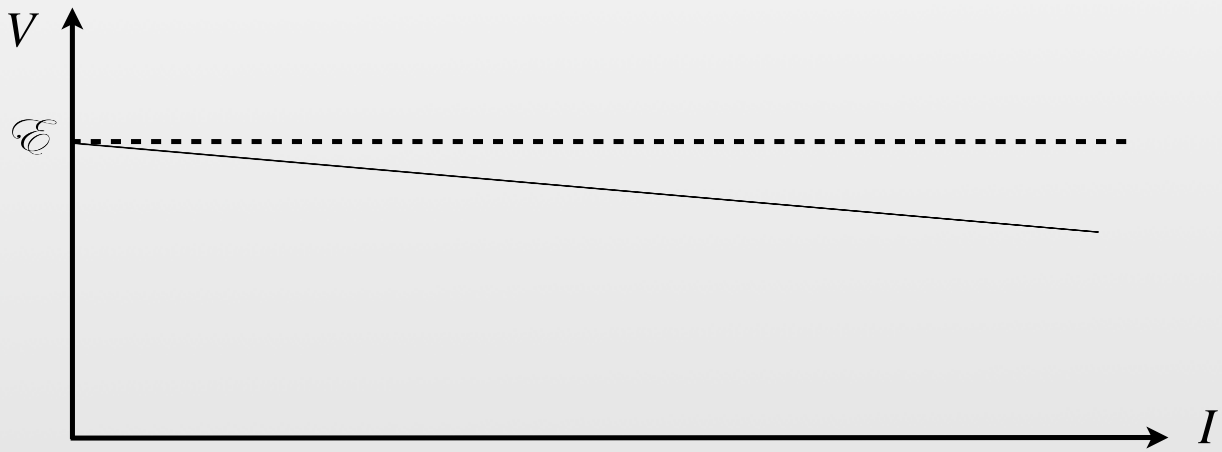

2.18A
8.72V
I=3.27V/3Ω=1.09A
1.09A
4 Ω
12 V
3.27V
3 Ω
1 Ω +2 Ω =3 Ω
2.18A
8.72V
4 Ω
12 V
3 Ω//3 Ω = 1.5 Ω
V=(2.18A)(1.5Ω)=3.27V


2.18A
8.72V
1.09A
1.09A
4 Ω
12 V
3.27V
3 Ω
2 Ω
V=(1.09A)(2Ω)=2.18V
1 Ω
V=(1.09A)(1Ω)=1.09V
4 Ω
12 V
3 Ω
1 Ω +2 Ω =3 Ω

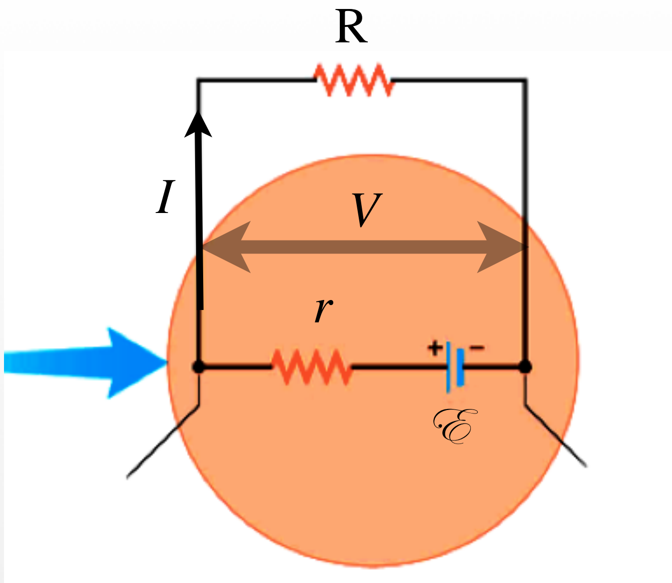

2.18A
8.72V
1.09A
4 Ω
1.09A
2 Ω
2.18V
12 V
+
−
3 Ω
3.27V
1 Ω
1.09V

6) *Internal Resistance*





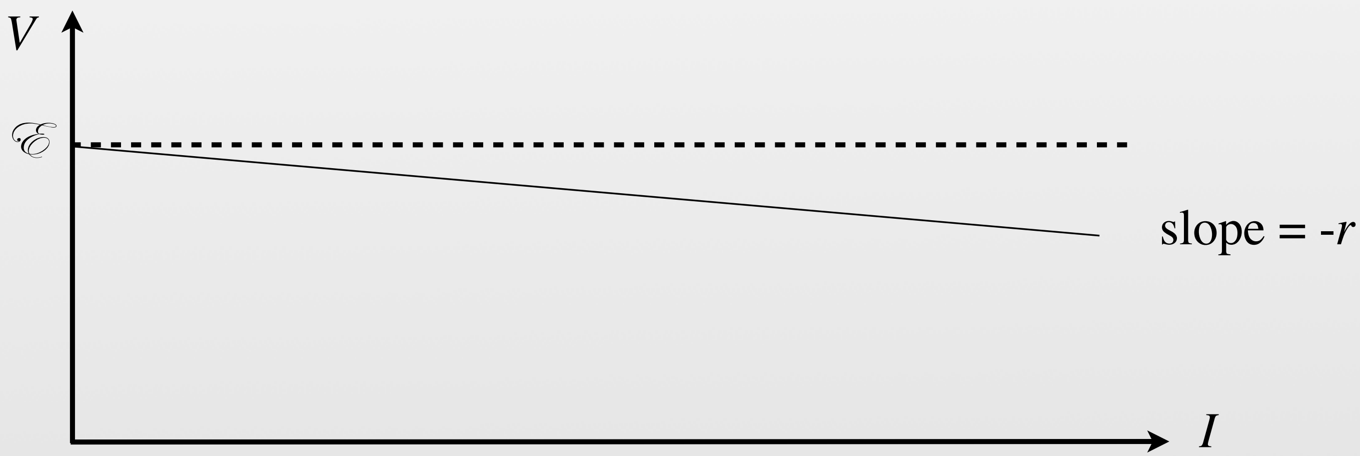
Terminal voltage (*V*) depends on current (linear)



$\mathcal{E} = I(r+R) \quad R = \mathcal{E}/I - r$

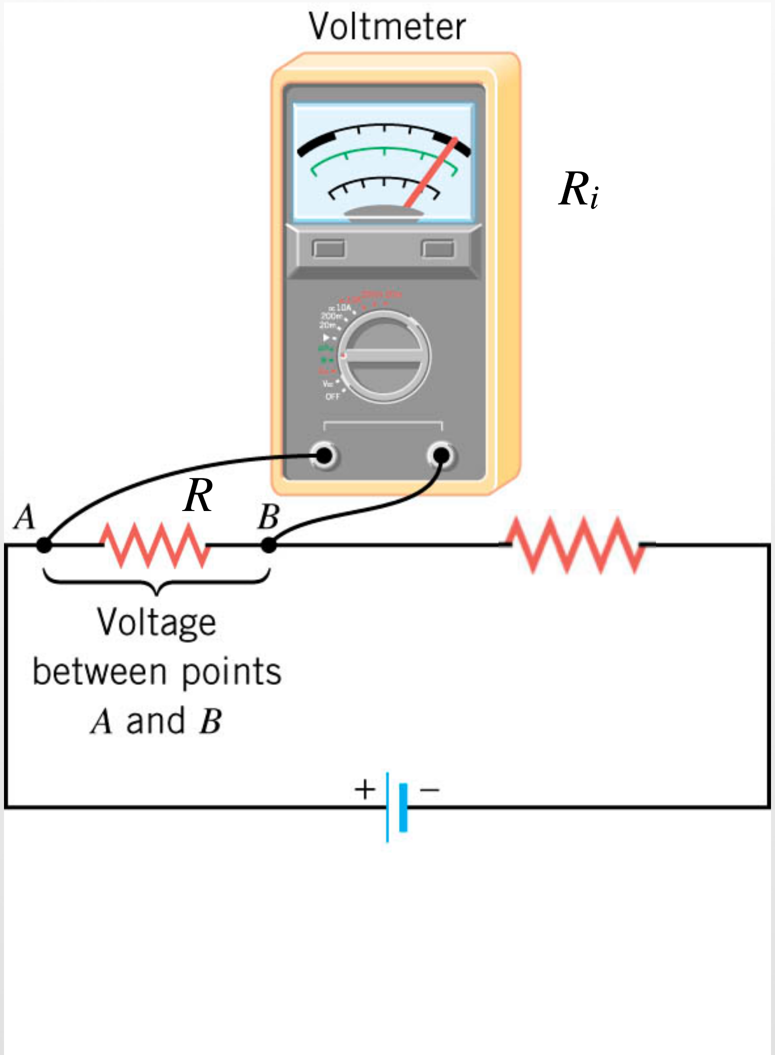
$V = IR$

$V = -rI + \mathcal{E}$



7) Current and volt meters

a) Voltmeter



Digital Multimeter

- measures voltage between 2 points (connect in parallel)

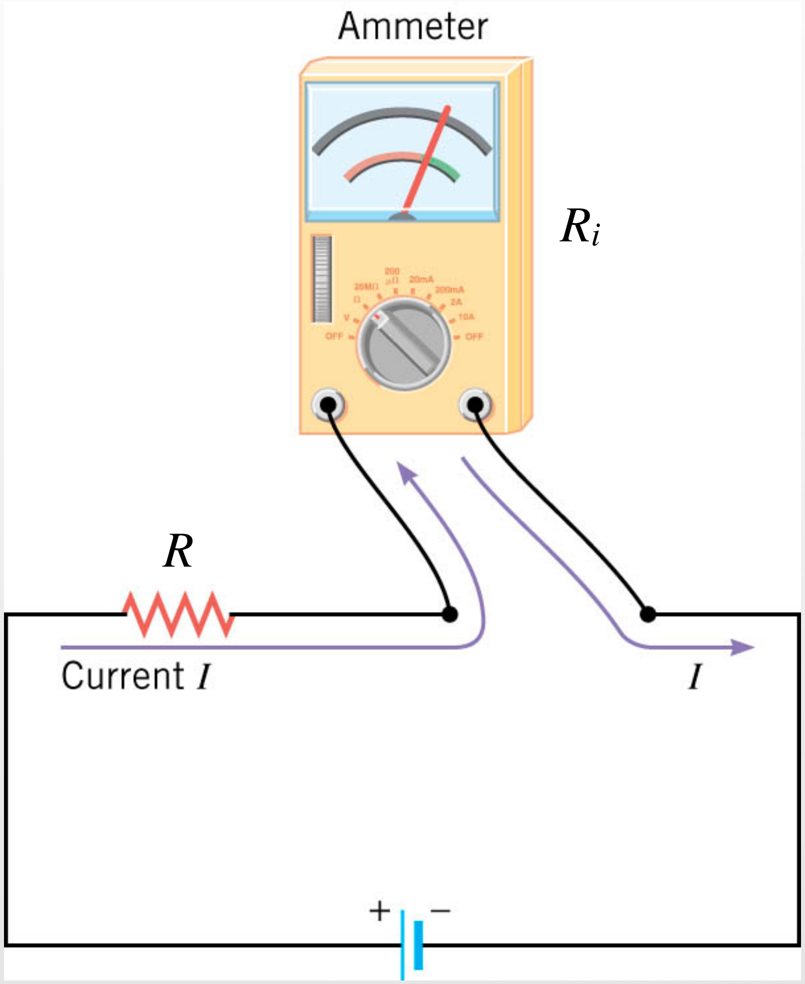
If $R_i \gg R$, then $R_i \,//\, R \cong R$

- high resistance better

  digital: $R_i > 10\ \mathrm{M\Omega}$

  analog: $R_i < 1\ \mathrm{M\Omega}$

b) Ammeter



- measures current *through* wire (connect in series)

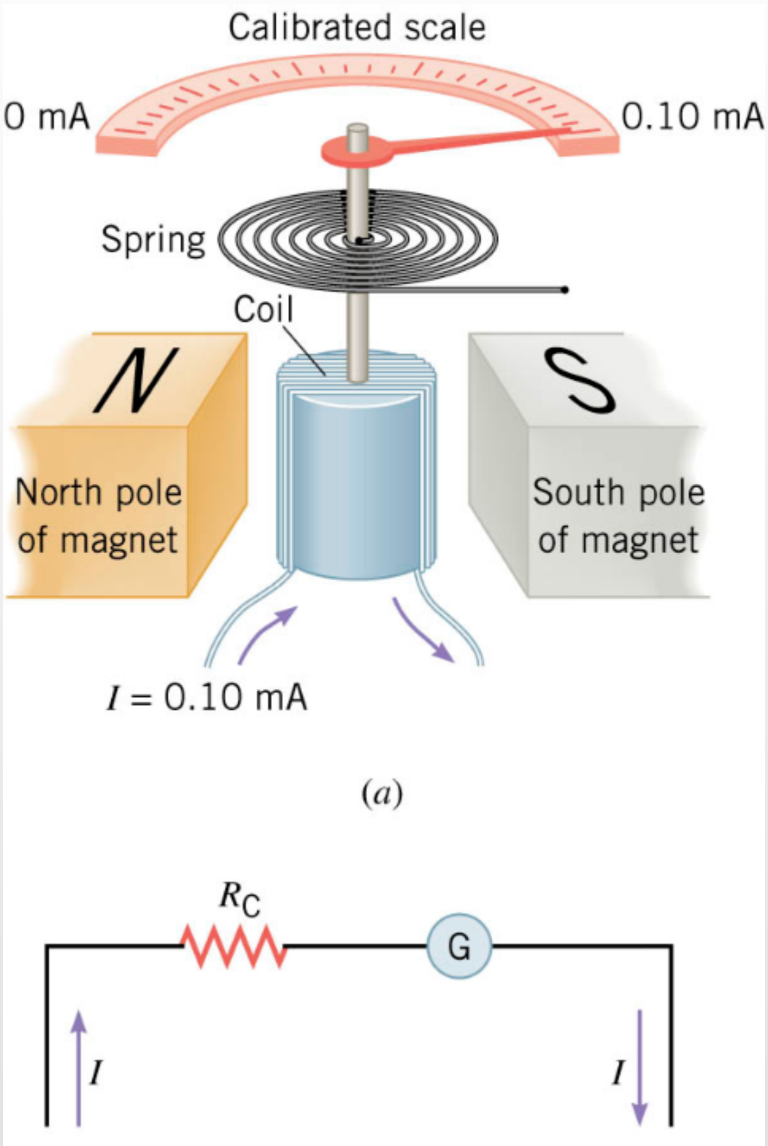
If $R_i \ll R$, then $R_i + R \cong R$

- low resistance better

$R_i < 1\ \Omega$

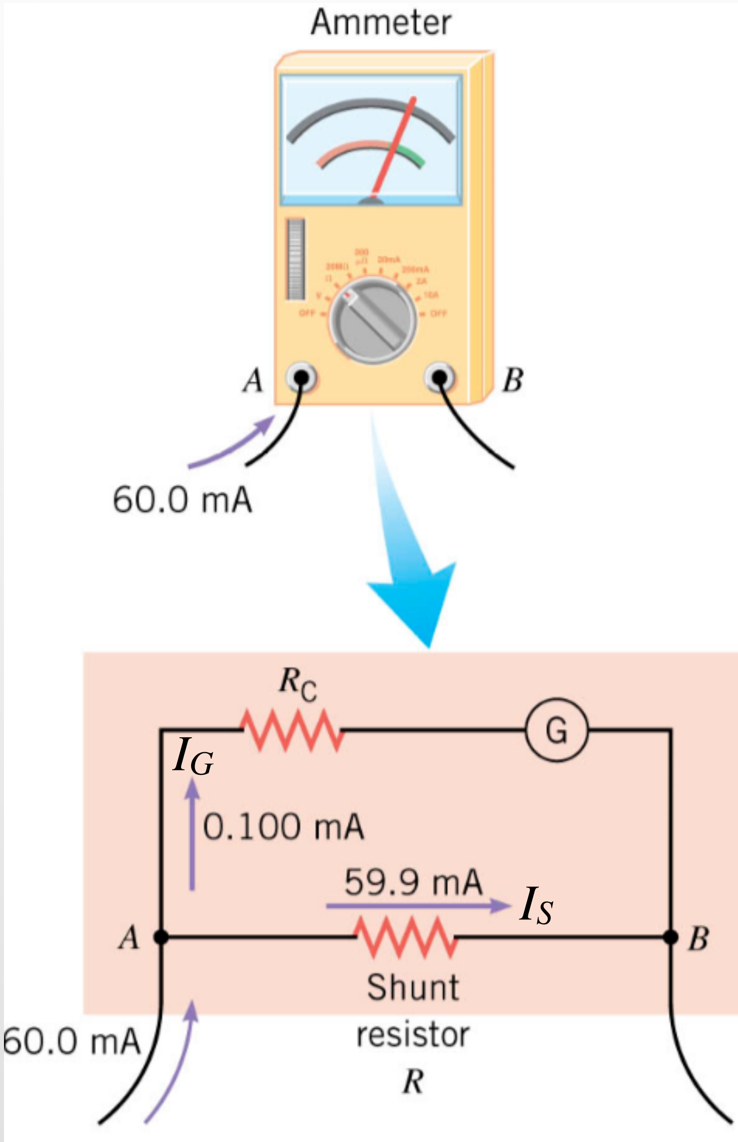

VC9805 MULTIMETER
POWER
LIGHT
HOLD
AUTO POWER OFF
CE
Ω
2k
20k
200k
2M
200kHz
200m
2
20
V
200
1000
TEMP
2m
20m
200m
20
A
2mH
20mH
200mH
2H
20H
L
CAT II
20A
mA
COM
VΩHz
MAX
200mA
FUSED
UNFUSED

c) Galvonometer



- $R_C \sim 50\ \Omega$
- Maximum current ~ 100 μA
- As is, not practical as an ammeter or voltmeter

d) Analog ammeter

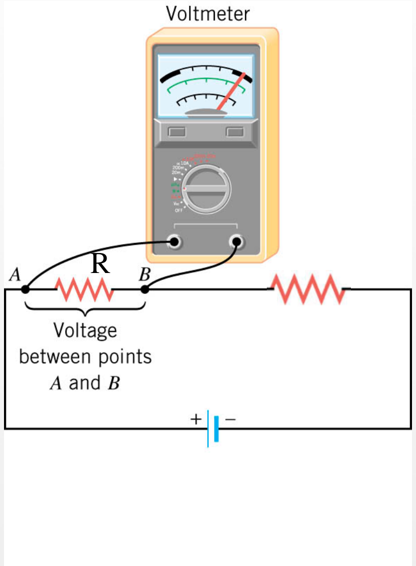


- Add parallel shunt resistor

$$I_G \ll I_S$$

$$R_S \ll R_C$$

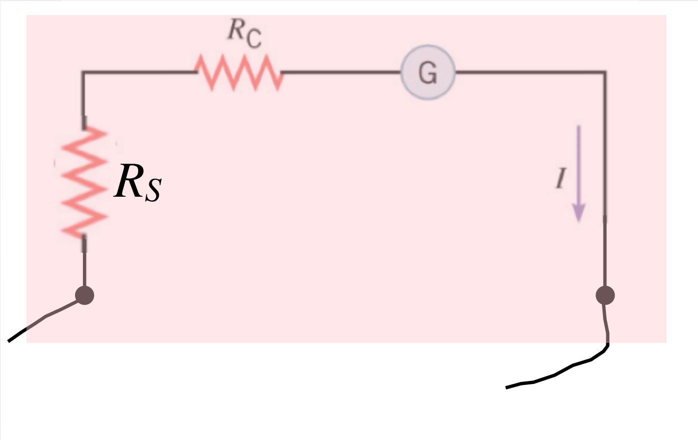
e) Analog voltmeter



- Add series resistor

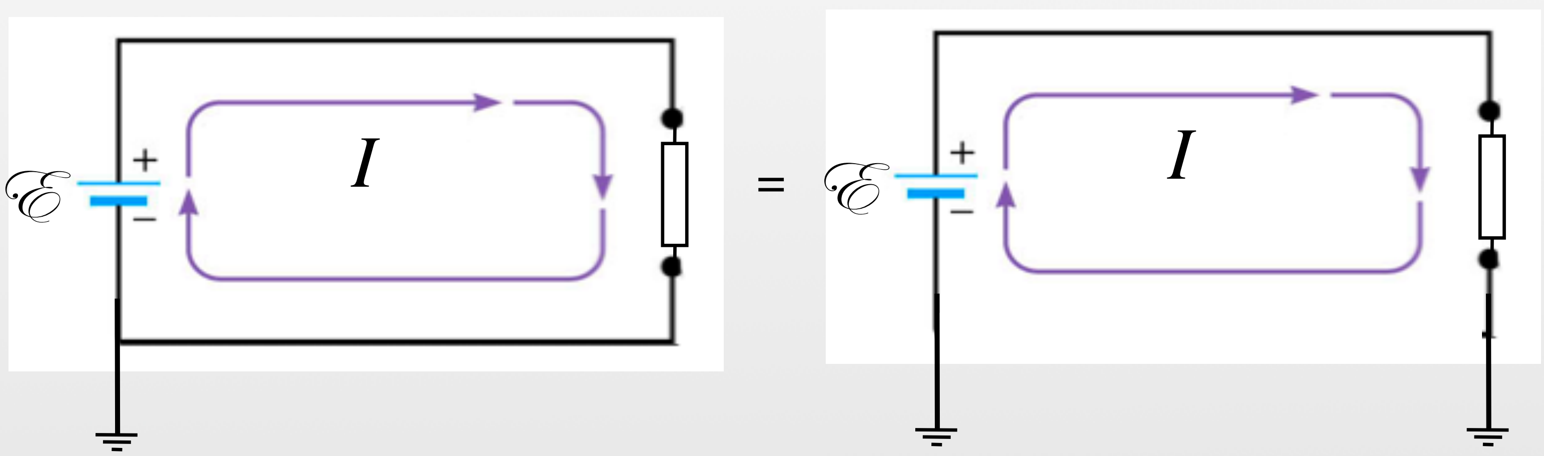
$$I_G << I_R$$

$$R_S >> R$$



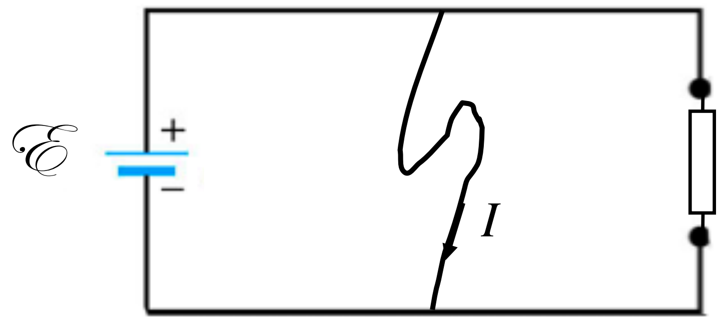
8) ground, open, short circuits

One point may be referred to as ground

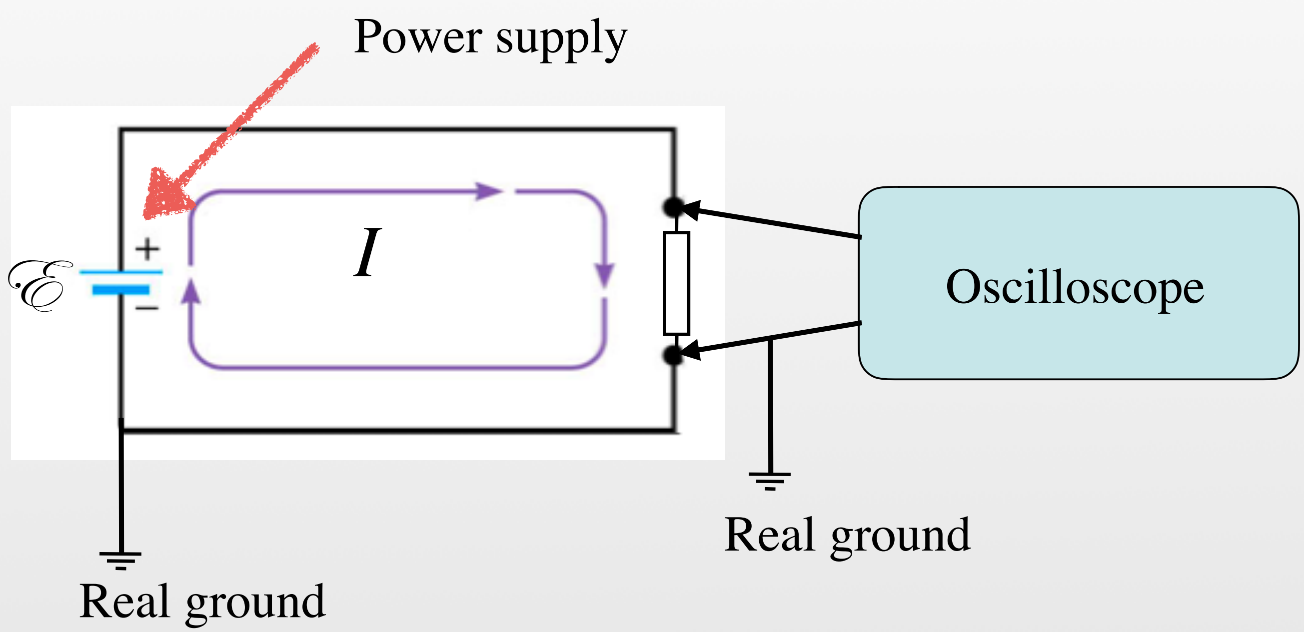


The ground may be connected to “true” ground through water pipes, for example.

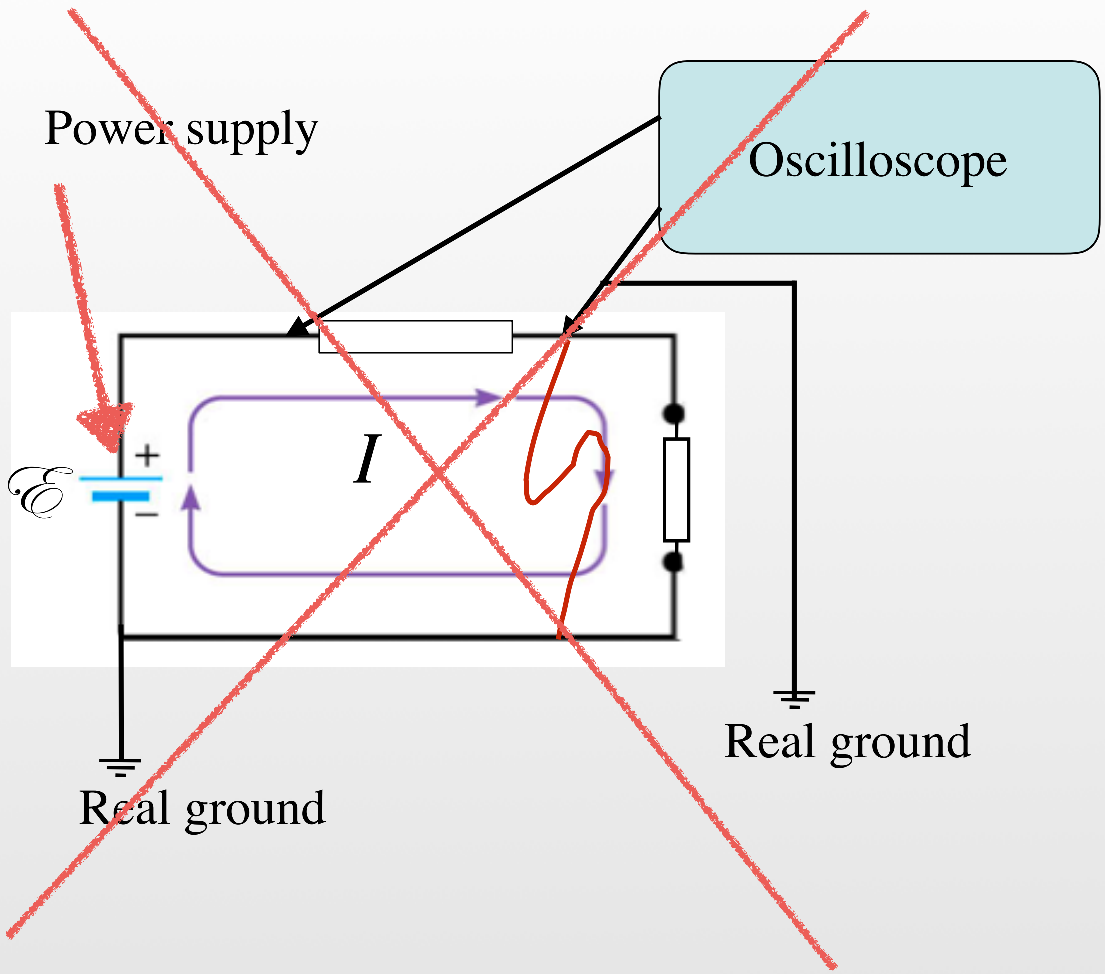
Short circuit



Open circuit

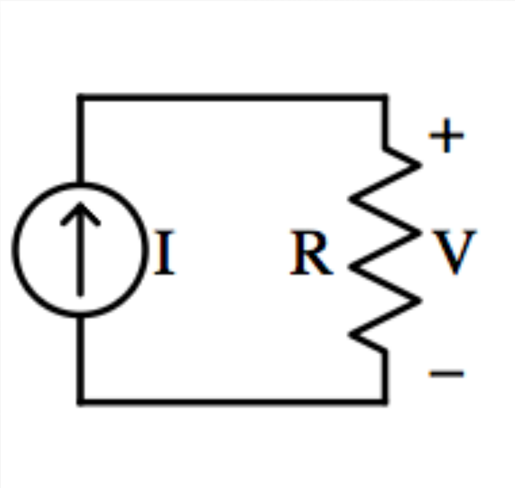


Power supply
$\mathcal{E}$
+
−
I
Oscilloscope
Real ground
Real ground


Power supply
Oscilloscope
I
ℰ
+
−
Real ground
Real ground

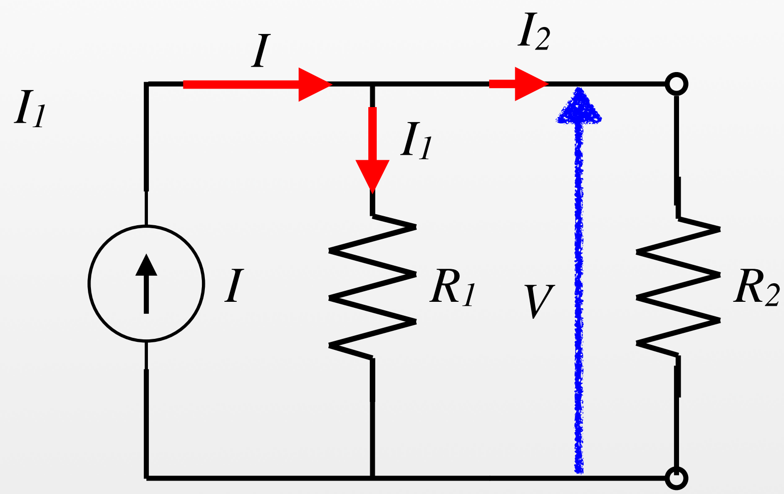
9) Equivalent circuits

a) constant current source



Current independent of R,
so larger R —> larger power

Current divider



Find I_2

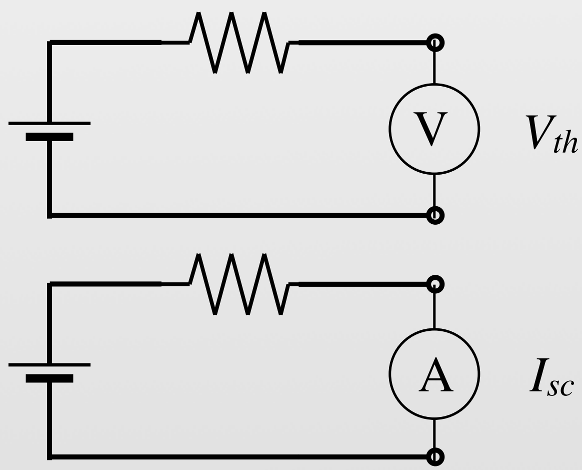
$$I_2 = \frac{V}{R_2} \qquad V = I(R_1//R_2) = I\frac{R_1 R_2}{R_1 + R_2}$$

$$I_2 = I\frac{R_1}{R_1 + R_2}$$

b) Thevenin's theorem

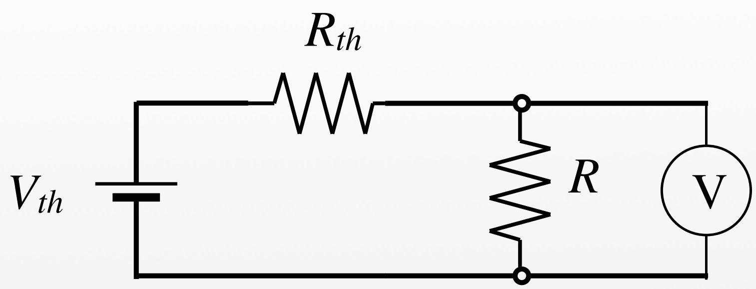
Any network of sources (voltage or current) and resistors with 2 terminals can be replaced by a combination of 1 ideal battery and 1 resistor:

where V_{th} is the open circuit voltage



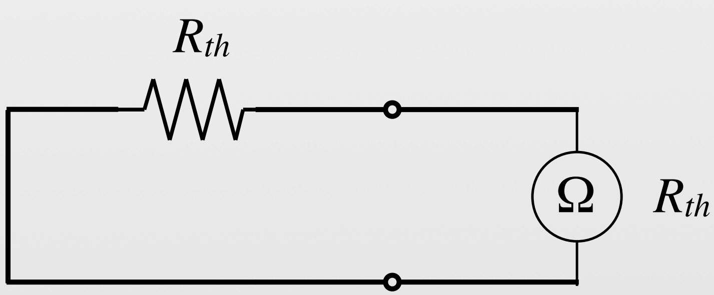
and $$R_{th} = \frac{V_{th}}{\text{closed circuit current}}$$

Also



when V = $V_{th}/2$, then $R = R_{th}$

Also

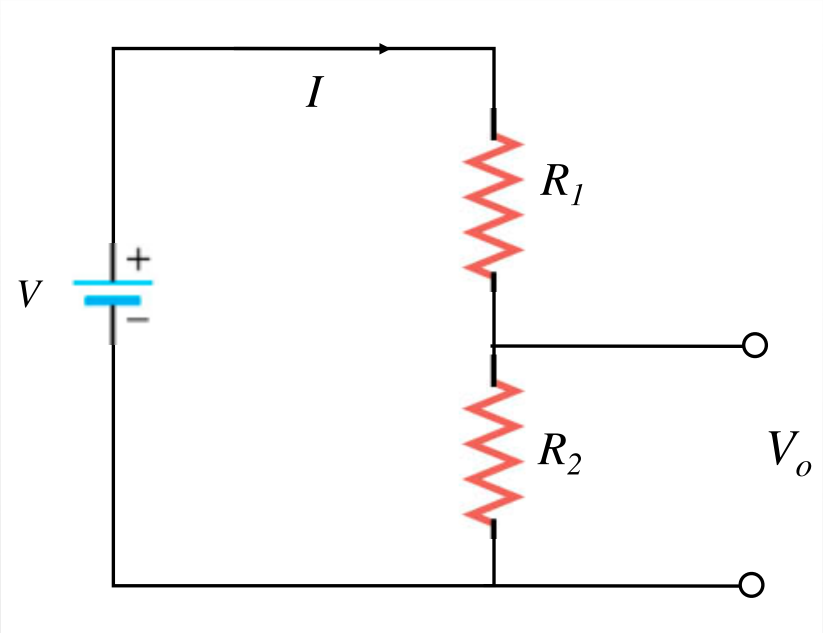


with voltage sources shorted (and current sources open), the resistance appearing between the terminals is the Thevenin resistance

- Thevenin voltage is open circuit voltage

- Thevenin resistance is resistance with voltage sources shorted.

$e.g.$ Voltage divider



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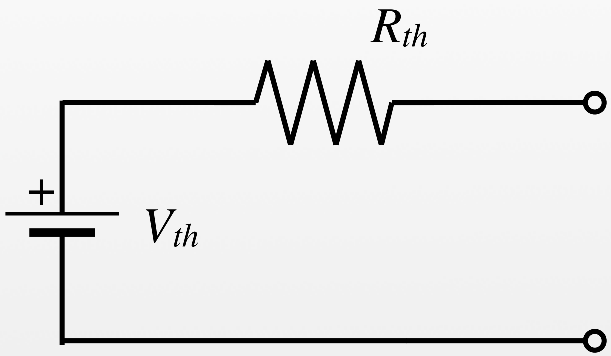


$$V_{th} = \frac{R_2}{R_1 + R_2} V$$

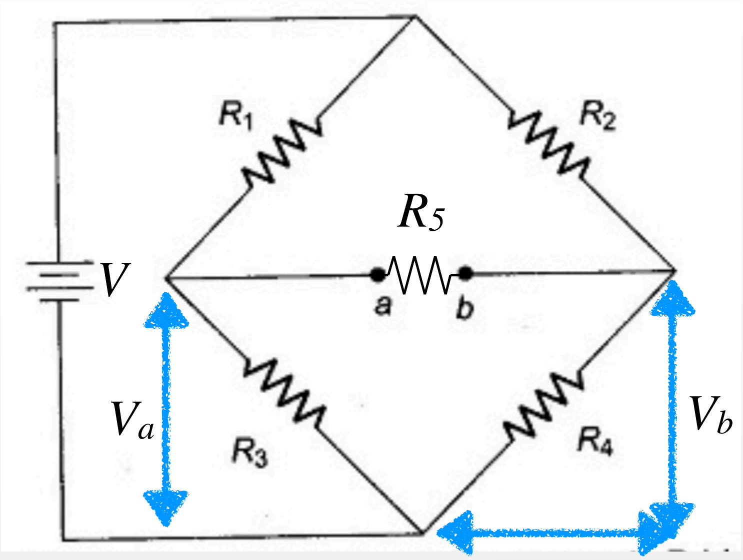
$$R_{th} = R_1 \,||\, R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

also

$$I_{sc} = \frac{V}{R_1}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{R_2}{R_1 + R_2} V \frac{R_1}{V} = R_1 \,||\, R_2$$

e.g. Wheatstone Bridge: Find voltage across and current through R_5



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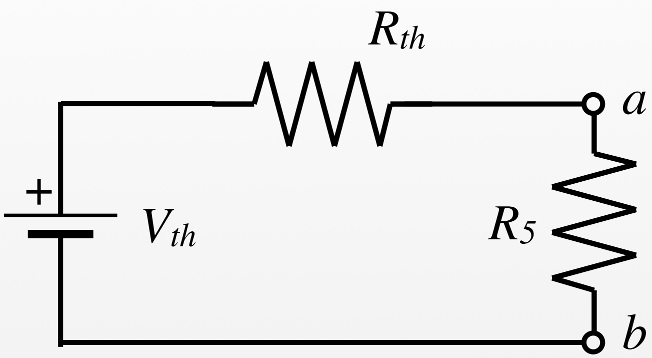
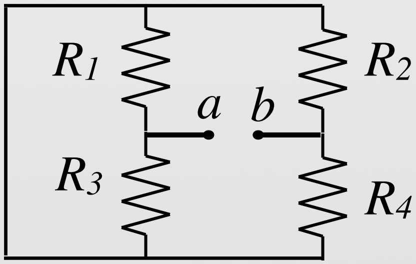
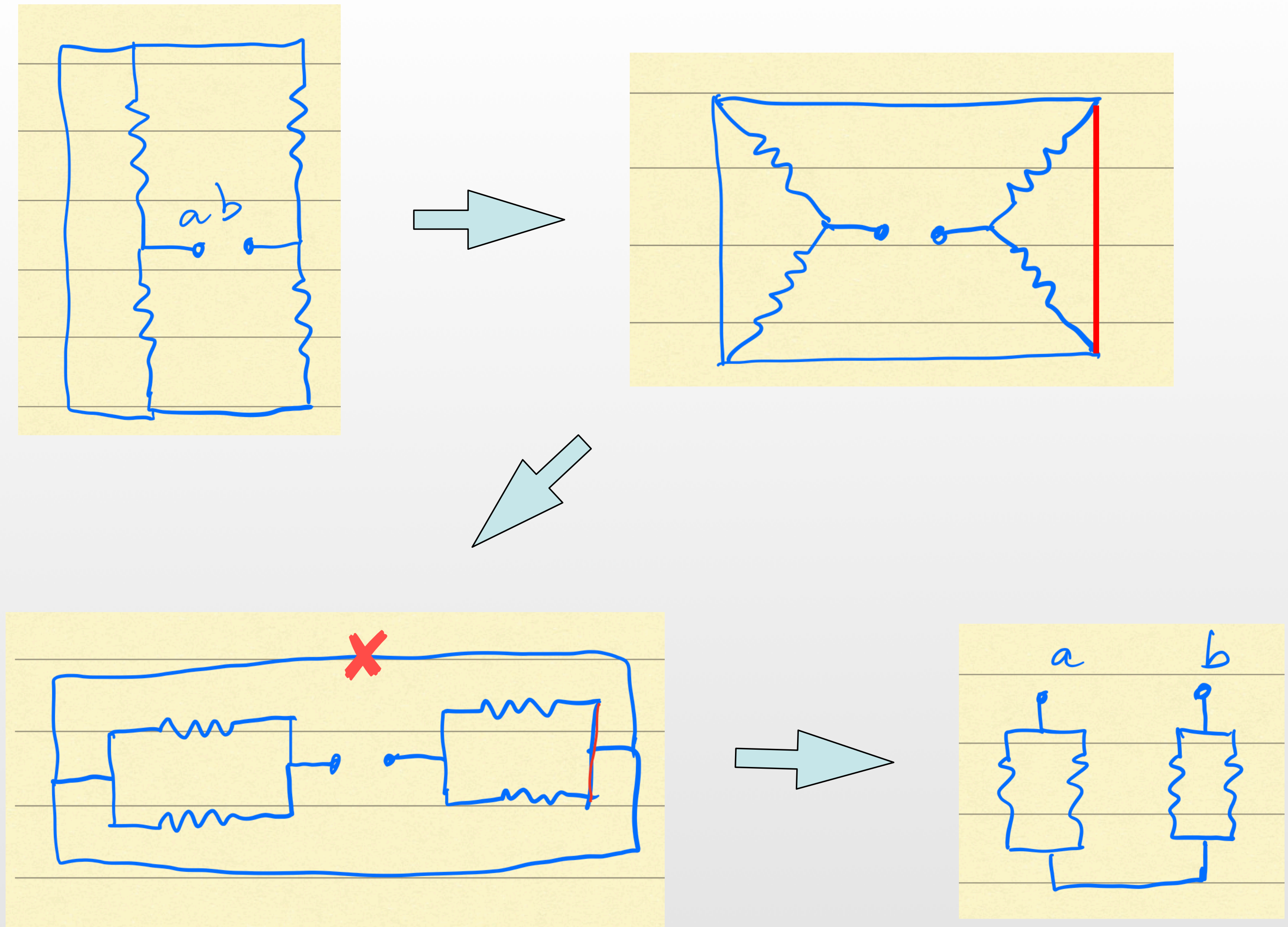


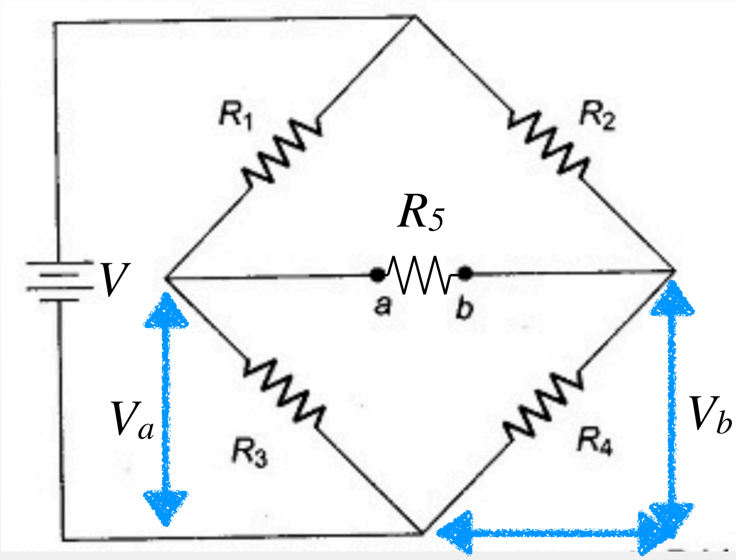
• Open circuit voltage: $$V_{th} = V_a - V_b = V\frac{R_3}{R_1 + R_3} - V\frac{R_4}{R_2 + R_4}$$

• Replace V with short circuit to give R_{th}




a
b
a
b

e.g. Wheatstone Bridge: Find voltage across and current through R_5



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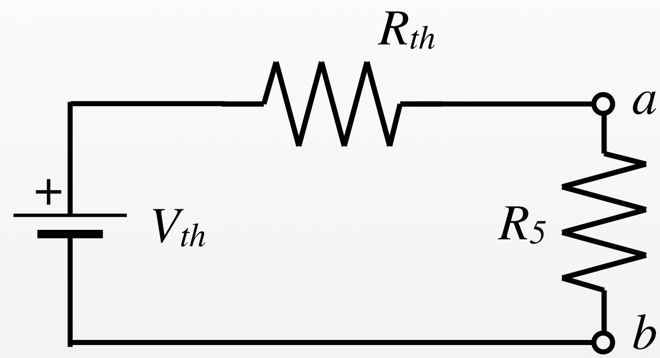
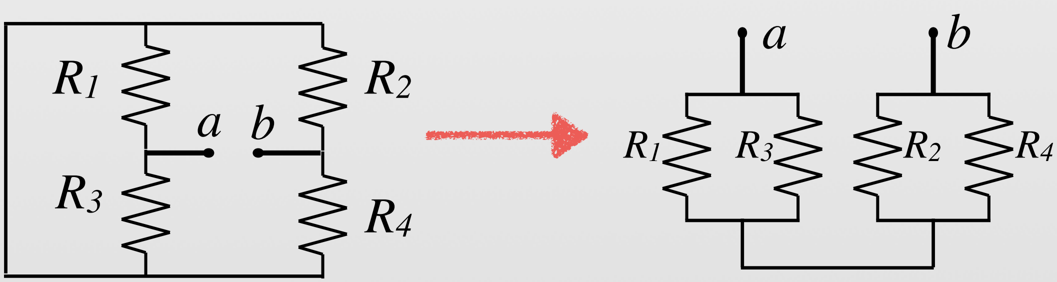
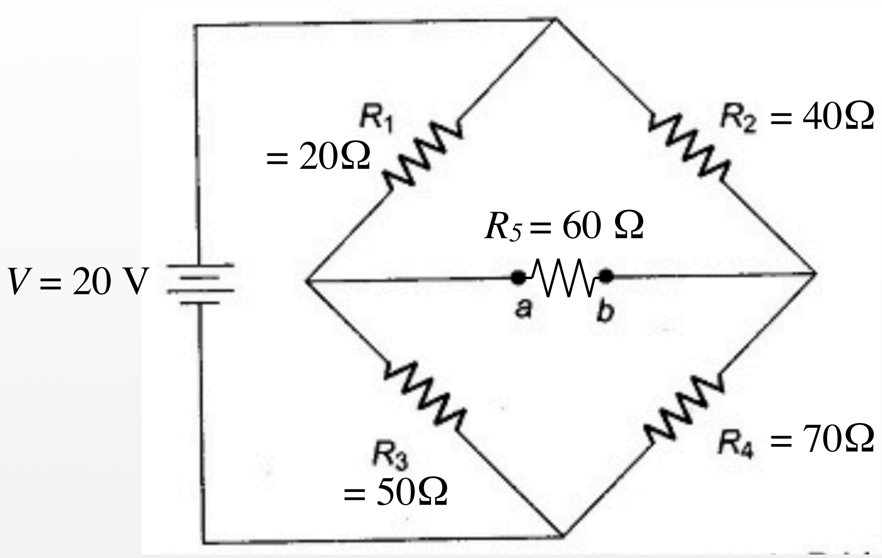


- Open circuit voltage: $$V_{th} = V_a - V_b = V\frac{R_3}{R_1+R_3} - V\frac{R_4}{R_2+R_4}$$

- Replace V with short circuit to give R_{th}



$$R_{th} = R_1 \,||\, R_3 + R_2 \,||\, R_4$$



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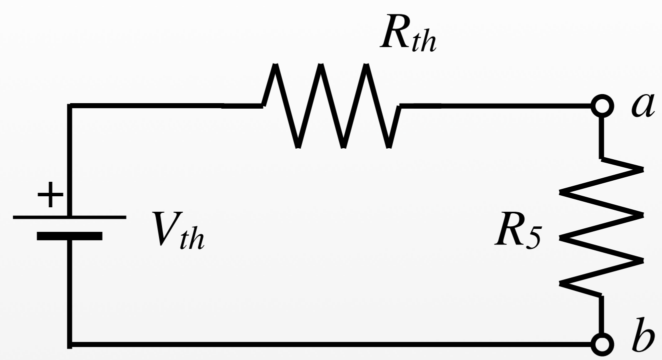


Find V_{ab} and the current through R_5

Using Thevenin equivalent cct:

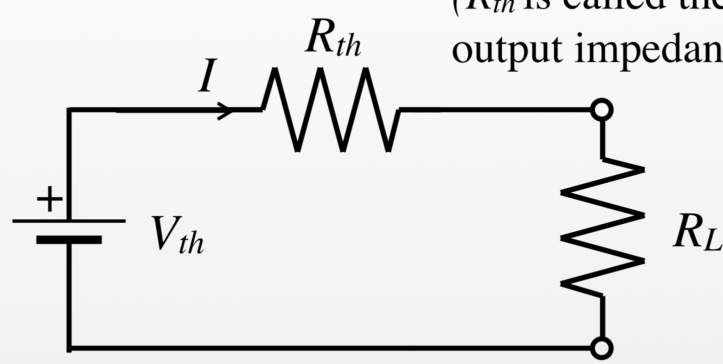
$$V_{th} = V\frac{R_3}{R_1 + R_3} - V\frac{R_4}{R_2 + R_4} = 1.56\ \text{V}$$

$$R_{th} = R_1 \parallel R_3 + R_2 \parallel R_4 = 39.7\Omega$$

$$I_5 = \frac{V_{th}}{R_{th} + R_5} = 0.0156\ \text{A}$$

$$V_5 = V_{ab} = I_5 R_5 = 0.936\ \text{V}$$

Maximum power transfer for a Thevenin circuit:



(R_{th} is called the output resistance (or output impedance) of the circuit.)

The power delivered to the load resistance is:

$$P = I^2 R_L = \left(\frac{V}{R_{th}+R_L}\right)^2 R_L \quad \text{—> 0 for } R_L = 0$$

$$= \frac{V^2 / R_L}{(1+R_{th}/R_L)^2} \quad \text{—> 0 for } R_L \text{ —> } \infty$$

Maximum power corresponds to $\dfrac{dP}{dR_L} = 0$

$$\frac{dP}{dR_L} = V^2 \frac{\left((R_{th}+R_L)^2 - R_L 2(R_{th}+R_L)\right)}{(R_{th}+R_L)^2} = 0 \quad \rightarrow (R_{th}+R_L)^2 = R_L 2(R_{th}+R_L)$$

$$\rightarrow R_{th}^2 + 2R_{th}R_L + R_L^2 = 2R_L R_{th} + 2R_L^2$$

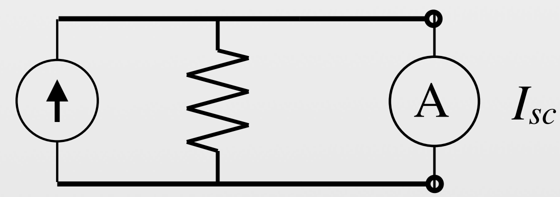
$$\rightarrow \boxed{R_{th} = R_L}$$

c) Norton 's theorem

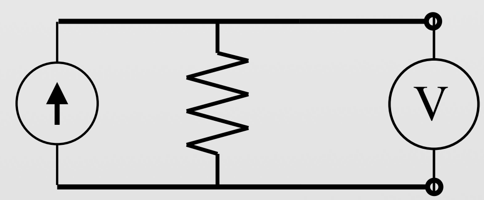
Any network of sources (voltage or current) and resistors with 2 terminals can be replaced by a combination of 1 current source and 1 resistor:

where I_n is the short circuit current

$$I_n = \frac{V_{th}}{R_{th}}$$



and $$R_n = \frac{V_{oc}}{I_n} = \frac{V_{th}}{I_{sc}} = R_{th}$$

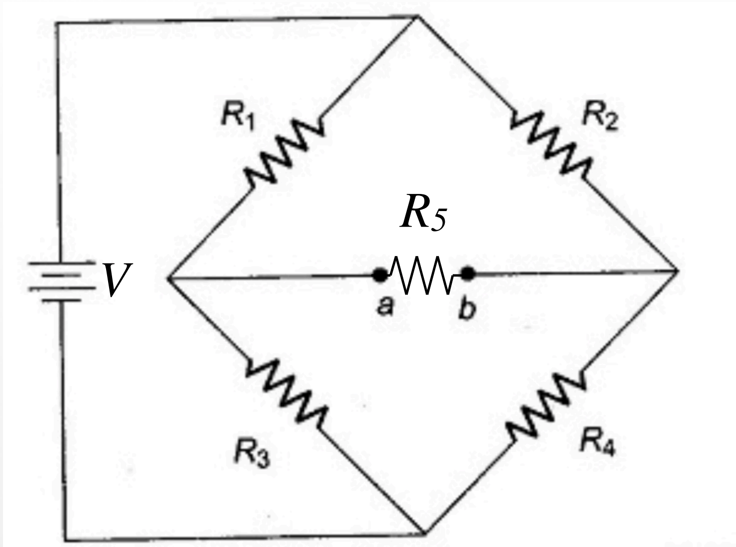


which is the resistance with the current source open

- Norton resistance = Thevenin resistance (resistance with voltages shorted and current sources open)

- Norton current = short cct current (or $I_n = \dfrac{V_{oc}}{R_{th}}$)

e.g. Wheatstone Bridge: Find voltage across and current through R_5



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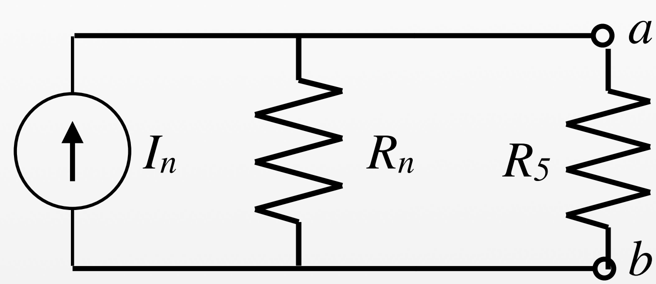
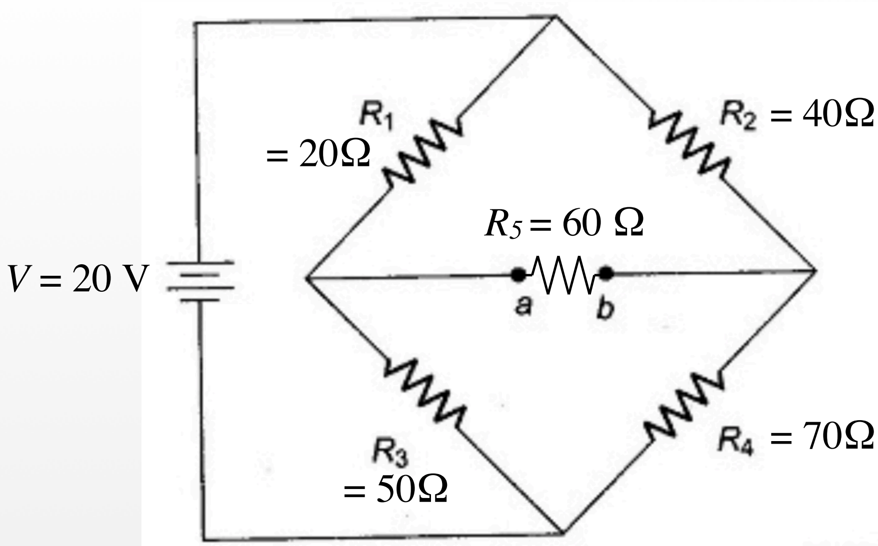


- Norton resistance: $R_n = R_{th} = R_1 \parallel R_3 + R_2 \parallel R_4$

- Norton current: $$I_n = \frac{V_{th}}{R_n} = \frac{V\left(\frac{R_3}{R_1 + R_3} + \frac{R_4}{R_2 + R_4}\right)}{R_n}$$



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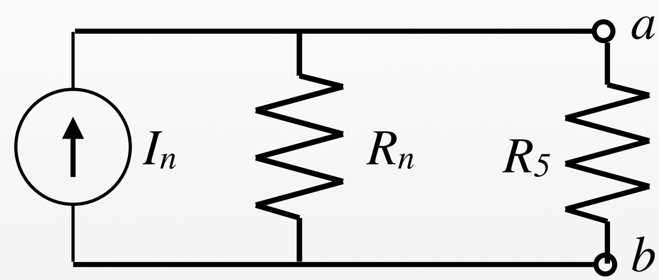


Find V_{ab} and the current through R_5

Using Norton equivalent cct:

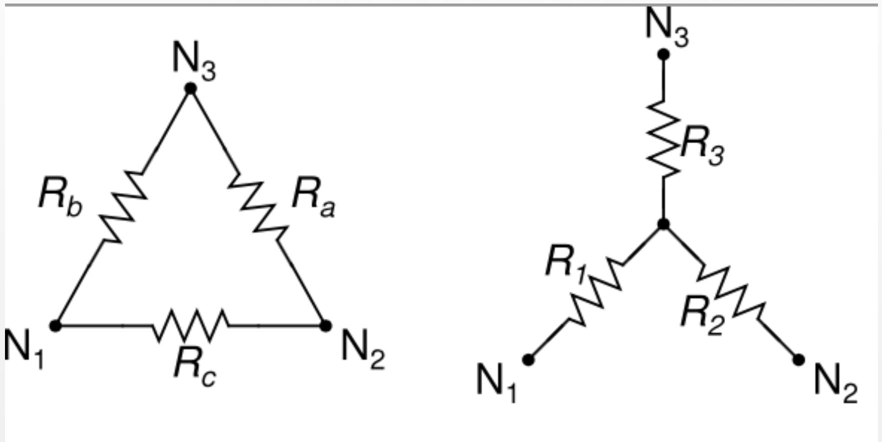
$$R_n = R_{th} = R_1 \parallel R_3 + R_2 \parallel R_4 = 39.7\Omega$$

$$I_n = \frac{V_{th}}{R_n} = \frac{V\left(\frac{R_3}{R_1 + R_3} + \frac{R_4}{R_2 + R_4}\right)}{R_n} = 0.0393 \text{ A}$$

$$I_5 = I_n \frac{R_n}{R_5 + R_n} = 0.0156A$$

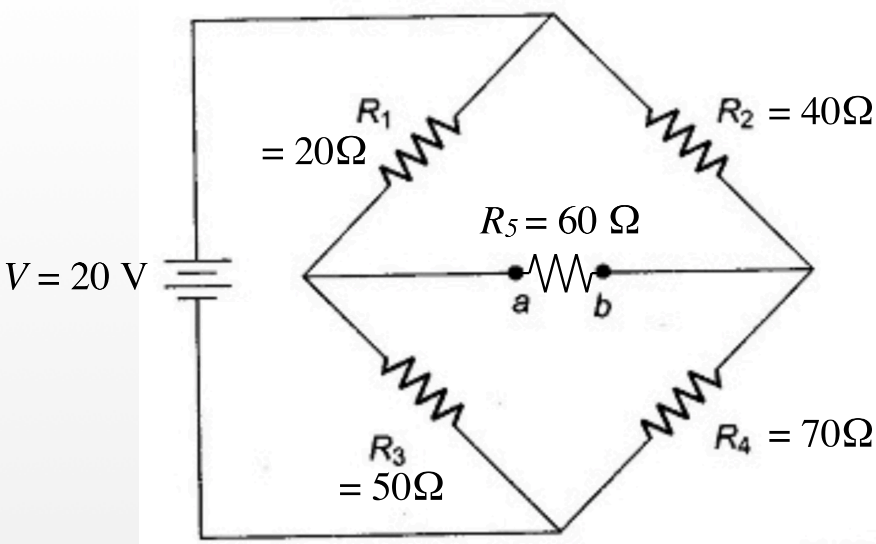
$$V_{ab} = I_5 R_5 = 0.943V$$

d) Y-Δ transforms



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



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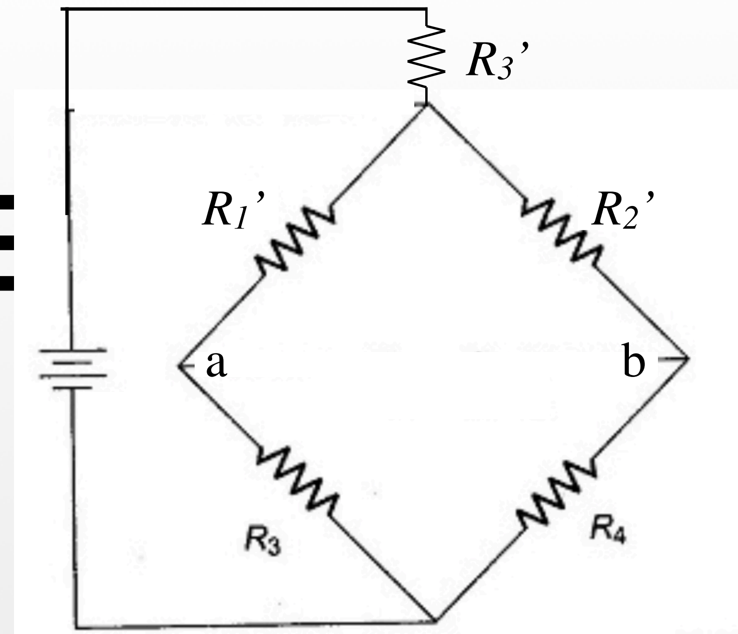
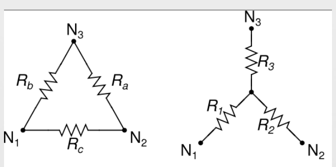


Find V_{ab} and the current through R_5

<u>Using Δ-Y transform:</u>

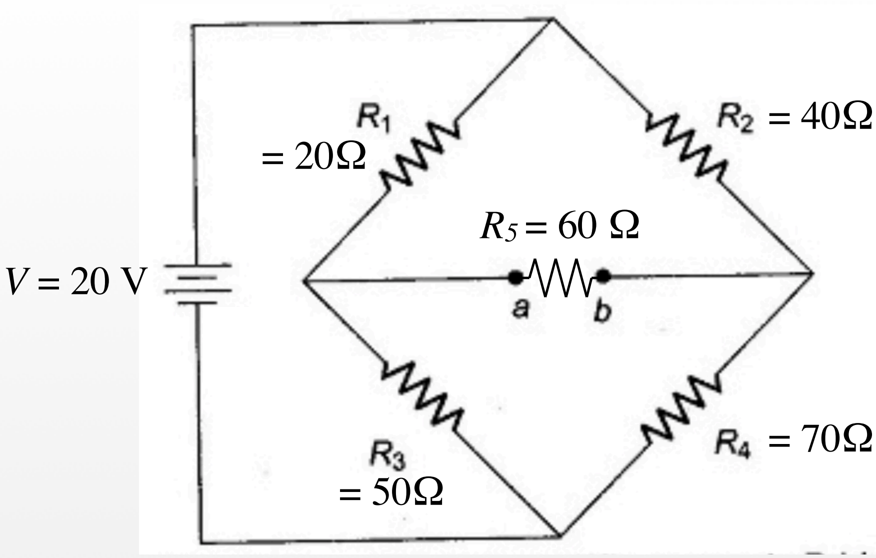


$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_1' = \frac{R_1 R_5}{R_1 + R_2 + R_5} = 10\Omega$$

$$R_2' = \frac{R_2 R_5}{R_1 + R_2 + R_5} = 20\Omega$$

$$R_3' = \frac{R_1 R_2}{R_1 + R_2 + R_5} = 6.67\Omega$$



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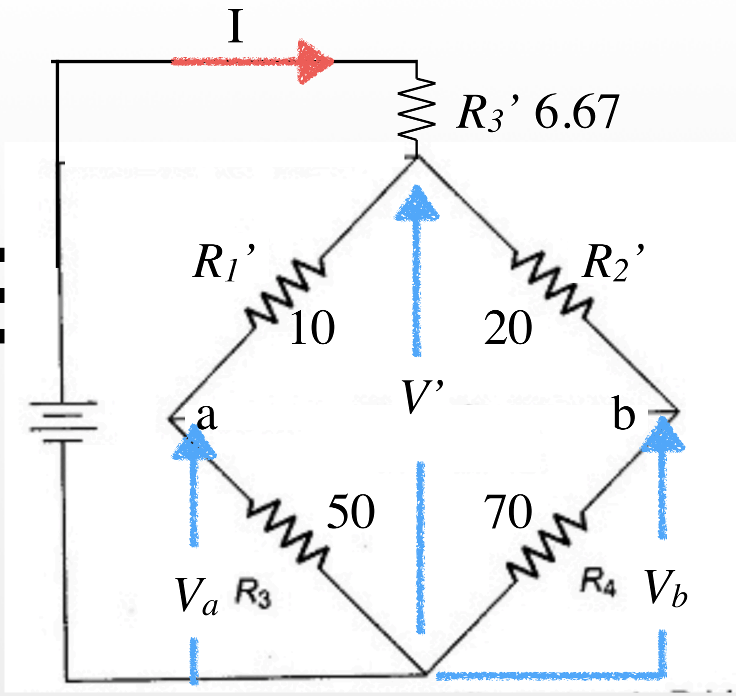
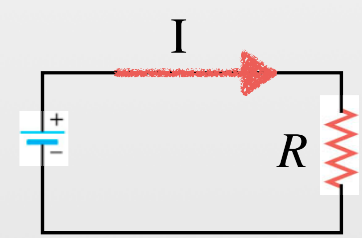


Find V_{ab} and the current through R_5

<u>*Using Δ-Y transform:*</u>



$$R = R_3' + (R_1' + R_3) \,||\, (R_2' + R_4) = 6.67\Omega + 60\Omega \,||\, 90\Omega = 42.67\Omega$$

$$I = \frac{V}{R} = 0.469 \text{ A} \longrightarrow V' = V - IR_3' = 16.87 \text{ V}$$

$$V_a = V' \frac{R_3}{R_1' + R_3} = 14.06 \text{ V}$$

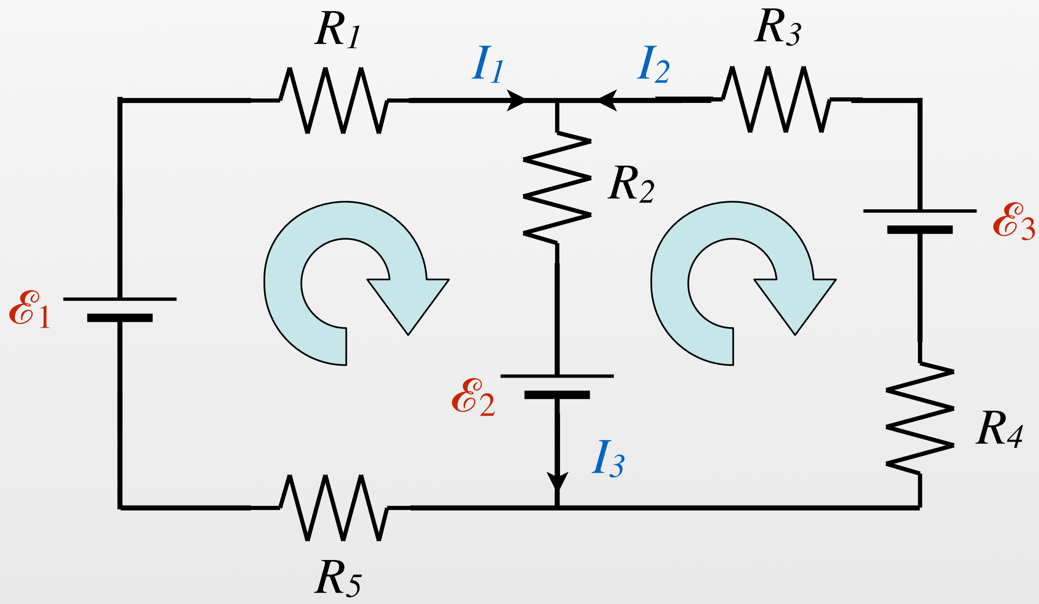
$$V_b = V' \frac{R_4}{R_2' + R_4} = 13.12 \text{ V}$$

$$V_{ab} = V_a - V_b = 0.94 \text{ A}$$

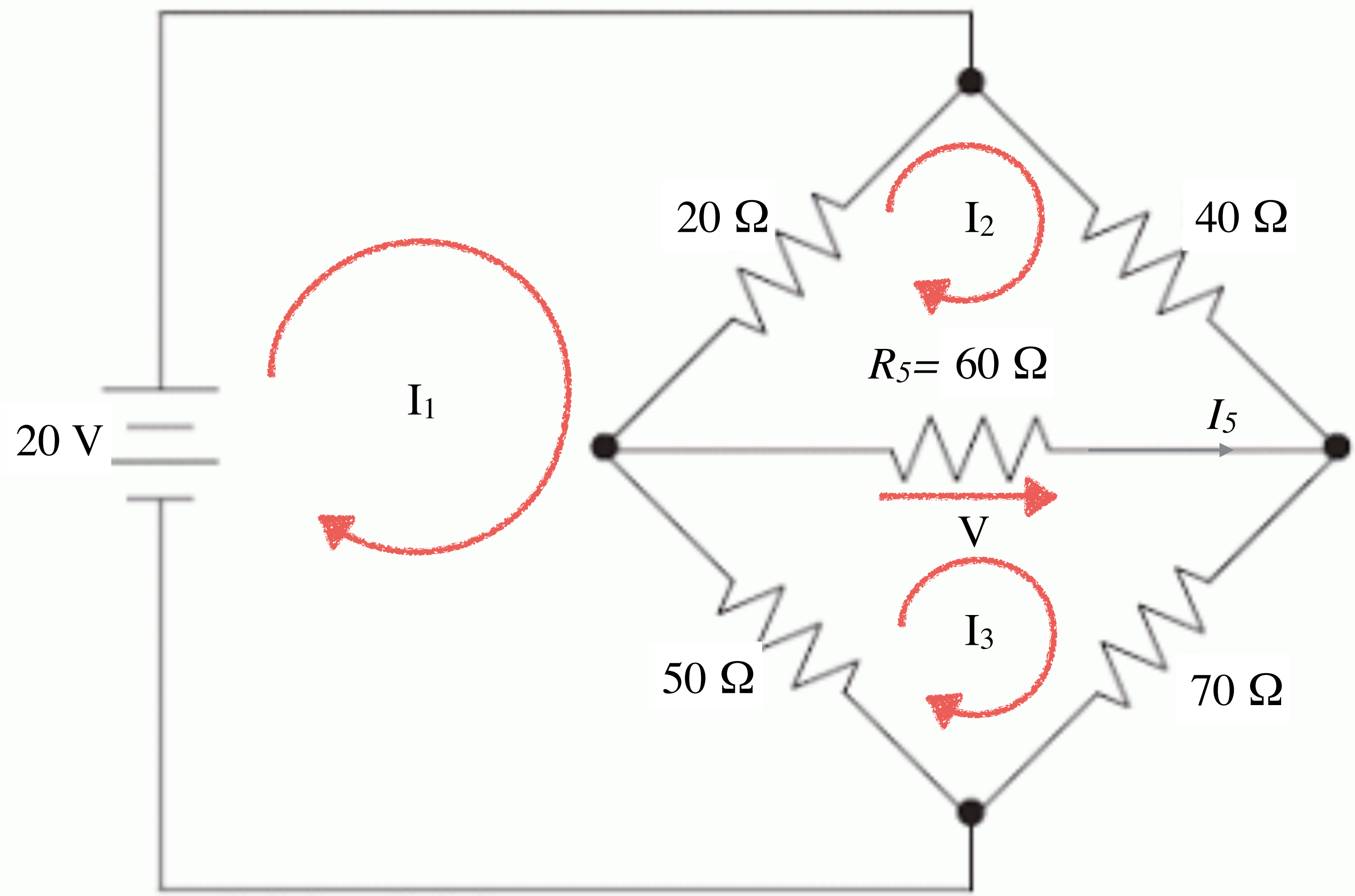
$$I_5 = \frac{V_{ab}}{R_5} = 0.016 \text{ A}$$

10) Circuit analysis *(using K's laws)*

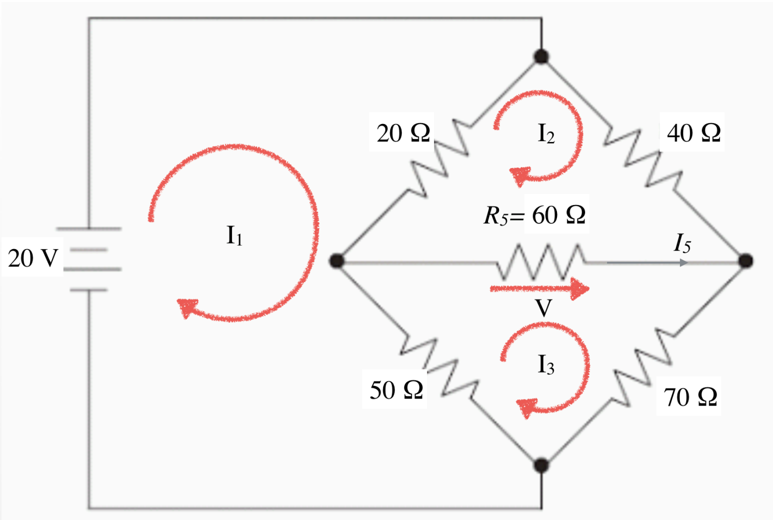
example



Typically, voltages and resistors are known. Solve for currents.



Find V_{ab} and the current through R_5
using Kirchhoff's loop rule



$$-70\Omega I_1 + 20\Omega I_2 + 50\Omega I_3 = -20 \text{ V}$$

$$20\Omega I_1 - 120\Omega I_2 + 60\Omega I_3 = 0$$

$$50\Omega I_1 + 60\Omega I_2 - 180\Omega I_3 = 0$$

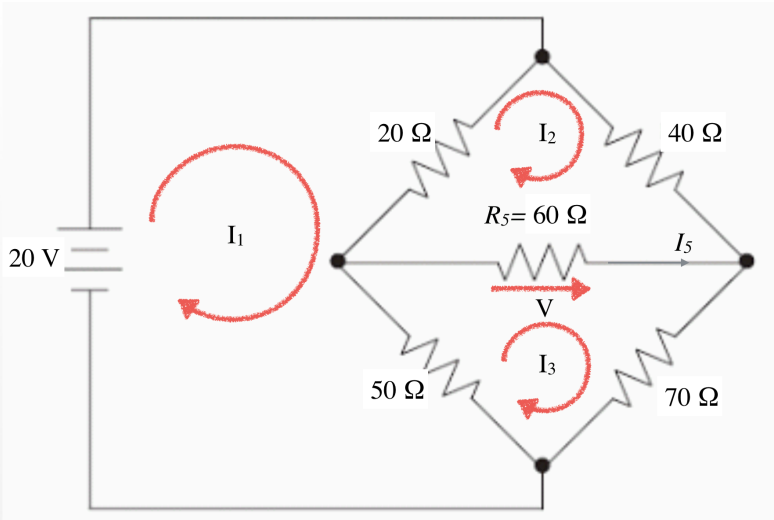
Determinants:

$$I_1 = \frac{\begin{vmatrix} -20 & 20 & 50 \\ 0 & -120 & 60 \\ 0 & 60 & -180 \end{vmatrix}}{\begin{vmatrix} -70 & 20 & 50 \\ 20 & -120 & 60 \\ 50 & 60 & -180 \end{vmatrix}} \text{A} = \tfrac{15}{32}\text{A} = 0.469\text{A}$$

$$I_2 = \tfrac{11}{64}\text{A} = 0.172\text{A} \qquad I_3 = \tfrac{3}{16}\text{A} = 0.1875\text{A}$$

$$I_5 = I_3 - I_2 = 0.155\text{A}$$

$$V_{ab} = I_5 R_5 = 0.93\text{V}$$



$$-70\Omega I_1 + 20\Omega I_2 + 50\Omega I_3 = -20\ \text{V}$$

$$20\Omega I_1 - 100\Omega I_2 + 60\Omega I_3 = 0$$

$$50\Omega I_1 + 60\Omega I_2 - 180\Omega I_3 = 0$$

using matrix inversion:

$$\begin{pmatrix} -70 & 20 & 50 \\ 20 & -100 & 60 \\ 50 & 60 & -180 \end{pmatrix} \Omega \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -20V \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{R}\,\underline{I} = \underline{V}$$

$$\underline{I} = \underline{R}^{-1}\underline{V}$$

	A	B	C	D	E	F
1	R=	-70	20	50		
2		20	-120	60		
3		50	60	-180		
4						
5						
6	R-1 =	-0.023438	-0.008594	-0.009375		
7		-0.008594	-0.013151	-0.006771	{=MINVERSE(B1:D3)}	
8		-0.009375	-0.006771	-0.010417		
9						
10						
11	V=	-20				
12		0				
13		0				
14						
15	I=	0.46875	{=MMULT(B6:D8,B11:B13)}			
16		0.171875				
17		0.1875				
18						
19						
20						

WolframAlpha computational knowledge engine.

solve simultaneous equations: -70x+20y+50z=-20;20x-120y+60z=0;50x+60y-180z=0

Web Apps | Examples | Random

Input interpretation:

	$-70x + 20y + 50z = -20$
solve	$20x - 120y + 60z = 0$
	$50x + 60y - 180z = 0$

Result:

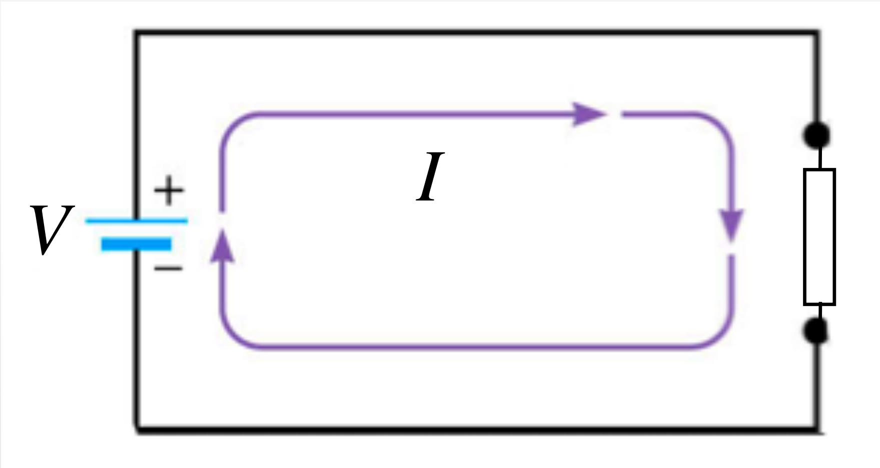
Decimal form | Step-by-step solution

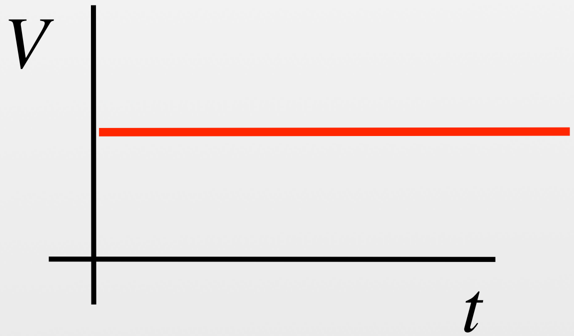
$$x = \frac{15}{32} \text{ and } y = \frac{11}{64} \text{ and } z = \frac{3}{16}$$

Open code

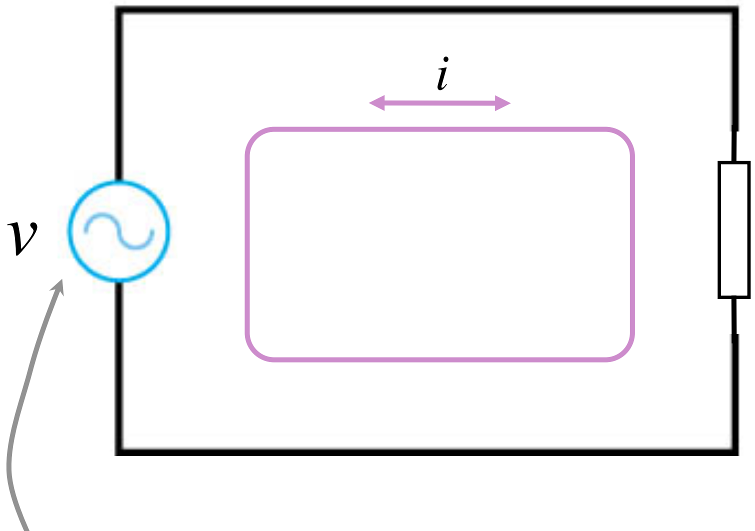
11) ac circuits

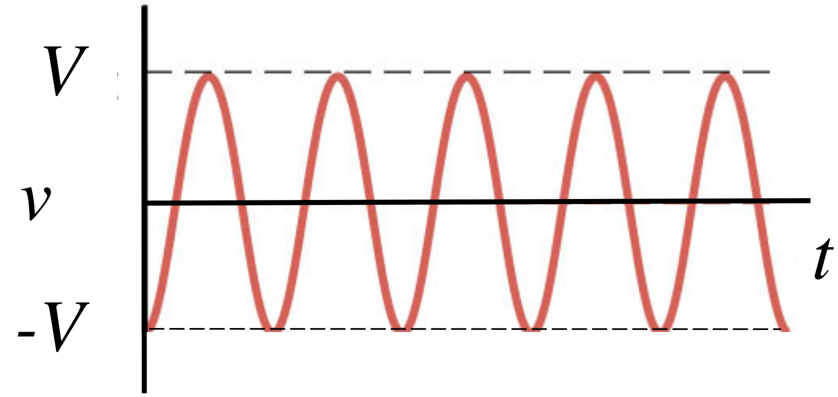
a) Direct (Constant) Current





b) Alternating Current (sinusoidal)





ac generator alternates polarity: *e.g.* $v = V\sin(\omega t + \phi)$

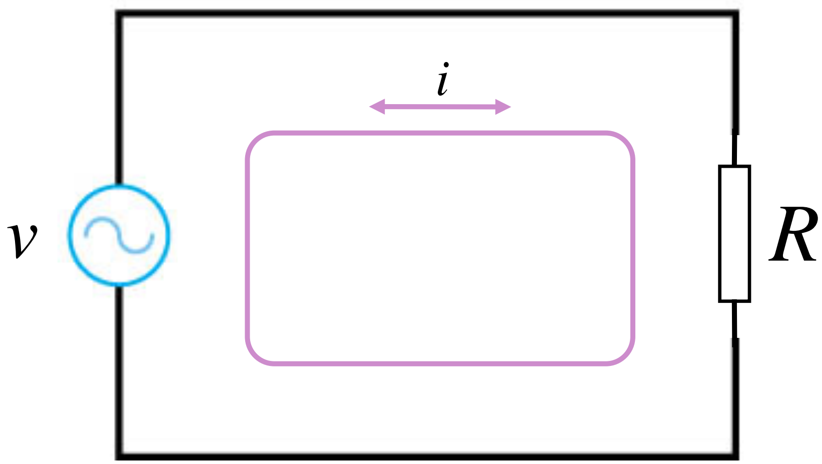
V (or V_0 or V_p) = peak voltage or amplitude

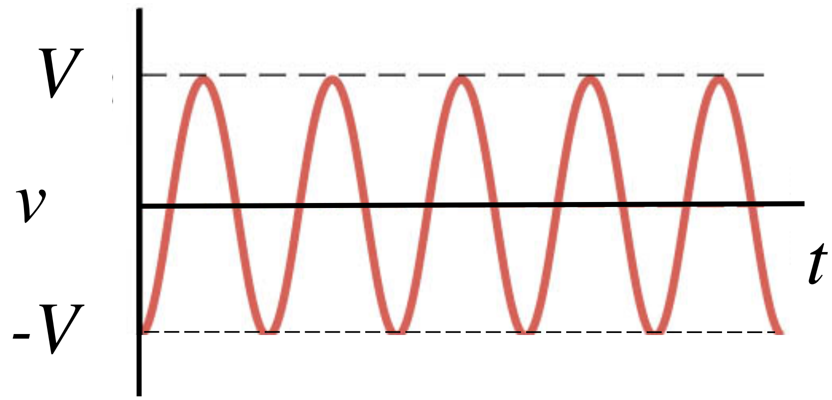
ω = angular frequency = $2\pi f$ (rad/s)

f = frequency (s^{-1} or Hz; cycles per second)

$T = 1/f$ = period

ϕ = phase angle (determines value of v at $t = 0$)

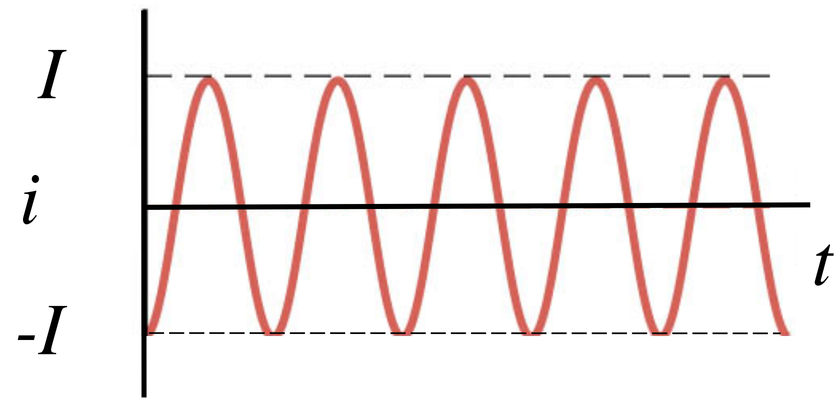




$$v = V\sin(\omega t + \phi)$$

$$v = iR$$

$$i = \frac{v}{R} = \frac{V}{R}\sin(\omega t + \phi)$$



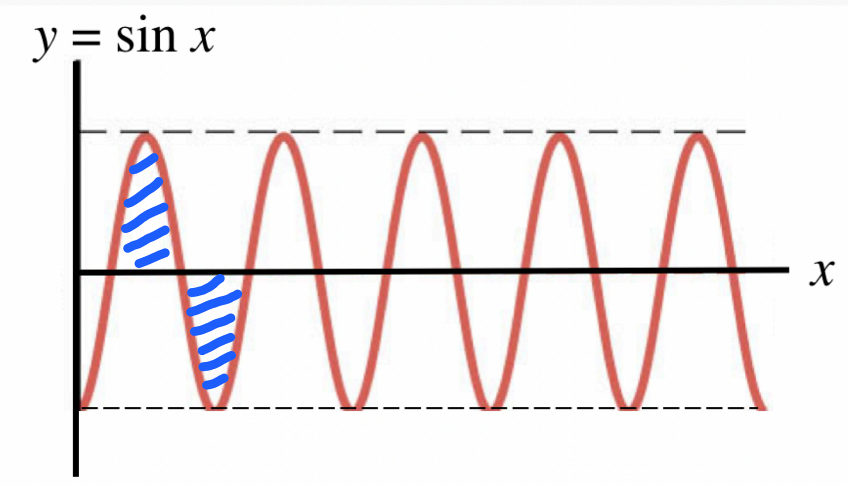
$$i = I\sin(\omega t + \phi)$$

$$V = IR$$

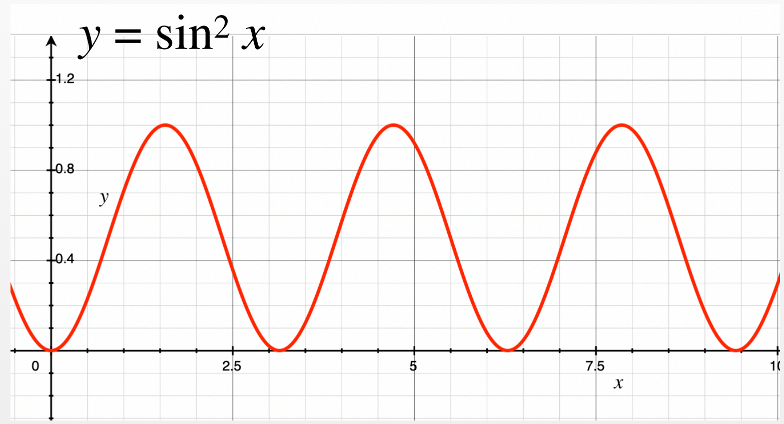
Ohm's law applies to amplitudes.

c)rms of a sinusoid

Average of sin x over one period is zero



Average of $\sin^2 x$?



$$< y^2 > = < \sin^2 x > \quad = \frac{1}{2\pi}\int_0^{2\pi} \sin^2 x dx \quad = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx \quad = \frac{1}{2\pi}\frac{x}{2}\Big|_0^{2\pi} \quad = \frac{1}{2}$$

$$y_{rms} = \sqrt{< \sin^2 x >} = \frac{1}{\sqrt{2}}$$

so if $v = V\sin(\omega t + \phi)$ $V_{rms} = \frac{V}{\sqrt{2}}$

$i = I\sin(\omega t + \phi)$ $I_{rms} = \frac{I}{\sqrt{2}}$

$$V_{rms} = I_{rms}R$$

d) Ave power in ac (resistor) circuit

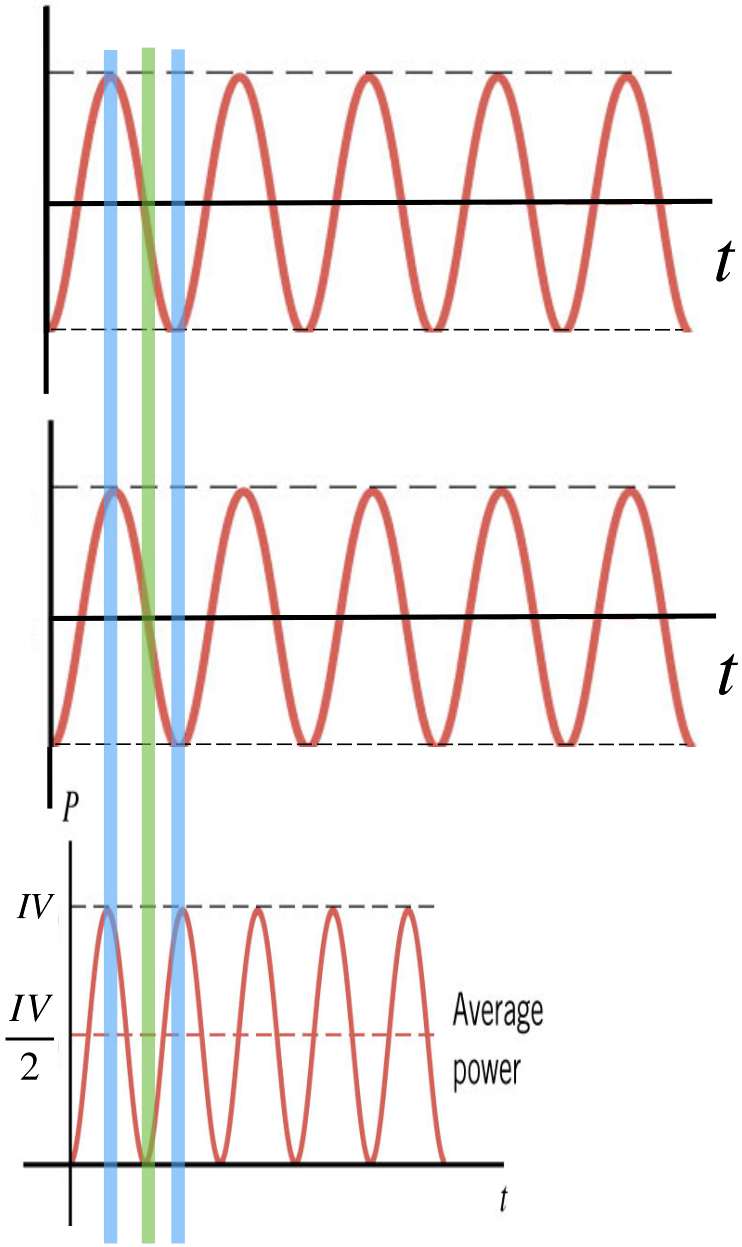
$$v = V\sin(\omega t + \phi)$$

$$i = I\sin(\omega t + \phi)$$

$$p = vi$$

$$= VI\sin^2(\omega t + \phi)$$

(for resistors)



Average voltage: zero

$$V_{rms} = \sqrt{< v^2 >} = \frac{V}{\sqrt{2}}$$

Average current: zero

$$I_{rms} = \sqrt{< i^2 >} = \frac{I}{\sqrt{2}}$$

Average power: $< P > = \frac{1}{2}VI$

$$< P > = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{rms}I_{rms}$$

For a resistive circuit (v and i in phase) $< P > = V_{rms} I_{rms}$ $= I_{rms}^2 R$ $= \frac{V_{rms}^2}{R}$

e) Power factor

Consider: $v = V \sin(\omega t)$

$i = I \sin(\omega t + \phi)$

(current and voltage out of phase)

Then,

$$p = vi = VI \sin(\omega t) \sin(\omega t + \phi) = VI \sin(\omega t)(\sin \omega t \cos \phi + \cos \omega t \sin \phi)$$

$$= VI(\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)$$

$$= VI(\underbrace{\sin^2 \omega t}_{\text{averages to } 1/2} \cos \phi + \tfrac{1}{2} \underbrace{\sin(2\omega t)}_{\text{averages to } 0} \sin \phi)$$

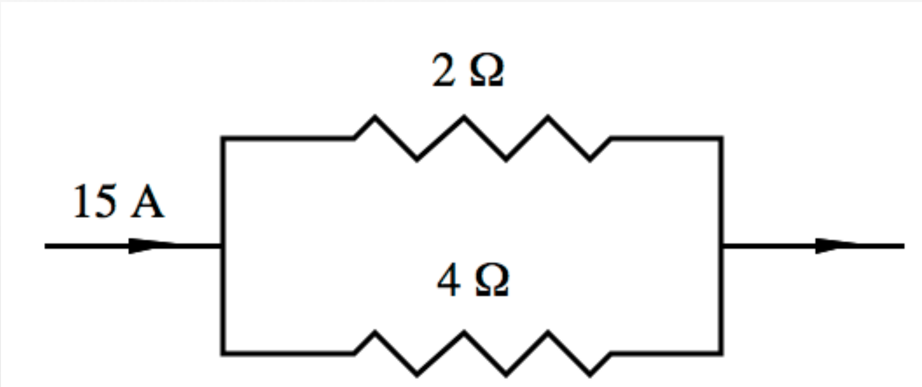
So, $< P > = \frac{VI}{2} \cos \phi$ or $< P > = V_{rms} I_{rms} \cos \phi$

$\cos \phi$ is the power factor

Complete the following statement: A simple circuit contains a resistance R and an ideal battery. If a second resistor is connected in parallel with R,

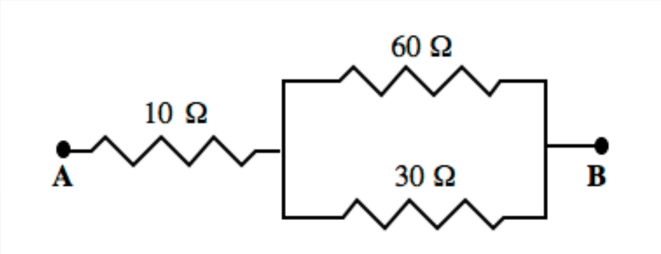
(a) the voltage across R will decrease.
(b) the current through R will decrease.
(c) the total current through the battery will increase.
(d) the rate of energy dissipation in R will increase.
(e) the equivalent resistance of the circuit will increase.

Two resistors are arranged in a circuit that carries a total current of 15 A as shown in the figure. Which one of the entries in the following table is correct?

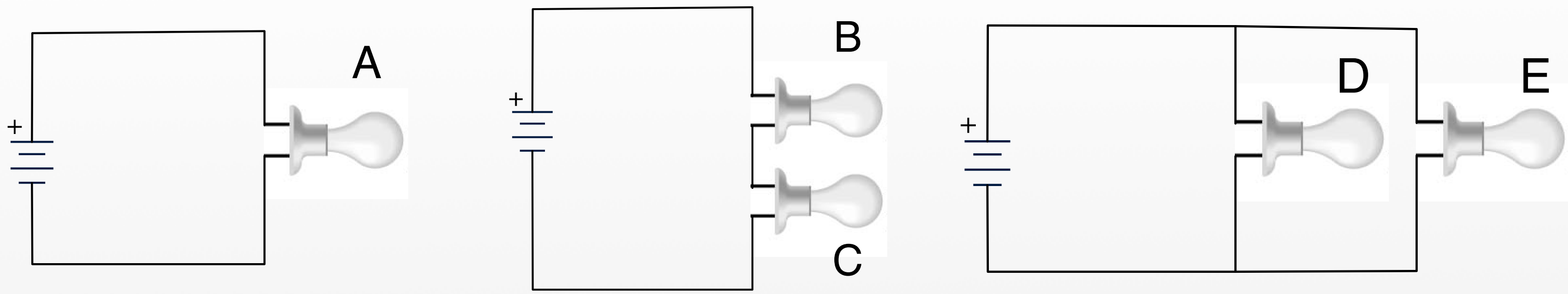


	Current through 2- Ω resistor	Voltage across 4- Ω resistor
(a)	5 A	10 V
(b)	5 A	20 V
(c)	10 A	20 V
(d)	15 A	15 V
(e)	10 A	10 V

What is the equivalent resistance between the points **A** and **B**?



(a) 10 Ω (b) 20 Ω (c) 30 Ω (d) 50 Ω (e) 100 Ω



Rank brightness of identical bulbs:

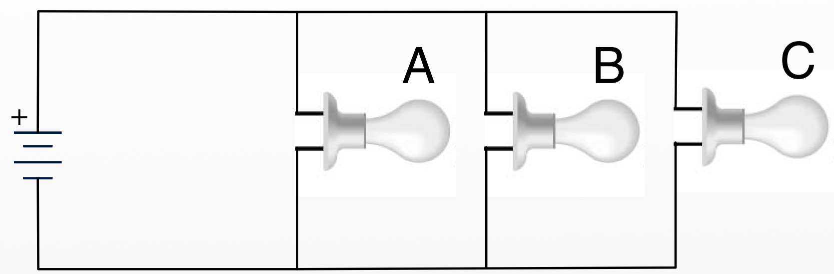
A) A > B = C > D = E

B) A > D = E > B = C

C) B = C > D = E > A

D) A = D = E > B = C

E) A = B = C = D = E



Rank brightness of bulbs A (60 W), B (100 W), C (150 W) in parallel:

A) A > B > C

B) C > B > A

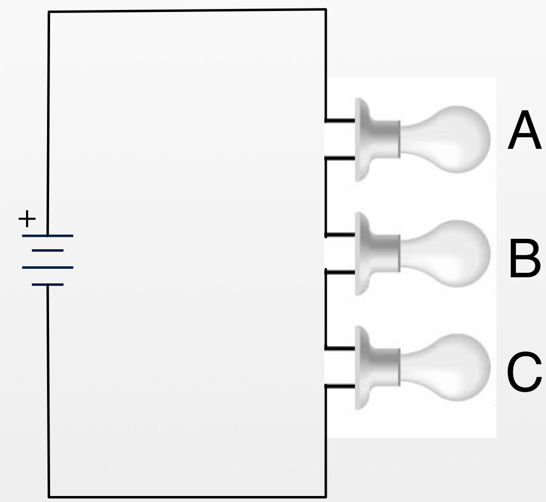
C) A > C > B

D) B > C > A

E) C > A > B

Rating (A) < Rating (B) < Rating (C)
Rating assumes 120 V.

Rank brightness of bulbs A (60 W), B (100 W), C (150 W) in series:



A) A > B > C

B) C > B > A

C) A > C > B

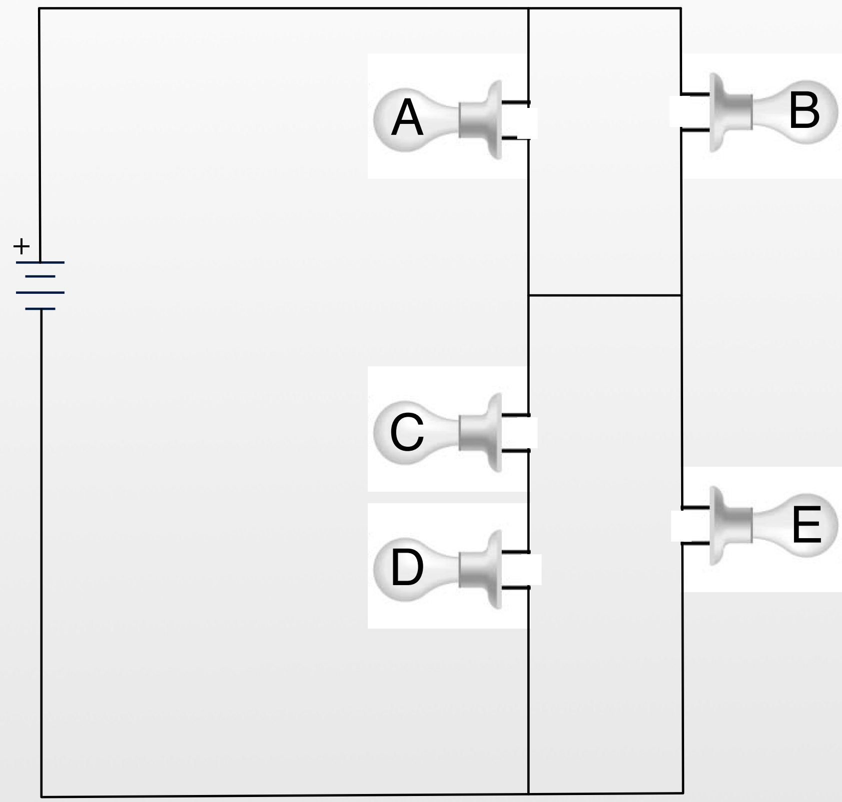
D) B > C > A

E) C > A > B

Rating (A) < Rating (B) < Rating (C)
Rating assumes 120 V.

Rank brightness of identical bulbs in the circuit:

(A // B) + [(C + D) // E]



A) A = B > C = D > E

B) E > C = D > A = B

C) E > A = B > C = D

D) A = B > C = D > E

E) C = D > A = B > E