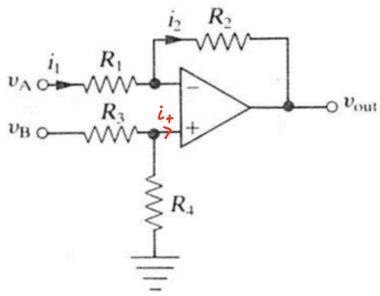
Phys 2610 (2019) Assignment 5 solutions

1. Find a general expression for the output of the following circuit, and show that it reduces to $v_{out} = \frac{R_2}{R_1}(v_B - v_A)$ when $\frac{R_1}{R_2} = \frac{R_3}{R_4}$.



Since current entering the op amp is zero, $i_1 = i_2$, so $\dfrac{v_A - v_-}{R_1} = \dfrac{v_- - v_{out}}{R_2}$

Solving for v_-: $v_-\left(\frac{1}{R_2} + \frac{1}{R_1}\right) = \frac{v_A}{R_1} + \frac{v_{out}}{R_2} \implies v_- = \left(\frac{v_A}{R_1} + \frac{v_{out}}{R_2}\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right)$

or

$$\boxed{v_- = \frac{v_A R_2 + v_{out} R_1}{R_1 + R_2}}$$

Also, since $i_+ = 0$, $\boxed{v_+ = v_B \dfrac{R_4}{R_3 + R_4}}$

Setting the inputs equal, and solving for v_{out}:

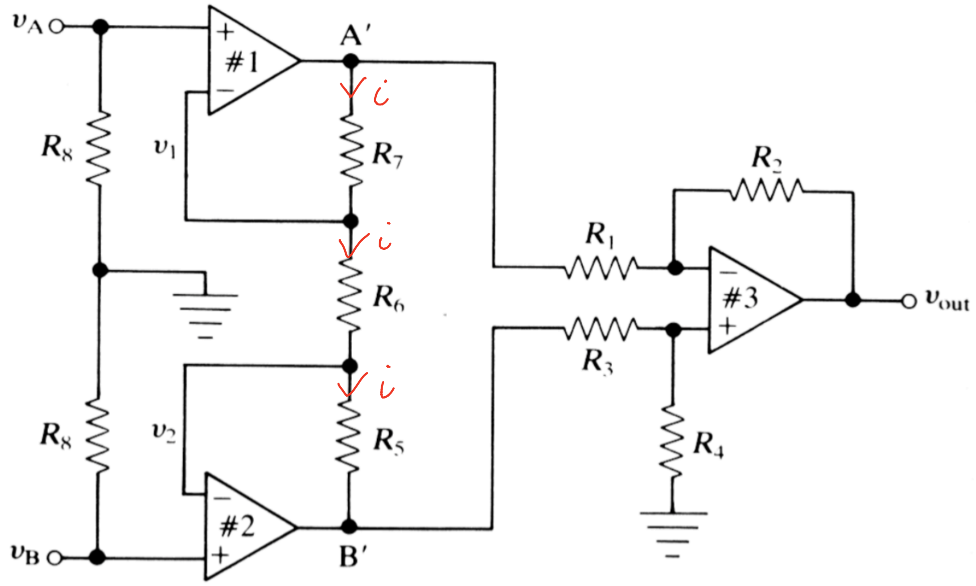
$$v_{out} = \left[v_B\left(\frac{R_4}{R_3 + R_4}\right) - v_A\left(\frac{R_2}{R_1 + R_2}\right)\right]\left(\frac{R_1 + R_2}{R_1}\right)$$

If $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$, then $\dfrac{R_4}{R_3 + R_4} = \dfrac{1}{R_3/R_4 + 1} = \dfrac{1}{R_1/R_2 + 1} = \dfrac{R_2}{R_1 + R_2}$

$$\boxed{v_{out} = (v_B - v_A)\frac{R_2}{R_1}}$$

2. Show that the gain for the following circuit, with $R_5 = R_7 = R$, and $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ is given by

$$a = \frac{v_{out}}{v_A - v_B} = -\frac{R_2}{R_1}\left(1 + \frac{2R}{R_6}\right)$$



The current through R_6 is determined from v_A and v_B because $v_+ = v_-$:

$$i = \frac{v_A - v_B}{R_6}$$

The same current flows in $R_5 + R_7$ because the current into the op amps is negligible.

$$\therefore\ i = \frac{v_{A'} - v_{B'}}{R_5 + R_6 + R_7} = \frac{v_A - v_B}{R_6} \Rightarrow v_{A'} - v_{B'} = (v_A - v_B)\left(\frac{R_5 + R_6 + R_7}{R_6}\right)$$

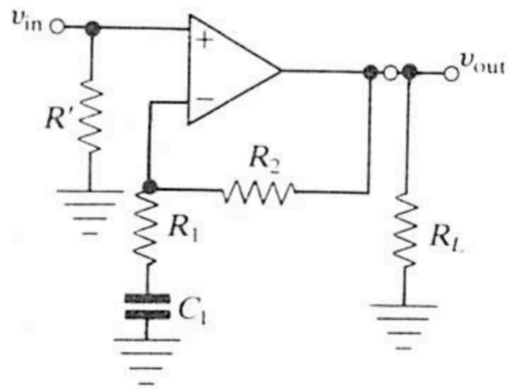
Using $R_5 = R_7 = R$, gives $\quad v_A' - v_B' = (v_A - v_B)\left(1 + \frac{2R}{R_6}\right)$

We have, from problem 1, $v_{out} = (v_B' - v_A')\frac{R_2}{R_1}$ if $\frac{R_2}{R_1} = \frac{R_3}{R_4}$, so

$$= -\left(1 + \frac{2R}{R_6}\right)\frac{R_2}{R_1}(v_A - v_B)$$

$$a_v = \frac{v_{out}}{v_A - v_B} = -\left(1 + \frac{2R}{R_6}\right)\frac{R_2}{R_1}$$

3. Find an expression for the complex gain of the following circuit, when $\omega = \omega_1 = 1/(R_1 C_1)$.



Using $v_+ = v_-$, $\quad v_{in} = v_{out}\left(\dfrac{R_1 + 1/j\omega C_1}{R_2 + R_1 + 1/j\omega C_1}\right)$

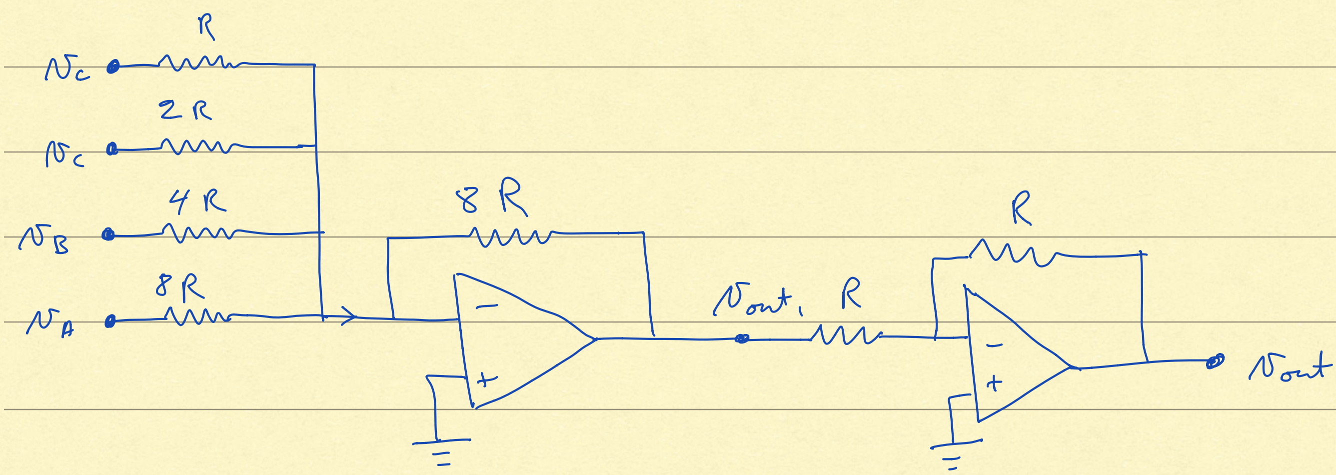
Then, for $\omega = 1/R_1C_1$,

$$a = \frac{v_{out}}{v_{in}} = \frac{R_1 + R_2 + R_1/j}{R_1 + R_1/j} = 1 + \frac{R_2}{R_1}\left(\frac{j}{1+j}\right)$$

or $a = 1 + \dfrac{R_2}{2R_1}(1+j)$

or $a = \left(1 + \dfrac{R_2}{2R_1}\right) + j\left(\dfrac{R_2}{2R_1}\right)$

4. Design a summing amplifier to sum from inputs v_A, v_B, v_C, v_D and to produce an output of $v_{out} = v_A + 2v_B + 4v_C + 8v_D$.



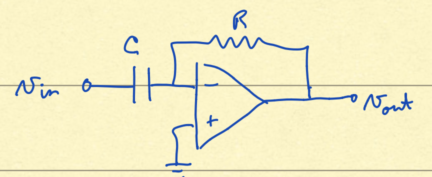
$$i = \frac{v_A}{8R} + \frac{v_B}{4R} + \frac{v_C}{2R} + \frac{v_D}{R} = -\frac{v_{out,1}}{8R}$$

$$\Rightarrow v_{out,1} = -(v_A + 2v_B + 4v_C + 8v_D)$$

$$\Rightarrow v_{out} = v_A + 2v_B + 4v_C + 8v_D$$ (not req'd for full credit)

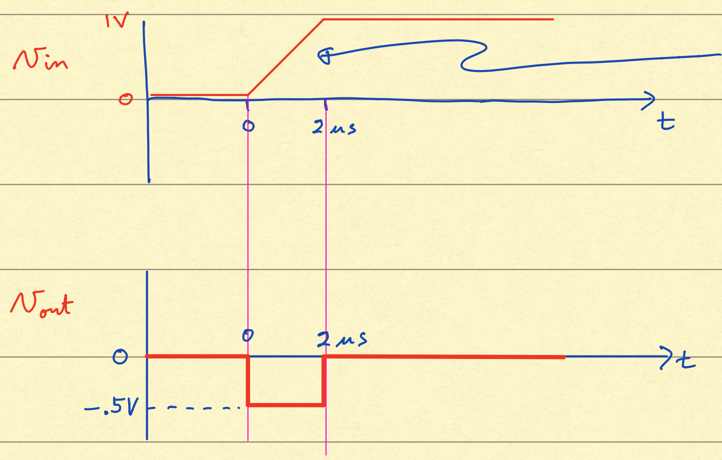
5. Design an op amp differentiator with an output given by $v_{out} = -(10^{-6}\text{s})\frac{d}{dt}v_{in}$. If an input step function rises from 0 to 1 V in 2 μs and then is constant at 1V, sketch the output.

Differentiator:



$v_{out} = -RC \frac{d}{dt} v_{in}$

For $RC = 10^{-6}$ s, take $R = 1\text{k}\Omega$ and $C = \frac{10^{-6}\text{s}}{1\text{k}\Omega} = 1\text{mF}$



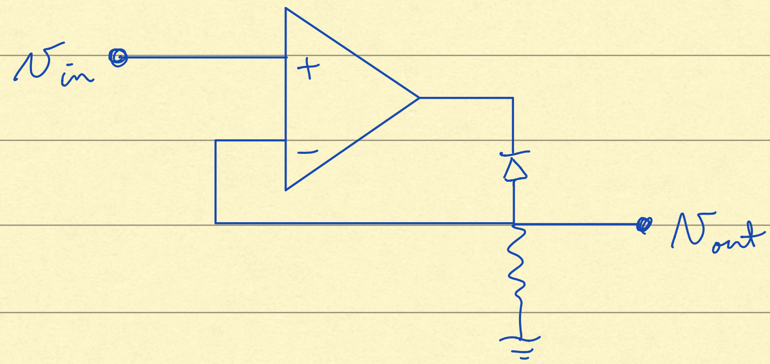
slope $= \frac{1\text{V}}{2\mu\text{s}} = 5 \times 10^5$ V/s

$\Rightarrow v_{out} = (-10^{-6}\text{s})(5 \times 10^5 \text{V/s})$

$= -0.5\text{V}$

The output is proportional to the slope, which is zero before $t=0$ and after $t = 2\mu$s. During the increase, the slope is 5×10^5 V/s, so the output is -0.5 V.

6. Design an ideal diode op amp circuit to produce a negative half-waveform from a sinusoidal input.



For $v_{in} > 0$, diode is off $\Rightarrow$ $v_{out} = 0$

For $v_{in} < 0$, diode is on $\Longrightarrow$ neg. feedback $\Rightarrow$ $v_{out} = v_{in}$