Power of Ten

Dealing with Numbers Great and Small

Everyday Lengths

1 meter = 1 m = 1 yard
1 kilometer = 1 km = 10^3 m ~ 5/8 mile
1 centimeter = 1 cm = 10^{-2} m ~ 2/5 inch (width of a pen)
1 millimeter = 1 mm = 10^{-3} m ~ 1/25 inch (width of lead in pencil)

Small Scales

The wavelength of yellow light in vacuum may be called 5750 Å or 575 m$_\mu$ or 575 nm or 0.575 $_\mu$m. The units used here are:

1 Å = 1 Ångstrom = 10^{-10} m

(This unit is often used because a typical atom is a few Ångstroms in size.)

1 nm = 1 nanometer = 10^{-9} m

1 $_\mu$m = 1 micrometer = 1 micron = 10^{-6} m

(This is a convenient unit for high-power microscopes, with which one can look at objects as small as a few micrometers. $_\mu$ is the Greek letter "mu")

Important Prefixes

Prefixes multiply the unit by some power of 10. For example, milli- always means one thousandth.

n = nano- (Greek "dwarf") = 10^{-9} (thousand millionth)
$_\mu$ = micro- (Greek "small") = 10^{-6} (millionth)
m = milli- (Latin "thousand") = 10^{-3} (thousandth)
k = kilo- (Greek "thousand") = 10^3 (thousand)
M = mega- (Greek "big") = 10^6 (million)

Thus we can now proudly state that 1 nm = 10^{-3} $_\mu$m.

Scientific Notation:

indentAstronomy is the subject of very large numbers. Microscopy is the subject of very small numbers. Consequently, scientific notation has been introduced to deal with such quantities. For example, the distance from the Earth to the Sun is referred to as the *Astronomical Unit (AU)* and is:

1 AU = 150,000,000 km = 1.5x 10^8 km = 9.2 x 10^7miles

indent Another dimension to illustrate this kind of notation is the mass of the Sun expressed in units of kilograms (kg):

Mass of the Sun = 1 Solar Mass = 2.0 x 10^{30} kg

indent Note that 2.0 is the *coefficient* and *30* is the *exponent.* Not only can one use scientific notation to express large numbers but also very small numbers. For example, the size of an atom is on the order (or about) a few Ångströms (Å) which is defined as:

$$1 \text{ Ångström} = 1 \text{ Å} = 1.0 \times 10^{-10} \text{ m} = 0.0000000001 \text{ meters}$$

indent aNote that the negative sign in the exponent means the value is less than one.

indent Here are some alternate forms of notation for the same numerical values:

$$3.2 \times 10^{8} = 32 \times 10^{7} = 320 \times 10^{6} = 320{,}000{,}000 = 320 \text{ million}$$

$$0.0002 = 2 \times 10^{-4} = 20 \times 10^{-5} = 200 \times 10^{-6}$$

Using a pocket calculator with an exponent key, usually labeled EX or EE or EX, the number: 4.3×10^{5} is entered as 4.3 EX 5.

Examples of Scientific Notation, Metric Prefixes, and Symbols

Word	Sci. Notation	Metric Prefix	Symbol
one billionth	1×10^{-9}	nano	n
one millionth	1×10^{-6}	micro	μ
one thousandth	1×10^{-3}	milli	m
one	1		
one thousand	1×10^{3}	kilo	k
one million	1×10^{6}	Mega	M
one billion	1×10^{9}	Giga	G
one trillion*	1×10^{12}	Tera	T

*in American English, 1×10^{12} = one thousand million in British English.

Examples of simple matlh using scientific notation:

Addition:

$$3.2 \times 10^{8} + 0.40 \times 10^{8} = 3.6 \times 10^{8}$$

Subtraction:

$$4.52 \times 10^{5} - 3.32 \times 10^{5} = 1.20 \times 10^{5}$$

Multiplication:

$$(3.2 \times 10^{-8}) \times (2 \times 10^{4}) = 6.4 \times 10^{-4}$$

Division:

$$(3.2 \times 10^{8}) / (2 \times 10^{-4}) = 1.6 \times 10^{12}$$

Note that when you multiply, the coefficients are multiplied together while the exponents (or powers of ten) are added. Similarly, when you divide, the coefficients are divided and the exponents are subtracted.

Also see:
http://www.powersof10.com
http://microcosm.web.cern.ch/microcosm/P10/english/what.html

Powers! $(2 \times 10^{3})^{2} = 2^{1\times2} \times 10^{3\times2} = 2^{2} \times 10^{6} = 4 \times 10^{6}$