2015 Final

1) $a = \frac{1}{2}t \quad \therefore v = v_0 + \frac{1}{4}t^2 \; ; \; v_0 = 16\frac{m}{s}$.

$$x = x_0 + v_0 t + \frac{1}{12}t^3 \; ;$$

At $t = 2.0s$ $\therefore \Delta x = x - x_0 = 16 \times 2 + \frac{1}{12}(2.0)^3$

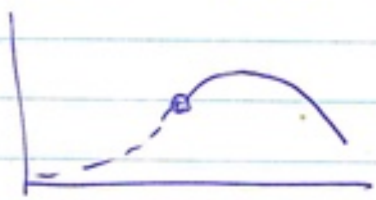
$$= 32.7 \text{ m.}$$

E

2) At point $v_y > 0$, $a_y = -g$

$y(t)$ vs. t must be continuous

$\therefore$



B

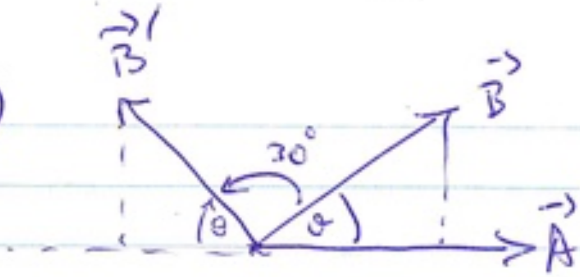
3) $y(t) = -\frac{1}{2}gt^2$

$y(3) = -44.1$ m

$y(4) = -78.4$ m.

$\therefore \Delta y = 34.3$ m

D

4) 

$$2\theta + 30° = 180°$$
$$\therefore 2\theta = 150°$$
$$\theta = 75°$$

(C)

5) $\hat{j} \times \hat{k} = \hat{i}$, and $\hat{i} \cdot \hat{i} = +1$.

(B)

6) $\vec{v}_1 = 5.4\hat{i} - 4.8\hat{j}$ (m/s)

$\vec{v}_2 = 1.7\hat{i} + 5.9\hat{j}$ (m/s) $\Delta t = 2s$.

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{(1.7 - 5.4)\hat{i} + (5.9 + 4.8)\hat{j}}{2\ s}$$
$$= -1.9\hat{i} + 5.4\hat{j} \quad (m/s^2)$$

(B)

7) 

$g_{moon} < g_{earth}$

$-h = -\frac{1}{2}gt^2 \therefore t^2 = \frac{2h}{g}$ $\therefore t_{moon} > t_{earth}$

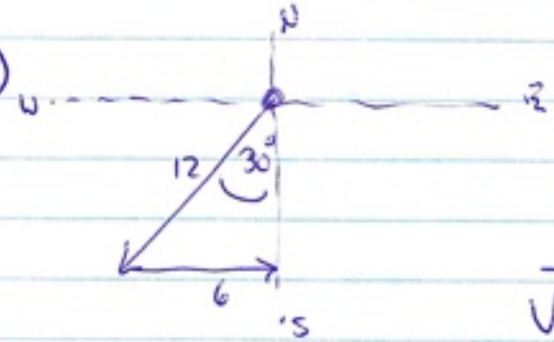
Also $v^2 = 2gh$ $\therefore v_{moon} < v_{earth}$

$\therefore$ I and II only (B)

8) $a_c = \frac{v^2}{R}$ $\therefore v^2 = a_c R$

$= 25 \times (9.8 \frac{m}{s^2})(0.5 m)$

$\therefore v = 11$ m/s (A)

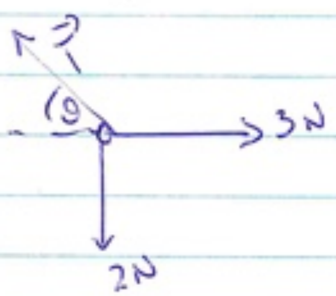
9) 

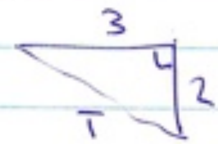
$\vec{v}_{bg} = \vec{v}_{br} + \vec{v}_{rg}$

b: boat
r: river
g: ground

$\vec{v}_{br} = -12\sin 30° \hat{i} - 12\cos 30° \hat{j}$
$= -6\hat{i} - 10.4\hat{j}$
$\vec{v}_{rg} = +6\hat{i}$

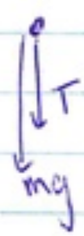
$\therefore \vec{v}_{bg} = -10.4\hat{j}$ (ie south) (E)

10) 



$T^2 = (3)^2 + (2)^2$
$= 13$

$\therefore T = \sqrt{13}$ N
$= 3.6$ N (B)

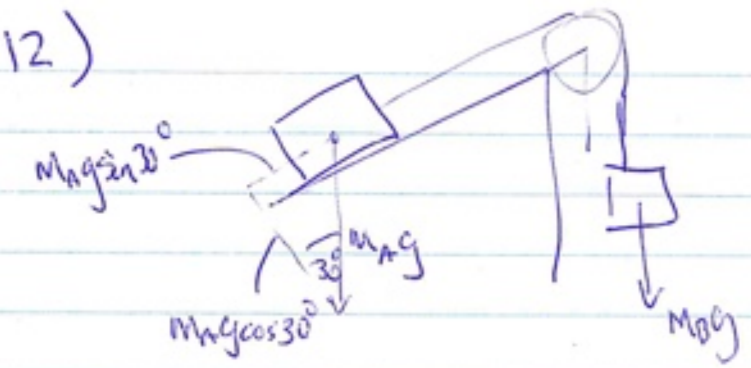
11) 

$T + mg = m\frac{v^2}{R}$

$T = 0 \Rightarrow v^2 = gR = (9.8 \frac{m}{s})(1.2 m)$

$\therefore v = 3.4$ m/s (D)

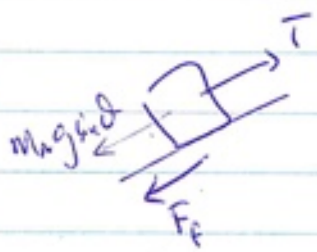
12)



$m_B g > m_A g \sin 30^\circ$

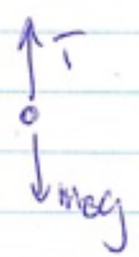
and $m_B g > m_A g \sin 30^\circ + \mu_s m_A g \cos$

$\therefore$ B accelerates <u>down</u>



$$T - F_f - m_A g \sin 30^\circ = m_A a_A \quad (1)$$

$$F_f = \mu_k m_A g \cos 30^\circ \quad (2)$$



$$T - m_B g = m_B a_B \qquad (a_B = -a_A)$$

$$= - m_B a_A \quad (3)$$

Put into (1): $\therefore (m_B g - m_B a_A) - \mu_k m_A g \cos 30^\circ - m_A g \sin 30^\circ$

$$= m_A a_A$$

$$\therefore (m_A + m_B)\, a_A = g\left(m_B - m_A \sin 30^\circ - m_A \mu_k \cos 30^\circ\right.$$

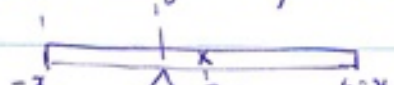
$$\therefore 7.0\,\text{kg}\ a_A = \left(9.8\,\frac{m}{s^2}\right)\Big(3.0\,\text{kg} - 4.0\,\text{kg} \times \tfrac{1}{2}$$

$$- 4.0\,\text{kg}\,(0.1)(0.87)\Big)$$

$$= 6.4\ \text{kg}\cdot\text{m/s}^2$$

$$\therefore a_A = 0.92\ \text{m/s}^2 \qquad \text{A}$$

13) The COM is at the support point. Taking this to be the origin,



$$x_{com} = \frac{m_A(-x) + m_B(6-x) + m_{board}(3-x)}{m_A + m_B + m_{board}}$$

$$= 0$$

$m_A = 12 \quad m_B = 16 \quad m_{board} = 20$

$$\therefore -12x + 16\cdot 6 - 16\cdot x + 20\cdot 3 - 20\cdot x =$$

$$\therefore \quad 48x = 156$$

$$\therefore x = 3.25 \text{ m.}$$ (B)

14) B is initially at rest. (unstated assumption).

For an elastic collision, $v_{Bf} = \frac{2m_A}{m_A + m_B} v_{Ai}$

$$= \frac{2\times 5\,kg}{6\,kg}\, 20\, m/s$$

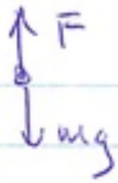
$$= 33.3\, m/s.$$

Time to fall is $-h = -\frac{1}{2}gt^2 \quad \therefore t^2 = \frac{2h}{g} = \frac{2\times 6.0\,m}{9.8\,m/s^2}$

$$\therefore t = 1.11\, s$$

$$\therefore x = \left(33.3\, \frac{m}{s}\right) \times (1.11\, s) = 36.9\, m$$ (D)

15) $F = -ky$



$= mg$

$\therefore k = -\dfrac{mg}{y} = -\dfrac{6.0\,kg \times 9.8\,m/s^2}{(-0.10\,m)} = 590\ N/m$

$W_s = -\Delta U = -\frac{1}{2}k\left(y_f^2 - y_i^2\right)$ $\quad y_i = -0.10\,m$

$y_f = -0.20\,m$

$= -\frac{1}{2}\left(590\,\frac{N}{m}\right)\left((0.20\,m)^2 - (0.10\,m)^2\right)$

$= -8.8\ J.$ (A)

16) $W = \Delta K = -\frac{1}{2}mv^2 = -\frac{1}{2}(11.0\,kg)\left(30\,\frac{km}{h} \times \frac{1\,h}{3600\,s} \times \frac{10^3\,m}{km}\right)^2$

$= -382\ J.$

$P = -\dfrac{W}{\Delta t} = \dfrac{382\,J}{12\,s} = 32\ W$

17) $m_A V_A + m_B V_B = 0$ $\quad \therefore V_B = -\dfrac{m_A V_A}{m_B}$

$= -\ 0.2\,V_A$

$K_A = \frac{1}{2}m_A V_A^2$; $K_B = \frac{1}{2}m_B V_B^2$

$= 22\,J$ $\quad = \frac{1}{2}\left(\frac{m_B}{m_A}\right)m_A\left(0.2\,V_A\right)^2$

(D) $\quad = 5 \times (0.2)^2\,K_A = 0.2\,K_A = 4.4$

18) $\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$

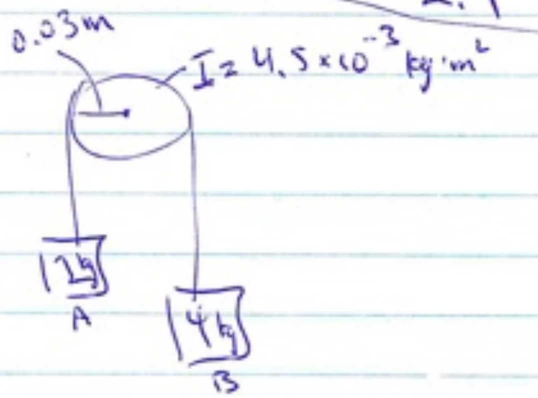
$\omega_f = 6.0$ rad/s
$\omega_i = 5.0$ rad/s
$\Delta\theta = 5 \times 2\pi$ rad
$= 31.4$ rad

$$\therefore \alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = 0.175 \text{ rad/s}^2$$

$$\tau = I\alpha = (12\,\text{kg}\cdot\text{m}^2) \times (0.175\,\text{rad/s}^2)$$

$$= 2.1\ \text{N}\cdot\text{m} \qquad \text{D}$$

19)



$R = 0.03$ m

$v = \omega R \qquad \therefore \omega = \frac{v}{R}$

$$K_{pulley} = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2}(4.5\times10^{-3}\,\text{kg}\cdot\text{m}^2)\left(\frac{2.0\text{m}}{0.03\text{m}}\right.$$

$$= 10\ \text{J}$$

D

20) α is constant, so $\omega = \alpha t = 2.0 \frac{rad}{s^2} \times 5.0 s$

$W = \Delta K$ $= 10$ rad/s

$= \frac{1}{2} I \omega^2 = \frac{1}{2} (6.0 \text{ kg}\cdot\text{m}^2)(10 \text{ rad/s})^2$

$= 300 \text{ J}.$ D

21) This requires cross products. $\vec{L} = m\vec{r} \times \vec{v}$

$\vec{v} = \frac{d\vec{r}}{dt} = -\omega \times 0.5 \sin(\omega t)\,\hat{i} + \omega \times 0.5 \cos \omega t\,\hat{j}$

At $t = 4.0 s$, $\vec{r} = -0.32\hat{i} - 0.38\hat{j} + 0.75\hat{k}$
$\vec{v} = 4.6\hat{i} - 3.8\hat{j}$

$\therefore \underline{L} = 5.8\hat{i} + 6.9\hat{j} + 6\hat{k}$ E

22) $I_i \omega_i = I_f \omega_f$ $\quad I_i = 600 \text{ kg m}^2 \quad I_f = 680 \text{ kg m}^2$
$\omega_i = 0.8 \text{ rad/s} \quad \omega_f = ?$

$\omega_f = \frac{I_i \omega_i}{I_f} = 0.71$ rad/s. B

23) $\Delta t = \dfrac{0.250 \times 10^{-3}\,m}{0.95 \times 3 \times 10^{8}\,m/s} = 8.77 \times 10^{-13}\,s$

$\Delta t = \gamma \Delta t_0 \qquad \gamma = \dfrac{1}{\sqrt{1-(0.95)^2}} = 3.2$

$\therefore \Delta t_0 = \dfrac{\Delta t}{3.2} = 2.74 \times 10^{-13}\,s$

24) $\overset{-0.5c}{\leftarrow}$ S2 $\qquad$ us $\qquad$ S1 $\overset{+0.5c}{\rightarrow}$

Frame S: S2 $\qquad v = +0.5c \qquad$ (us wrt S2)
Frame S′: us $\qquad u' = +0.5c \qquad$ (S1 wrt us)

$$u = \frac{u'+v}{1+u'v/c^2} = \frac{1.0c}{1+(0.5)(0.5)} = \frac{1.0c}{1.25} = 0.8c$$

B

25) $K = (\gamma - 1)mc^2 = 2mc^2$

$\therefore \gamma \cancel{mc^2} = 3\cancel{mc^2}$

$\therefore \gamma^2 = 9 \qquad \therefore 1 - \dfrac{v^2}{c^2} = \dfrac{1}{9}$

$\therefore \dfrac{v^2}{c^2} = \dfrac{8}{9}$

$\therefore v = 0.94c$ $\quad$ A