PHYS1050 Midterm 2016 (Winter)

1. Assume MKS units throughout. For constant acceleration we have

$$\begin{aligned} x(t) &= x_0 + v_0 t + \frac{1}{2}at^2, \\ v(t) &= v_0 + at, \end{aligned}$$

   with $x_0 = 1$. Therefore we have two equations

$$v_0 + \frac{1}{2}a = 3, \qquad 2v_0 + 2a = 4.$$

   The solutions are $v_0 = 4$, $a = -2$.

2. Assume MKS units throughout. Let y_1 and y_2 be the position of objects 1 and 2, respectively. Then

$$\begin{aligned} y_1 &= -\frac{1}{2}gt^2, \\ y_2 &= -\frac{1}{2}g(t-2.0)^2 = -\frac{1}{2}gt^2 + 2.0gt - 2.0g. \end{aligned}$$

   Define $\Delta y = y_2 - y_1$ as the distance between the two objects. Then

$$\Delta y = 2.0g(t-1.0) = 98., \qquad t = 98./(2.0g) + 1.0 = 6.0.$$

3. Let T be the tension in the cable, and a the upward acceleration of the elevator.

$$\sum F_y = T - mg = ma, \qquad \therefore T = m(a+g).$$

   Now $a = \Delta v/\Delta t = 2.5\ \mathrm{m/s^2}$, so $T = 18$ kN.

4. We have

$$\begin{aligned} v(t) &= \frac{dx(t)}{dt} = 54. - 6.0t^2, \\ a(t) &= \frac{dv(t)}{dt} = -12.0t. \end{aligned}$$

   We find $v(t) = 0$ when $t = 3.0$ s, at which time $a(t) = -36.0\ \mathrm{m/s^2}$.

5. Leting T be the period, we have

$$\begin{aligned} T &= \frac{60 \text{ s}}{1500} = 0.04 \text{ s}, \qquad v = \frac{2\pi r}{T} = 23.6 \text{ m/s}, \\ a_c &= \frac{v^2}{r} = 3700 \text{ m/s}^2. \end{aligned}$$

6. $F_x = (20 \text{ N}) \cos 20° = 18.8 \text{ N} = ma$. Therefore $a = 0.75 \text{ m/s}^2$.

7. If there is no acceleration, the tension in the string is $T = m_B g$. At the point of slipping, the frictional force is $F_f = \mu_s N = \mu_s m_A g = T$. Therefore $m_B = \mu_s m_A = 20$ kg.

8. All three blocks have a common acceleration a. The system has a total mass $M = 6m$, so applying $F = Ma$ we find $a = F/(6m)$. The net force on block B is $F_\text{B} = m_B a = (2m)(F/(6m)) = F/3$.

9. At the top of the loop, the centripetal acceleration is directed downward. The forces on the pilot are gravity (downward) and the normal force from the seat (also downward). The equations are

$$\begin{aligned} \sum F_y &= -mg - N = ma_y = -m\frac{v^2}{r}, \\ \therefore N &= m(\frac{v^2}{r} - g) = 690 \text{ N} = 0.69 \text{ kN}. \end{aligned}$$

10. The static friction between tires and road is what provides the centripetal acceleration. Taking the radially inward direction as positive, we have at the point of slipping

$$\begin{aligned} \sum F_r &= \mu_s N = \mu_s mg = ma = m\frac{v^2}{r}, \\ \therefore \mu_s &= \frac{v^2}{gr} = 0.46. \end{aligned}$$

As this expression does not depend on mass, the weight of the car is irrelevant.

11. When paddling downstream, the canoeist's speed with respect to (wrt) ground is $v + v_\text{river}$. The time t_1 it takes to paddle a distance L is therefore $t_1 = L/(v + v_\text{river})$. When paddling upstream, the canoeist's speed wrt ground

is $v - v_{\text{river}}$, and the time t_2 it takes to paddle a distance L is therefore $t_2 = L/(v - v_{\text{river}})$. Solving $t_2 = 4t_1$ we find

$$\begin{aligned} \frac{L}{v - v_{\text{river}}} &= \frac{4L}{v + v_{\text{river}}}, \\ \therefore v + v_{\text{river}} &= 4v - 4v_{\text{river}}, \qquad v = \frac{5}{3}v_{\text{river}}. \end{aligned}$$

12. The weight (downward), horizontal force, and tension form a closed right-angle triangle with sides of magnitude 1, 2 and T (in units of N). Clearly $T^2 = 1^2 + 2^2 = 5$, so $T = \sqrt{5}$.

13. If $W = 25$ N is the weight, then the component of W acting down the plane is $W \sin\theta$. The force of friction F_f acts up the plane, and the mass is $m = W/g$. Taking down the plane as the positive direction, we have

$$\begin{aligned} \sum F &= W \sin\theta - F_f = ma = W\frac{a}{g}, \\ \therefore F_f &= W\left(\sin\theta - \frac{a}{g}\right) = 5.5 \text{ N}. \end{aligned}$$

14. Let $\vec{r}_1$ be the first displacement, and $\vec{r}_2$ be the second. The net displacement is $\vec{r} = \vec{r}_1 + \vec{r}_2$. In terms of components we have

$$\begin{aligned} \vec{r}_1 &= -(250 \text{ km})\cos 30^\circ \hat{\imath} - (250 \text{ km})\sin 30^\circ \hat{\jmath} \\ &= -(217 \text{ km})\hat{\imath} - (125 \text{ km})\hat{\jmath}, \\ \vec{r}_2 &= (300 \text{ km})\hat{\jmath}, \\ \therefore \vec{r} &= -(217 \text{ km})\hat{\imath} + (175 \text{ km})\hat{\jmath}. \end{aligned}$$

The angle associated with this vector is in the second quadrant (negative x, positive y), and satisfies

$$\tan\theta = \frac{r_y}{r_x} = -0.808, \qquad \therefore \theta = 141^\circ.$$

This value of θ corresponds to 51° W of N.

15. Call the 90 kg block object 1, and the 110 kg block object 2. Let T be the tension in the string. Summing the forces on each block gives the following equations:

$$\begin{aligned} T - m_1 g &= m_1 a_1, \\ T - m_2 g &= m_2 a_2, \end{aligned}$$

with the constraint that $a_2 = -a_1$. Hence

$$\begin{aligned} T &= m_1 a_1 + m_1 g = m_2(-a_1) + m_2 g, \\ \therefore a_1 &= \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g = 0.98 \text{ m/s}^2. \end{aligned}$$

From this we find $T = m_1(a_1 + g) = 970$ N.

16. Relative to the straight line trajectory in the absence of gravity, the projectile falls a distance $\frac{1}{2}gt^2$. For $t = \{1, 2, 3\}$ (in units of s), and taking $g = 10$ m/s^2, these distances are $\{5, 20, 45\}$ m.

17. As drawn, the x and y-components of $\vec{P}$ satisfy $0 < P_x < P$ and $0 < P_y < P$. For constant speed there is no net force in either the x or y directions, so $P_x = f$ and $P_y + N = F_g$. The first equation requires $P > f$, while the second requires $N < F_g$.

18. Let v_x and v_y be the x and y components of $\vec{v}$ when the atoms leave the oven. The trajectory of the atoms obeys the equations

$$x = v_x t, \qquad y = v_y t - \frac{1}{2}gt^2.$$

Because the angle is very shallow, we can take $v_x \approx v$, as suggested in the problem. The time to reach slit 2 can be determined by setting $y = 0$, giving $t = (2v_y/g)$. We also have $L = vt$, so that $t = (L/v)$. Hence

$$\frac{2v_y}{g} = \frac{L}{v}, \qquad \therefore v_y = \frac{Lg}{2v}.$$

The atoms reach the detector at time $2t$, at which point, using the above expression for v_y, and $t = L/v$,

$$\begin{aligned} y = v_y(2t) - \frac{1}{2}g(2t)^2 &= \left(\frac{Lg}{2v}\right)\left(\frac{2L}{v}\right) - \frac{1}{2}g\left(\frac{2L}{v}\right)^2 \\ &= -g\frac{L^2}{v^2}. \end{aligned}$$

Therefore

$$v^2 = -g\frac{L^2}{y} = 49,000 \text{ (m/s)}^2, \qquad \therefore v = 220 \text{ m/s}.$$