

Constants and Units

$$k = 10^3, \mu = 10^{-6}, n = 10^{-9}$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$g = 9.80 \text{ m/s}^2$$

Particle Dynamics

Mathematics

Quadratic equation:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry

$$x^2 + y^2 = r^2$$

$$\sin \theta = y / r$$

$$\cos \theta = x / r$$

$$\tan \theta = y / x$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Calculus:

$$\frac{d}{dt}(a \cdot t^n) = a \cdot n t^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Translational Kinematics

Three dimensions:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

One dimension:

$$v_{x,av} = \frac{\Delta x}{\Delta t}$$

$$a_{x,av} = \frac{\Delta v_x}{\Delta t}$$

Constant acceleration in one dimension:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

Uniform circular motion:

$$a = \frac{v^2}{r}$$

$$\left. \begin{array}{l} f_s \leq \mu_s N \\ f_k = \mu_k N \end{array} \right\} \begin{array}{l} N = \text{normal} \\ \text{force} \end{array}$$

Relative Motion

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \quad (PA \text{ means } P \text{ relative to } A, \text{ etc.})$$

Work, Kinetic Energy, Potential Energy

$$W = \vec{F} \cdot \vec{s}$$

$$W = \int_A^B \vec{F} \cdot d\vec{s}$$

$$\sum \vec{F} = m\vec{a}$$

$$W = mg \quad \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$K = \frac{1}{2}mv^2$$

$$W = \Delta K = K_f - K_i \quad E = K + U$$

$$\Delta E = E_f - E_i = W_{nc}$$

$$U_s = \frac{1}{2}kx^2$$

(spring)

$$F_x = -kx$$

$$U_g = mgy$$

(gravity)

$$\text{Power} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Momentum and Collisions

$$\vec{p} = m\vec{v} \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} \equiv \int \vec{F} dt = \vec{F}_{av} \Delta t = \Delta \vec{p} \quad (\text{impulse})$$

$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i \quad \vec{v}_{cm} = \frac{1}{M} \sum_i m_i \vec{v}_i$$

$$\vec{P} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M \vec{v}_{cm} \quad \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

(conservation of momentum)

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

(elastic collision)

Rotational Kinematics

$$\begin{aligned}\omega &= \frac{d\theta}{dt} & \alpha &= \frac{d\omega}{dt} \\ v &= \omega r & a_T &= \alpha r \\ \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ \omega &= \omega_0 + \alpha t \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0)\end{aligned}$$

} constant acceleration α

Torque and Angular Momentum

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\ell} = \vec{r} \times \vec{p}$$

$$\begin{aligned}\vec{\tau} &= \frac{d\vec{\ell}}{dt} \\ |\vec{A} \times \vec{B}| &= |\vec{A}| |\vec{B}| \sin\theta \\ L &= I\omega \\ \tau &= I\alpha \\ I &= \sum_i m_i r_i^2\end{aligned}$$

} (rotating rigid object)

Special Relativity

$$\left. \begin{aligned}x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2)\end{aligned}\right\}$$

Lorentz transformation

$$L = L_0 / \gamma$$

Relative velocity formula for motion in one dimension:

$$u = \frac{u' + v}{\left(1 + \frac{vu'}{c^2}\right)}$$

Energy and momentum:

m = rest mass

$$\vec{p} = \gamma m \vec{v}$$

$$E = K + mc^2 = \gamma mc^2$$

$$E^2 = c^2 p^2 + m^2 c^4$$