Muon decay $\Delta t_0 = 2.2 \times 10^{-6}$ s

$\gamma = 7.23$ $(v = 0.9904c)$

(a) $\Delta t = \gamma \Delta t_0$

$= 15.9 \times 10^{-6}$ s

(b) $L_0 = v \Delta t$

$= (0.9904)(3 \times 10^{8} \frac{m}{s})(15.9 \times 10^{-6} s)$

$= 4720$ m.

(c) According to muon, it decays in

$\Delta t_0 = 2.2 \times 10^{-6}$ s.

So it travels a distance of

$v \Delta t_0 = 653$ m.

This is $\frac{L_0}{\gamma} = \frac{4720 \text{ m}}{7.23}$

$= 653$ m.

Earth: Muon decays in 15.9 μs
and travels 4720 m.

Muon: muon decays in 2.2 μs
and travels 653 m.

$$v = \frac{4720\,m}{15.9 \times 10^{-6}\,s} = \frac{653\,m}{2.2 \times 10^{-6}\,s}$$

$$= 2.97 \times 10^{8}\,\frac{m}{s}$$

$$= 0.9904 \times c$$

1 ly = 1 light-year = distance travelled by light in 1 year.
$\therefore c = 1\,ly/y$

Example

(a) Sam: $\Delta t = \dfrac{d}{v} = \dfrac{26.0\ ly}{(0.99c)}$

But $0.99c = 0.99\,\dfrac{ly}{y}$

$\therefore \Delta t = \dfrac{26.0\ ly}{0.99\ ly/y}$

$= 26.3\ y.$

(b) It takes 26.0 y for signal to return.

Total is $26.3\,y + 26.0\,y = 52.3\,y.$

(c) Sally measures proper time interval.

$\Delta t_0 = \dfrac{\Delta t}{\gamma}$ $\qquad \gamma = \dfrac{1}{\sqrt{1-(0.99)^2}} = 7.09$

$= \dfrac{26.3\,y}{7.09} = 3.71\,y.$

Another way to look at this:

Sally says distance is

$$L = \frac{26.0}{\gamma} \text{ ly}$$

$$= 3.67 \text{ ly}.$$

$$\Delta t_0 = \frac{3.67 \text{ ly}}{0.99 \text{ ly/y}} = 3.71^{\text{ш}} \text{ y}.$$