Example 1

$$f = 33.3\ \frac{rev}{min} \times \frac{1\ min}{60\ s}$$
$$= 0.556\ \frac{rev}{s}$$

$$\omega_0 = 2\pi f$$
$$= 2\pi\ \frac{rad}{\cancel{rev}} \times 0.556\ \frac{\cancel{rev}}{s}$$
$$= 3.49\ \frac{rad}{s}$$

(a) $\omega = 0$ $\qquad$ $\Delta\omega = 0 - 3.49\ rad/s$
$$= -3.49\ rad/s$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{-3.49\ rad/s}{20\ s}$$
$$= -0.174\ rad/s^2$$

(b) Two ways:

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$= \left(3.49\ \frac{rad}{s}\right)(20\ s) + \frac{1}{2}\left(-0.174\ \frac{rad}{s^2}\right)(20$$
$$= 35.0\ rad$$

or

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$
$$0 = \left(3.49\ \frac{rad}{s}\right)^2 + 2\left(-0.174\ \frac{rad}{s^2}\right)\Delta\theta$$

$$\therefore \Delta\theta = -\frac{\omega_0^2}{2\alpha} = 35.0 \text{ rad.}$$

$$\#\text{rotations} = \frac{1 \text{ rev}}{2\pi \text{ rad}} \times 35.0 \text{ rad.}$$

$$= \frac{35}{2\pi} \text{ rev.}$$

Example 2

$$r = 12\,cm = 0.12\,m$$

$$\omega_0 = 0 \qquad \theta_0 = 0.$$

$$\alpha = 5.0\ rad/s^2$$

At $t = 2.0\,s$

(a) $\omega = \alpha t = 5.0\frac{rad}{s^2} \times 2\,s = 10.0\frac{rad}{s}$

(b) $v = \omega r = 10.0\frac{rad}{s} \times 0.12\,m = 1.2\frac{m}{s}$

(c) $\Delta\theta = \theta = \frac{1}{2}\alpha t^2$

$$= \frac{1}{2}\left(5.0\frac{rad}{s^2}\right) \times (2.0\,s)^2$$

$$= 10.0\,rad.$$

OR $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$$\left(10\frac{rad}{s}\right)^2 = 0 + 2\left(5.0\frac{rad}{s^2}\right)\theta$$

$$\therefore \theta = \frac{100\ rad}{10.} = 10.0\ rad.$$

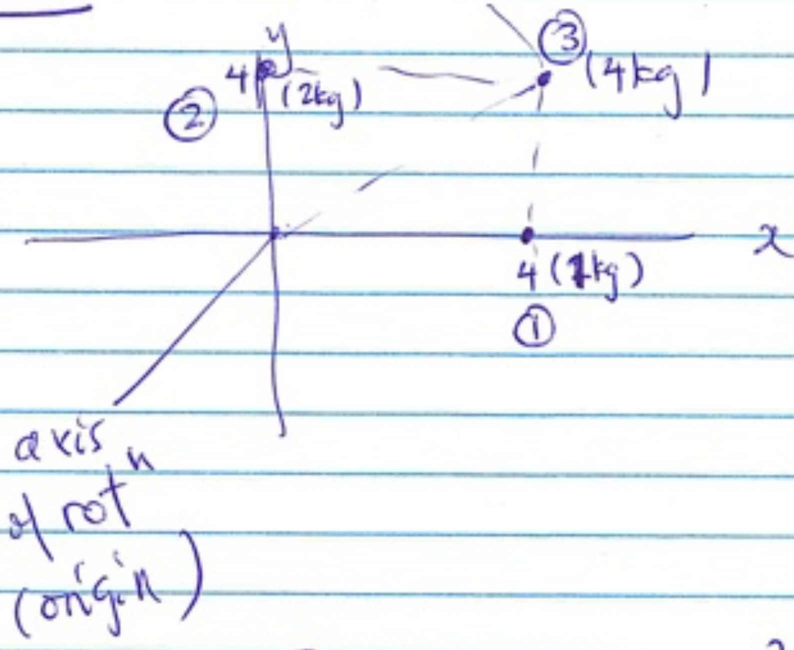
(d) $a_t = \alpha r = 5.0\frac{rad}{s^2} \times 0.12\,m = 0.6\frac{m}{s^2}$ (constant)

$$a_r = \frac{v^2}{r} = \omega^2 r = \left(10.0\frac{rad}{s}\right)^2 (0.12\,m) = 12\frac{m}{s^2}$$

$$a^2 = a_r^2 + a_t^2 \qquad \therefore a = \sqrt{a_r^2 + a_t^2} = 12.01\frac{m}{s}$$

Example 3

$\omega = 2$ rad/s



$v_1 = \omega r_1 = 8\,\frac{m}{s}$

$v_2 = \omega r_2 = 8\,\frac{m}{s}$

$v_3 = 8\sqrt{2}\,\frac{m}{s}$

①: $I_1 = (1.0\,kg)(4\,m)^2 = 16\ kg\cdot m^2$

$I_2 = (2.0\,kg)(4\,m)^2 = 32\ kg\cdot m^2$

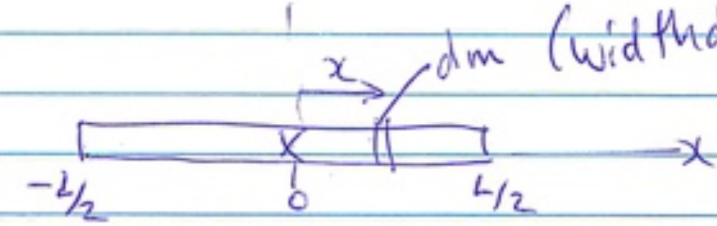
$I_3 = (4.0\,kg)(4^2+4^2)\,m^2 = 128\ kg\cdot m^2$ (same as $\frac{1}{2} m_3 v_3^2$)

Total $I = I_1 + I_2 + I_3 = 176\ kg\cdot m^2$

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2}(176\,kg\cdot m^2)\left(2\,\frac{rad}{s}\right)^2 = 352\ kg\cdot\frac{m^2}{s^2} = 352\ J$$

Simple examples:

Thin rod about CM:



$$I = \int r^2 \, dm \qquad dm = \frac{M}{L} dx$$

$$= \frac{M}{L} \int_{-\frac{L}{2}}^{+\frac{L}{2}} x^2 \, dx = \frac{M}{L} \left. \frac{x^3}{3} \right|_{-L/2}^{L/2}$$

$$= \frac{M}{3L} \left(\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3 \right)$$

$$= \frac{1}{12} M L^2$$

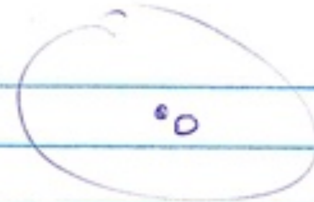
Thin rod about end!

$$I = \frac{M}{L} \int_0^L x^2 \, dx$$

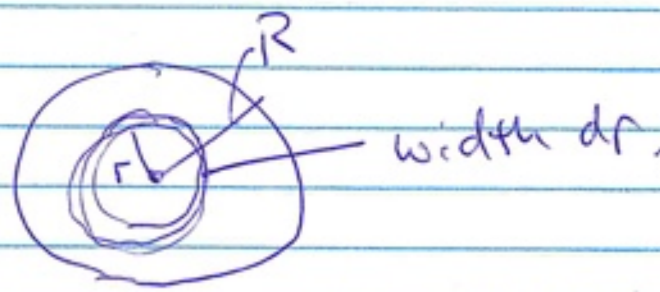
$$= \frac{M}{L} \left. \left(\frac{1}{3} x^3 \right) \right|_0^L = \frac{1}{3} M L^2$$

Hoop $\quad I = MR^2$

(about O)



Circular disc !



$$dm = \frac{M(2\pi r . dr)}{\pi R^2}$$

$$\therefore I = \frac{M}{\cancel{\pi} R^2} . 2\cancel{\pi} \int_0^R r^2 r . dr$$

$$= \frac{2M}{R^2} \int_0^R r^3 dr = \frac{2M}{R^2} \frac{R^4}{4}$$

$$= \frac{1}{2} MR^2$$