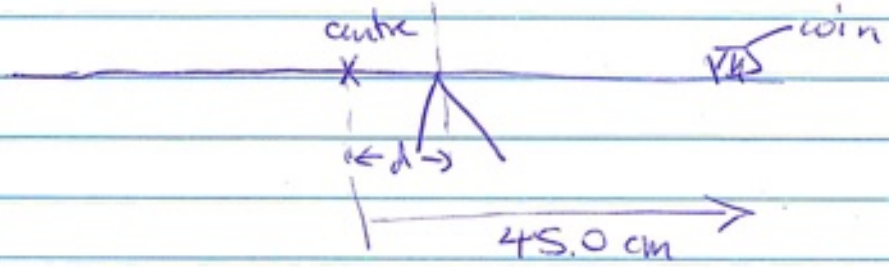
Given $m_{coin} = 7.3\,g$ M = mass of metre stick

COM of metre stick is at centre.

Put origin of coordinate system at fulcrum.



$x_{centre} = -d = -2.3$ cm.

$x_{coin} = 45.0\,cm - 2.3\,cm$

$x_{com} = M x_{centre} + m x_{coin} = 0$

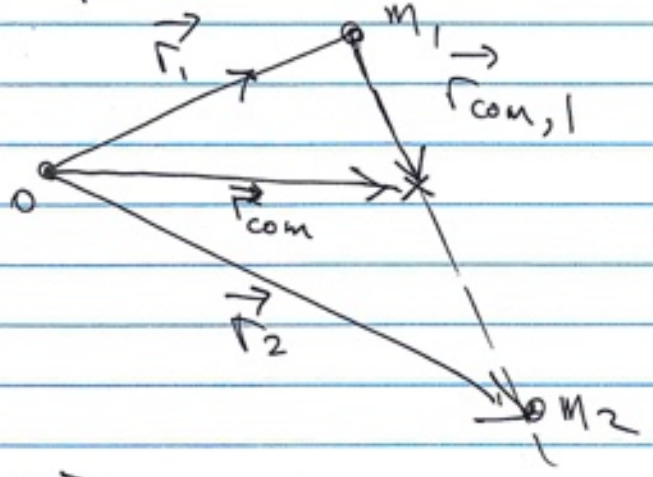
(to put x_{com} above fulcrum)

$$\therefore M = \frac{(45.0 - 2.3)}{2.3} \times 7.3\,g$$

$$= 135\,g.$$

(actual mass is 132 g).

Example



$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$\vec{r}_{com,1} = -\vec{r}_1 + \vec{r}_{com}$$

$$= -\vec{r}_1 + \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$= \frac{-(m_1+m_2)\vec{r}_1 + m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$= \frac{m_2}{m_1+m_2}\underbrace{(-\vec{r}_1+\vec{r}_2)}$$

position of 2 wrt 1.

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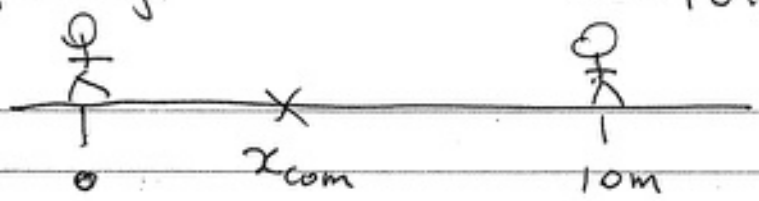


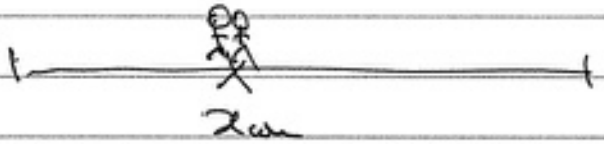
$x_1 = 0$ $\qquad$ $x_2 = 10$ m.

before $\qquad x_{com} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$= \dfrac{m_2}{m_1 + m_2} x_2 = \dfrac{40\text{ kg}}{105\text{ kg}} \times 10\text{ m}$

$= 3.8$ m.

after



x_{com} does not change

because $v_{com} = 0$

and no external forces.

Note: Internal forces cannot change the position of x_{com}.

Example

Assume boat is symmetric.

$\therefore x_{cm} = 2.5$ m relative to end.

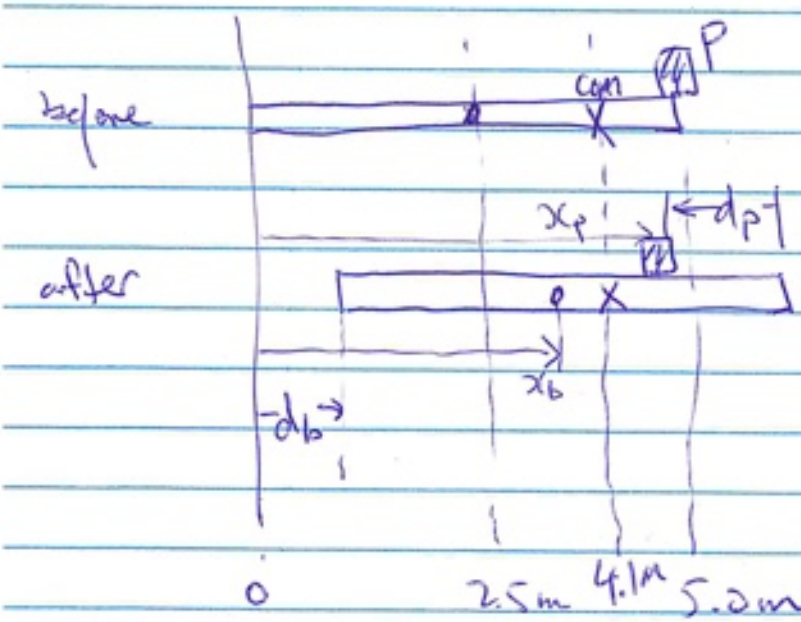
boat $m_b = 30$ kg $\quad x_b = 2.5$ m.

person $m_p = 50$ kg. $\quad x_p = 5$ m.

before $\quad x_{com} = \dfrac{m_b x_b + m_p x_p}{m_b + m_p}$

$= \dfrac{30 \times 2.5\,m + 50 \times 5\,m}{80}$

$= 4.1$ m



- boat has moved d_b to right

$\therefore x_b = 2.5\,m + d_b$

- person has moved $-d_p$ from end of boat

$\therefore x_p = 5.0\,m + d_b - d_p$

x_{com} has not changed.

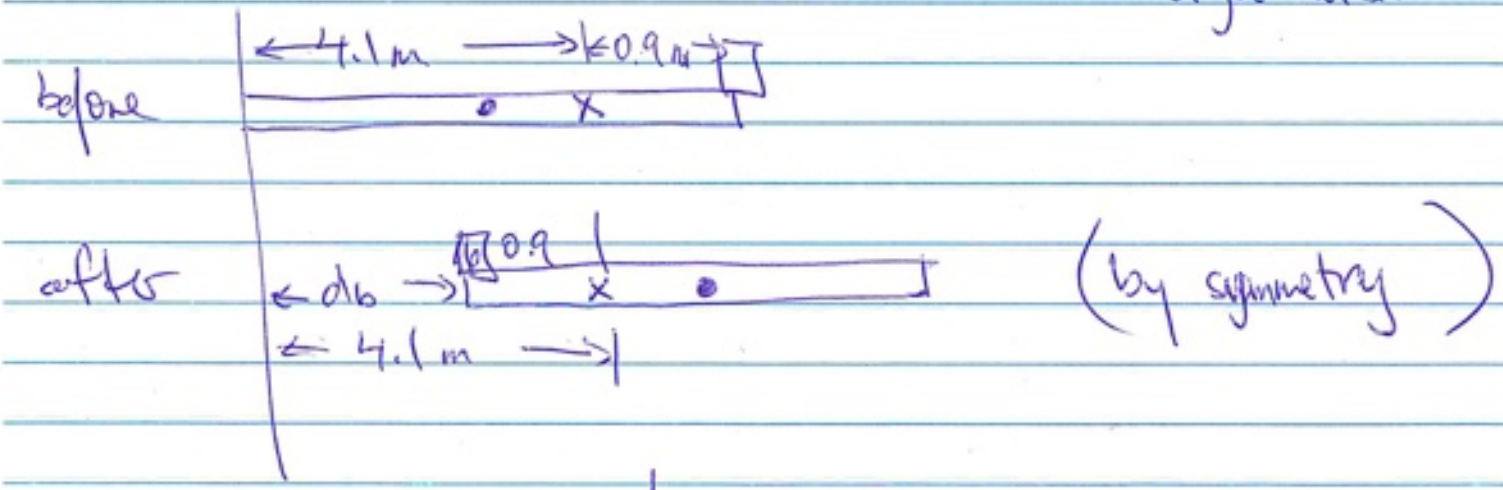
$$\therefore 4.1\ m = \frac{30\,x_b + 50\,x_p}{80}$$

$$= \frac{30(2.5 + x_b) + 50(5.0_m + d_b - d_p)}{80}$$

Solving we get $\boxed{d_b = \frac{5}{8} d_p}$

If $d_p = 5.0$ m, person is now at front of boat. Use symmetry arguments.

Therefore d_b = 3.2 m



$$d_b + 0.9\ m = 4.1\ m$$

$$\therefore d_b = 3.2\ m$$