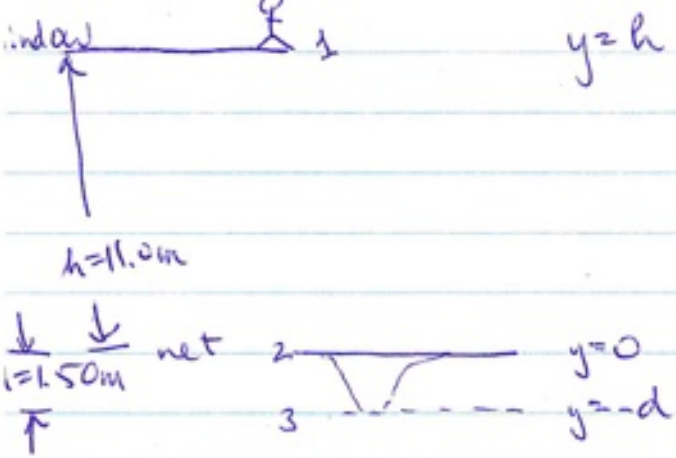


Choose $y=0$ at net

There are two approaches to use: (i) look at equating E_{mech} at each time
(ii) look at changes $\Delta E_{mech} =$

I'll show both approaches. Which one you prefer is a matter of taste.

(i) $E_{mech} = K_i + U_i$

At top $\overset{(y=h)}{1}$, $K_1 = 0$; $U_1 = mgh$ $\quad (U_{spring} = 0)$

At net $(y=0)$, $K_2 = \frac{1}{2}mv_2^2$; $U_2 = 0$ $\quad (U_{spring} = 0)$

At bottom $(y=-d)$, $K_3 = 0$; $U_3 = \frac{1}{2}kd^2 - mgd$

$\therefore K_1 + U_1 = K_3 + U_3 = E_{mech}$

$$\therefore mgh = \frac{1}{2}kd^2 - mgd$$

$$\therefore mg(h+d) = \frac{1}{2}kd^2$$

Alternatively we can look at $\Delta E_{mech} = 0$.

$\Delta K = 0, \quad \Delta U = 0$

but U is made up of gravitational (U_g)

plus spring (U_s) potential energy.

$\Delta U_g = mg\Delta y = -mg(h+d)$ (since $\Delta y = -(h+d)$

$\Delta U_s = \frac{1}{2}kd^2$

$\therefore -mg(h+d) + \frac{1}{2}kd^2 = 0$

OR $mg(h+d) = \frac{1}{2}kd^2$

$$\therefore k = \frac{2mg(h+d)}{d^2} = 7620\ N/m$$

(b)

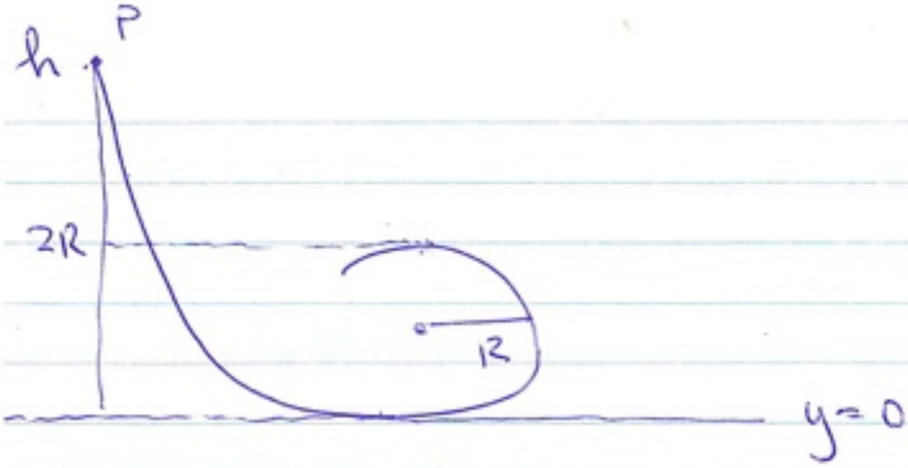
$F_s = -ky = +kd$ (up). at $y = -d$.

$F_g = -mg$

$\therefore F_{net} = kd - mg$

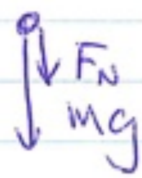
$$= 7620\ \frac{N}{m} \times 1.50\ m - 70\ kg \times 9.8\ \frac{m}{s^2}$$

$$= 10{,}700\ N = 15.7 \times (mg)$$



At top of loop

free-body diagram



$$F_N + mg = m\frac{v^2}{R}$$

Smallest v is when

$$F_N = 0.$$

$$\therefore\ mg = \frac{mv^2}{R} \qquad \therefore \text{ require } v^2 \geq gR$$

Find h. $E = mgh = \frac{1}{2}mv^2 + mg(2R)$

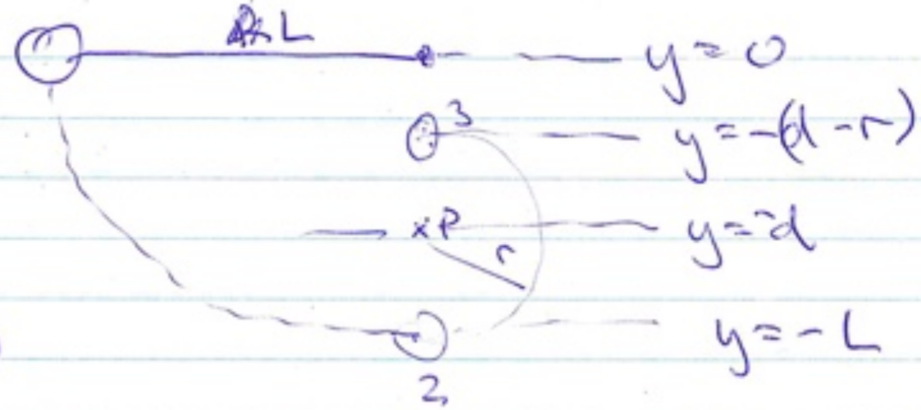
$$\therefore\ \cancel{m}g(h-2R) = \frac{1}{2}\cancel{m}v^2$$

$$\therefore\ v^2 = 2g(h-2R)$$

$$\geq gR$$

$$\therefore\ 2\cancel{g}h - 4\cancel{g}R = \cancel{g}R \qquad \text{at min.}$$

$$\therefore\ h = \frac{5}{2}R$$



(a)

$K_1 = 0 \quad U_1 = 0$ $\qquad (L = d + r)$

$K_2 = \frac{1}{2} m v^2 \qquad U_2 = -mgL$

$E_{total} = \frac{1}{2} m v^2 - mgL = 0$

$\frac{1}{2} m v^2 = mgL$

$\therefore v^2 = 2gL = 2gL + 2gr$

(b) $v_3^2 \geq gr$ (same argument as previous problem)

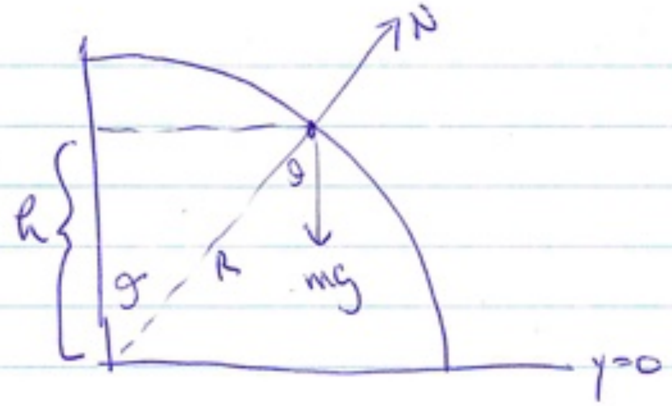
$K_3 = \frac{1}{2} m v_3^2 \qquad U_3 = -mg(d - r)$

$\frac{1}{2} m v_3^2 - mg(d - r) = 0$

Set $v_3^2 = gr \quad \therefore \frac{1}{2} gr - gd + gr = 0$

$\Rightarrow d = \frac{3}{2} r$

(This is obviously the same as the previous problem, ie release point is $\frac{5}{2} r$ above bottom.)



Sum of forces in radial direction:

$$\sum F_r = N - mg\cos\theta = ma_c = m\left(-\frac{v^2}{R}\right)$$

Lose contact when $N=0 \Rightarrow v^2 = gR\cos\theta$

$$= gh$$

since $h = R\cos\theta$

To find v^2 at this point, use conservation of energy.

At top: $K_1 = 0 \quad U_1 = mgR$

At θ: $K_2 = \frac{1}{2}mv^2 \quad U_2 = mgh$

$\therefore mgR = \frac{1}{2}mv^2 + mgh$

$\therefore mg(R-h) = \frac{1}{2}mv^2$ (same as $-\Delta U = \Delta K$

$\therefore v^2 = 2g(R-h) = gh$

$\therefore h = \frac{2}{3}R \qquad \therefore \cos\theta = \frac{h}{R} = \frac{2}{3}$