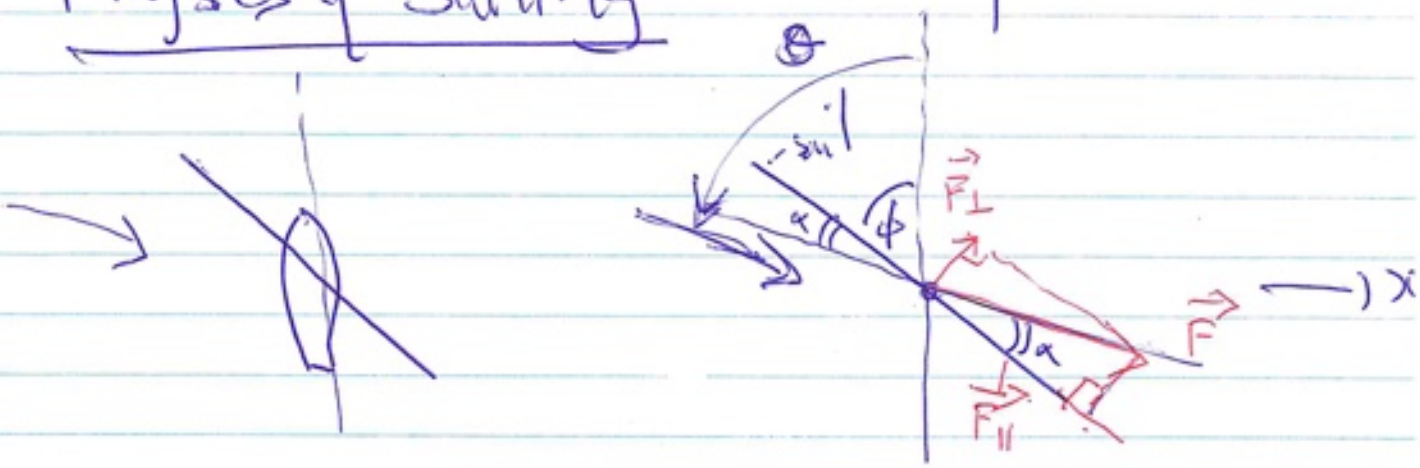
Physics of Sailing

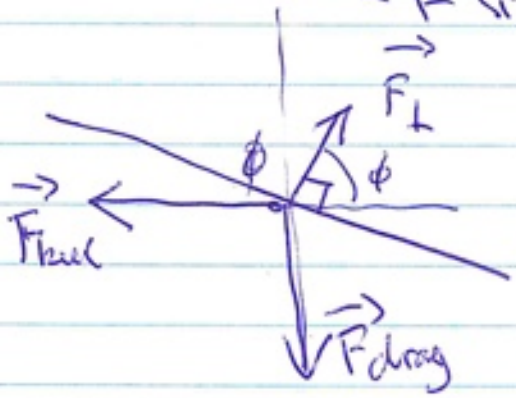


$\alpha = \theta - \phi$

$\vec{F}_\parallel$ does nothing (parallel to sail)

$\vec{F}_\perp$ sail force ($\perp$ to sail)

$= F \sin\alpha$



For constant velocity

$\vec{F}_\perp + \vec{F}_{keel} + \vec{F}_{drag} = \vec{0}$

x] $\Sigma F_x = F_\perp \cos\phi - F_{keel} = 0$

$\Sigma F_y = \underbrace{F_\perp \sin\phi} - F_{drag} = 0$

what moves boat forward

propulsive force is

$$F_t \sin\phi = F \sin\alpha \sin\phi$$
$$= F \sin(\theta - \phi) \sin\phi$$

If $\theta > \phi$ then $F_{prop} > 0$

What is optimal angle ϕ for a given θ?

$$F_{prop} = F \sin\phi \left[\sin\theta \cos\phi - \cos\theta \sin\phi \right]$$
$$= F \left[\sin\theta \sin\phi \cos\phi - \cos\theta \sin^2\phi \right]$$

$$\frac{dF_{prop}}{d\phi} = \sin\theta \left[\cos^2\phi - \sin^2\phi \right]$$
$$- \cos\theta \, 2 \sin\phi \cos\phi$$
$$= \sin\theta \cos 2\phi - \cos\theta \sin 2\phi = 0$$

$$\tan\theta = \tan 2\phi$$

$$\therefore \theta = 2\phi$$

Example

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos\phi$$

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta K = W$$

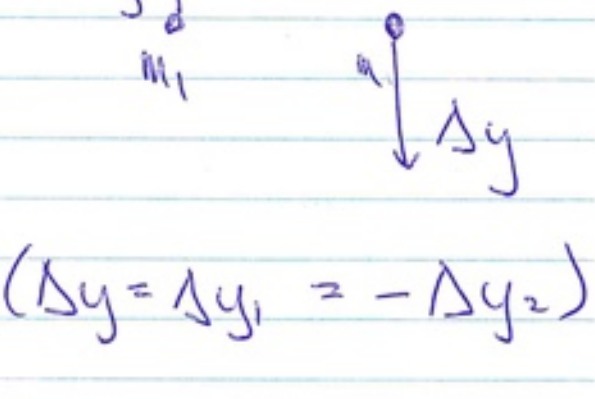
$$K_i = 0 \qquad K_f = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$= \frac{1}{2} (m_1 + m_2) v^2$$

$(W_g = -mg\Delta y)$

$$W = -m_1 g \Delta y - m_2 g (-\Delta y)$$

$$= (m_2 - m_1) g \Delta y$$



$$\frac{1}{2} (m_1 + m_2) v^2 = (m_2 - m_1) g \Delta y$$

$$v^2 = \left(\frac{m_2 - m_1}{m_2 + m_1} \right) 2 g \Delta y$$

Example

$$\Delta K = K_f - \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2$$

$$= \frac{1}{2}(m_A + m_B) v^2$$

$$W = -m_A g \Delta y_A - m_B g \Delta y_B$$

$$\Delta y_B = -s = -0.25 \text{ m}$$

$$\Delta y_A = s \sin 37^\circ$$

$$= 0.15 \text{ m}.$$

$$\therefore W = -50\text{kg}\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(0.15) - 100\text{kg}\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(-0.25)$$

$$= 171.3 \text{ J}$$

$$\therefore v^2 = 2.28 \frac{\text{m}^2}{\text{s}^2}$$

$$\therefore v = 1.51 \frac{\text{m}}{\text{s}}$$