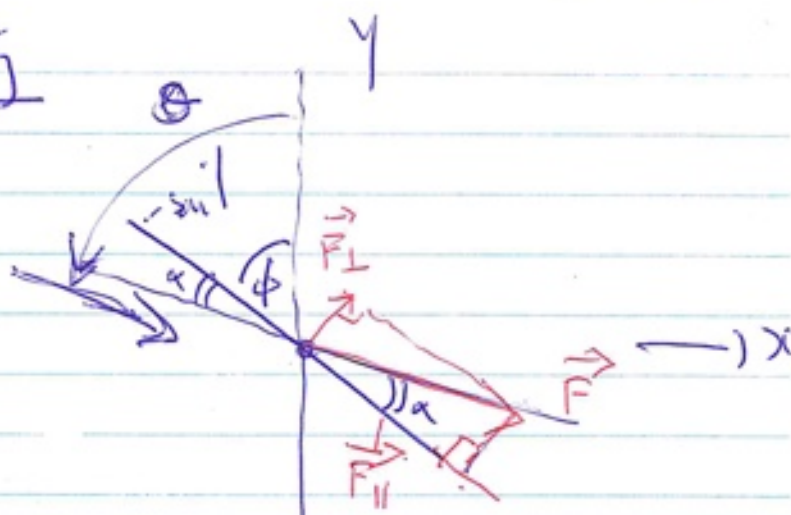
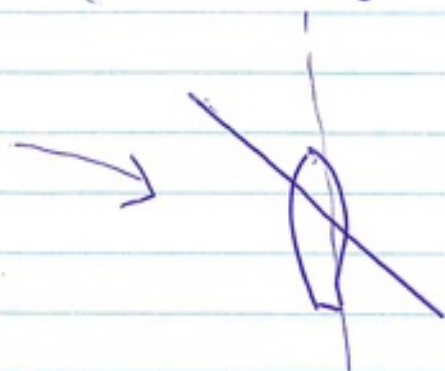


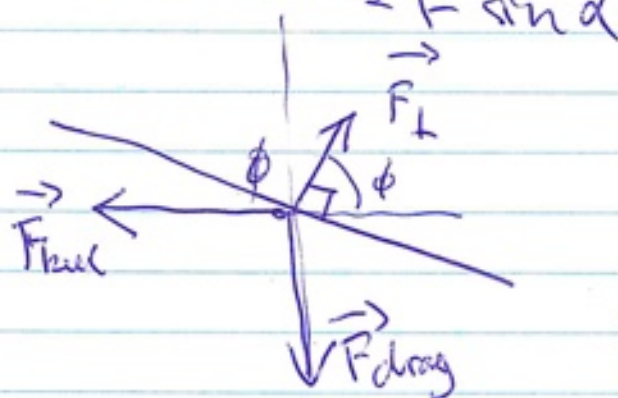
Physics of Sailing



$$\alpha = \theta - \phi$$

$\vec{F}_{\parallel}$ does nothing (parallel to sail)

$\vec{F}_{\perp}$ sail force ($\perp$ to sail)



For constant velocity

$$\vec{F}_{\perp} + \vec{F}_{\text{keel}} + \vec{F}_{\text{drag}} = \vec{0}$$

$$\sum F_x = F_{\perp} \cos \phi - F_{\text{keel}} = 0$$

$$\sum F_y = \underbrace{F_{\perp} \sin \phi}_{\text{what moves boat forward}} - F_{\text{drag}} = 0$$

what moves boat forward

propulsive force is

$$\begin{aligned} F_{\perp} \sin \phi &= F \sin \alpha \sin \phi \\ &= F \sin(\theta - \phi) \sin \phi \end{aligned}$$

If $\theta > \phi$ then $F_{\text{prop}} > 0$

What is optimal angle ϕ for a given θ ?

$$\begin{aligned} F_{\text{prop}} &= F \sin \phi \left[\sin \theta \cos \phi - \cos \theta \sin \phi \right] \\ &= F \left[\sin \theta \sin \phi \cos \phi - \cos \theta \sin^2 \phi \right] \end{aligned}$$

$$\begin{aligned} \frac{dF_{\text{prop}}}{d\phi} &= \sin \theta \left[\cos^2 \phi - \sin^2 \phi \right] \\ &\quad - \cos \theta \cdot 2 \sin \phi \cos \phi \\ &= \sin \theta \cos 2\phi - \cos \theta \sin 2\phi = 0 \end{aligned}$$

$$\tan \theta = \tan 2\phi$$

$$\therefore \theta = 2\phi$$

Example

$$\begin{aligned}\Delta K &= \frac{1}{2} m V_f^2 - \frac{1}{2} m v_i^2 \\ &= W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \phi\end{aligned}$$

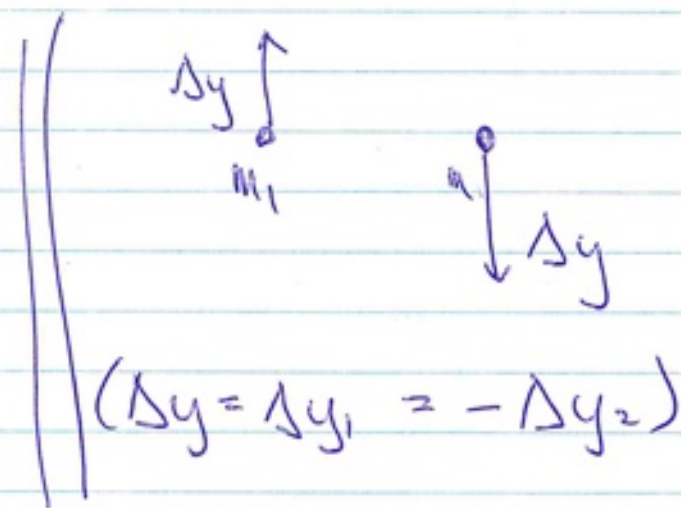
$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta K = W$$

$$K_i = 0 \quad K_f = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$(W_g = -mg\Delta y) \quad = \frac{1}{2} (m_1 + m_2) v^2$$

$$\begin{aligned}W &= -m_1 g \Delta y \\ &\quad - m_2 g (-\Delta y) \\ &= (m_2 - m_1) g \Delta y\end{aligned}$$



$$\begin{aligned}\frac{1}{2} (m_1 + m_2) v^2 &= (m_2 - m_1) g \Delta y \\ v^2 &= \left(\frac{m_2 - m_1}{m_2 + m_1} \right) 2g \Delta y\end{aligned}$$

example

$$\begin{aligned}\Delta K &= K_f = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2 \\ &= \frac{1}{2} (m_A + m_B) v^2\end{aligned}$$

$$W = -m_A g \Delta y_A - m_B g \Delta y_B$$

$$\Delta y_B = -s = -0.25 \text{ m}$$

$$\Delta y_A = s \sin 37^\circ$$

$$= 0.15 \text{ m.}$$

$$\begin{aligned}\therefore W &= -50 \text{ kg} \left(9.8 \frac{\text{m}}{\text{s}^2} \right) (0.15) \\ &\quad - 100 \text{ kg} \left(9.8 \frac{\text{m}}{\text{s}^2} \right) (-0.25) \\ &= 171.3 \text{ J}\end{aligned}$$

$$\therefore v^2 = 2.28 \frac{\text{m}^2}{\text{s}^2}$$

$$\therefore v = 1.51 \frac{\text{m}}{\text{s}}$$