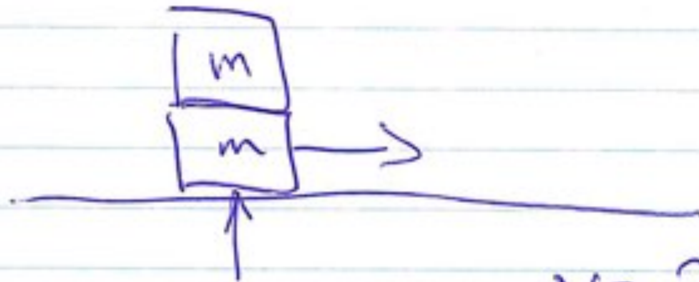
Friction model

(a)

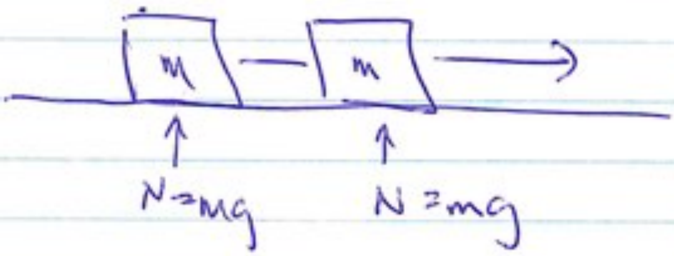


$$N = 2mg$$

$$f_{s,max} = \mu_s N = \mu_s 2mg$$

$$f_k = \mu_k N = \mu_k 2mg$$

(b)

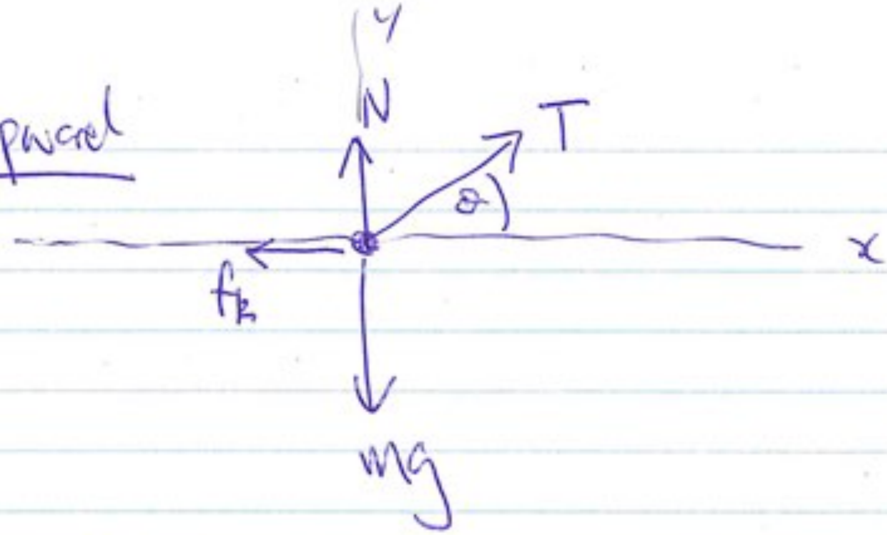


$$\text{Total } f_k = 2 \times \mu_k mg$$

f_k <u>Same</u> in both cases.

$\Rightarrow$ independent of shape of contact area

Pulling upward



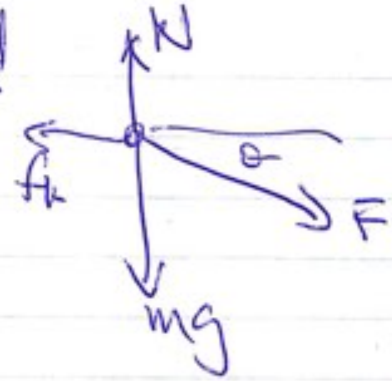
$\theta = 30^\circ$

$$\sum F_y = 0$$
$$= N + T\sin\theta - mg$$
$$\therefore N = mg - T\sin\theta$$
$$= 194\ N$$

$$f_k = \mu_k N$$
$$= 0.15 \times 194\ N$$
$$= 29.1\ N$$

$$\sum F_x = T\cos\theta - f_k = ma_x$$
$$\therefore a_x = \frac{T\cos\theta - f_k}{m}$$
$$= 4.8\ \frac{m}{s^2}$$

Pushing downward



$$\Sigma F_y = N - F\sin\theta - mg = 0$$

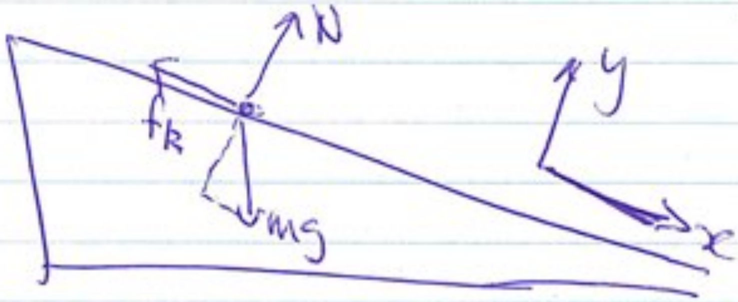
$$\therefore N = mg + F\sin\theta$$

$$= 394\,N$$

$$f_k = \mu_k N = 59.1\,N$$

$$ma_x = F\cos\theta - f_k$$

$$\therefore a_x = 3.8\,\frac{m}{s^2}$$



$$\sum F_x = mg \sin\theta - f_k = ma_x$$

$$\sum F_y = N - mg\cos\theta = 0$$

$$\therefore\ N = mg\cos\theta$$

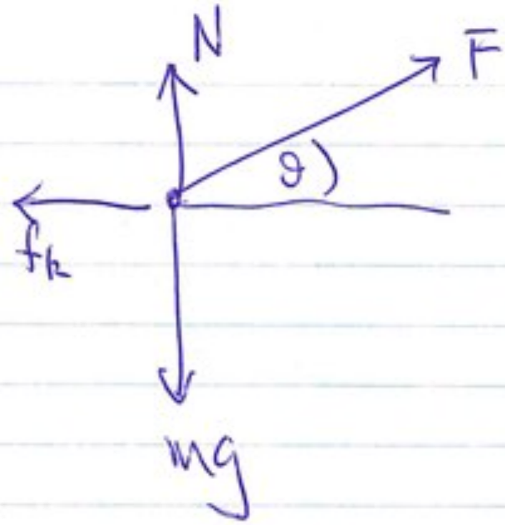
$$f_k = \mu_k N$$

$$= \mu_k mg\cos\theta$$

If $\quad a_x = 0, \quad \cancel{mg}\sin\theta = \mu_k \cancel{mg}\cos\theta$

$$\therefore\ \mu_k = \tan\theta.$$

Optimal ϑ:



$f_k = \mu_k N$

$\Sigma F_y = 0 \qquad N + F\sin\vartheta - mg = 0$

$\therefore N = mg - F\sin\vartheta$

$\Sigma F_x = F\cos\vartheta - \mu_k N = m a_x$

$\therefore a_x = \frac{F}{m}\cos\vartheta - \mu_k\left(g - \frac{F}{m}\sin\vartheta\right)$

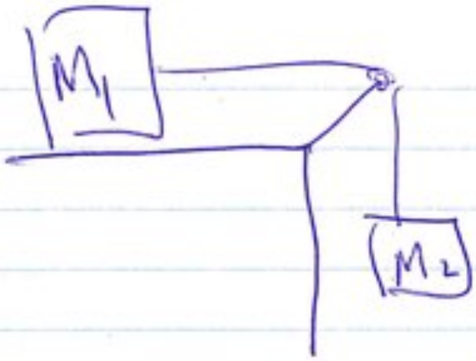
$\frac{da_x}{d\vartheta} = -\frac{\cancel{F}}{\cancel{m}}\sin\vartheta + \mu_k\frac{\cancel{F}}{\cancel{m}}\cos\vartheta = 0$

$\therefore \mu_k\cos\vartheta = \sin\vartheta$

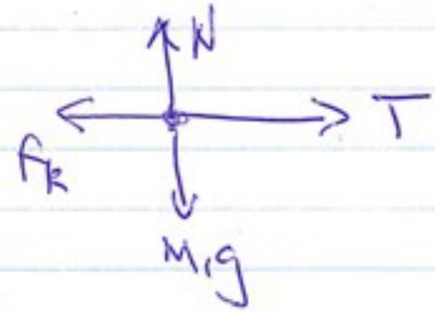
$\therefore \mu_k = \tan\vartheta$

$\therefore \mu_k = 0.4 = \tan\vartheta$

$\therefore \vartheta = 21.8^0$



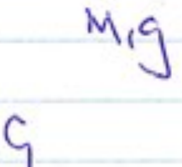
On m_1:



$$\sum F_y = 0 \Rightarrow N = mg$$

$$\sum F_x = T - f_k = T - \mu_k m_1 g$$
$$= m_1 a_1$$

On m_2:



$$T - m_2 g = m_2 a_2$$
$$= m_2(-a_1)$$

$$\boxed{a_2 = -a_1}$$

$$T = m_1(a_1 + \mu_k g)$$
$$= m_2(g - a_1)$$

$$m_2 g - m_2 a_1 = m_1 a_1 + m_1 \mu_k g$$

$$\therefore (m_1+m_2)a_1 = m_2 g - \mu_k m_1 g$$

Alternative: consider system

$$\Sigma F = m_2 g - \mu_k m_1 g$$
$$= (m_1+m_2)a \qquad a \text{ to "right"})$$

Same as previously.

Internal forces have not effect on system as a whole.