Projectile motion (motion in a 2D vertical plane)

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Here $\vec{a} = -g\hat{j}$, so

$$v_x = v_{0x}, \quad v_y = v_{0y} - gt$$

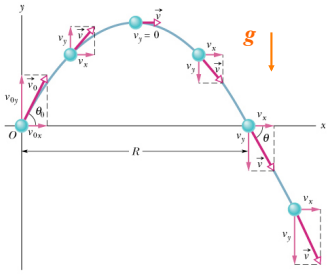
$$x - x_0 = v_{0x}t$$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

The projectile is launched with an initial velocity $\vec{v}_0$ at an angle θ_0.

$$v_{0x} = v_0 \cos\theta_0$$

$$v_{0y} = v_0 \sin\theta_0$$



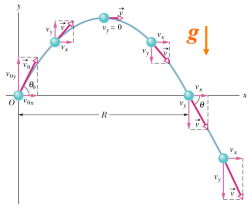
For $\theta_0 > 0$,

- time to reach maximum height:

$$v_y = v_{0y} - gt = 0, \quad \therefore t = \frac{v_{0y}}{g}$$

- maximum height:

$$y_{\max} = v_{0y}t - \tfrac{1}{2}gt^2 = \frac{v_{0y}^2}{2g}$$



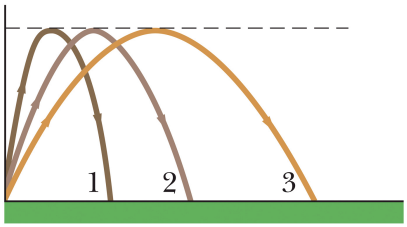
- time to reach ground level again: $t = \dfrac{2v_{0y}}{g}$
- range is: $R = v_{0x}t = \dfrac{2v_{0x}v_{0y}}{g} = \dfrac{2v_0^2}{g}\cos\theta_0 \sin\theta_0 = \dfrac{v_0^2}{g}\sin 2\theta_0$
- maximum range is $R_{\max} = \dfrac{v_0^2}{g}$ when $\theta_0 = 45^\circ$
- R is symmetric about 45°.
- In general there are 2 possible values of θ_0 for the same R.

Example 4.9

The figure shows 3 paths for a projectile. Rank the paths according to:

a) time of flight
b) initial vertical velocity component
c) initial horizontal velocity component
d) initial speed, greatest first

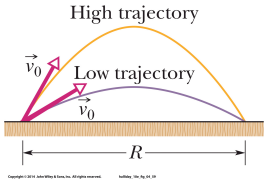
a) all same
b) all same
c) 3,2,1
d) 3,2,1



Problem 4.123

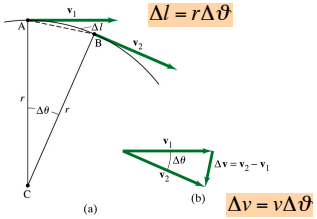
A projectile is fired with an initial speed v_0=82 m/s. A target is at a distance R=560 m on level ground, as shown.

(a) What are the two possible launch angles that will hit the target?
(b) What are the two times-in-flight for these angles?
(c) What is the maximum range of the projectile?



Uniform circular motion

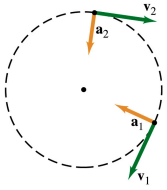
A particle moves in a circle (or a circular arc) of radius r at a **constant** speed v. Velocity $\vec{v}$ is tangent to the circular path, and therefore changes **direction**.



$$\Delta l = r\Delta\vartheta$$

$$\Delta v = v\Delta\vartheta$$

Centripetal acceleration has constant magnitude a, directed toward centre of circle.



$$v = \frac{\Delta l}{\Delta t} = r\frac{\Delta\vartheta}{\Delta t}$$

$$\therefore a = \frac{\Delta v}{\Delta t} = v\frac{\Delta\vartheta}{\Delta t} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

Example

A satellite orbits at an altitude of 200 km above the surface of the earth, where the acceleration due to gravity is g=9.20 m/s^2.

If the radius of the earth is R_E=6370 km, what is the velocity and period of the satellite?