4 Summary of kinematics in 3 dimensions

Position Vector

- Locates a particle in 3-space

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}},$$

Displacement

- Change in position vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}},$$

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}.$$

Average and Instantaneous Velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

Average and Instantaneous Acceleration

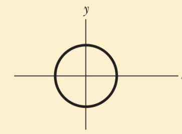
$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}.$$

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

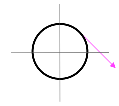
4-2 Average Velocity and Instantaneous Velocity

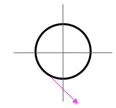
Checkpoint 1

The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{\mathrm{i}} - (2\text{ m/s})\hat{\mathrm{j}}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.



Answer: (a) Quadrant I (b) Quadrant III





Kinematics for constant acceleration in 3D

Only valid when $\vec{a}$ is a constant vector (*i.e.* in all 3 directions)!

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Remember this is really 3 equations, one for each component.

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$z - z_0 = v_{0z}t + \tfrac{1}{2}a_z t^2$$

So the equations for 1D motion apply separately to each component in 3.
Therefore motion in one direction in space is **independent** of motion in the other 2 directions!

Problem 4.15

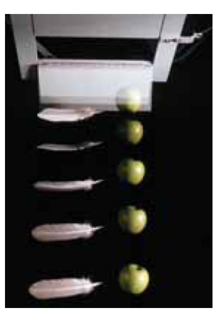
A particle leaves the origin with an initial velocity and acceleration

$\vec{v} = (3.00\ \hat{i})$ m/s, and $\vec{a} = (-1.00\hat{i} - 0.500\hat{j})$ m/s^2.

When it reaches its maximum x-coordinate, what is its
(a) velocity?
(b) position?

Free-Fall Acceleration

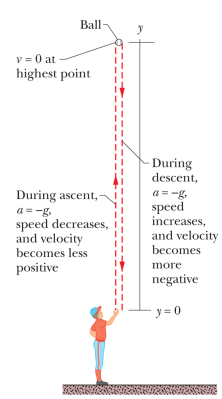
- Objects close to the earth's surface fall towards the surface of the earth.
- Rate of free-fall acceleration (no forces other than gravity) is constant, independent of the object's mass.
- Acceleration is toward the centre of the earth (locally vertical).
- If we take "up" as the positive y-direction, replace "a" by "-g", where $g \approx 9.8$ m/s^2.
- g varies slightly from equator to poles, and with height.



In vacuum, a feather and an apple will fall at the same rate.

Example

Positive y



$$v = v_0 - gt$$

$$y = v_0 t - \tfrac{1}{2}gt^2$$

$$y_0 = 0$$

Example (continued)

(a) What is the time to reach the maximum height?

$$v = v_0 - gt = 0, \quad \therefore t = t_1 \equiv \frac{v_0}{g}$$

(b) What is the maximum height y_{max}?

$$y_{max} = v_0 t_1 - \tfrac{1}{2} g t_1^2 = \frac{v_0^2}{2g}$$

(c) What is the time to return to initial height (*i.e.* $y = 0$)?

$$y = v_0 t - \tfrac{1}{2} g t^2 = t(v_0 - \tfrac{1}{2} g t) = 0, \quad \therefore t = 0 \text{ or } t = t_2 \equiv \frac{2v_0}{g} = 2t_1$$

(d) What is the velocity on return to initial height?

$$v = v_0 - g t_2 = -v_0$$

Example

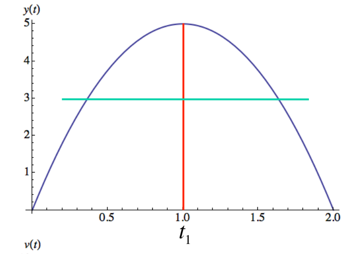
$v_0 = 9.8$ m/s

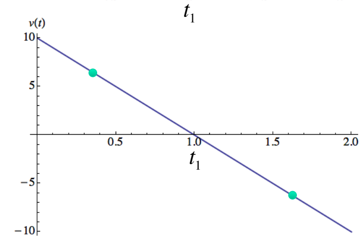
$\therefore t_1 = 1$ s and $y_{max} = 4.9$ m

$y(t)$ is symmetric about $t = t_1$

$v(t)$ is anti-symmetric about $t = t_1$

Therefore at a given height, $v(t)$ has same magnitude but opposite sign for $t > t_1$.





Example

Ball A is dropped from rest at a height h. At the same time, ball B is thrown vertically upward toward ball A at velocity v_0.

At what time do the balls meet?

$$y_A = h - \tfrac{1}{2} g t^2, \quad y_B = v_0 t - \tfrac{1}{2} g t^2$$

Solve

$$\begin{aligned} y_A &= y_B \\ \therefore h - \tfrac{1}{2} g t^2 &= v_0 t - \tfrac{1}{2} g t^2 \\ \therefore h &= v_0 t \end{aligned}$$

Therefore $t = \dfrac{h}{v_0}$ independent of g!

Both balls fall (accelerate downward) at the same rate.