Example

A plane flies East (E) for 300 km in 45 min, then South (S) for 600 km in 90 min.

Find: (a) $\Delta\vec{r}$

(b) $\vec{v}$

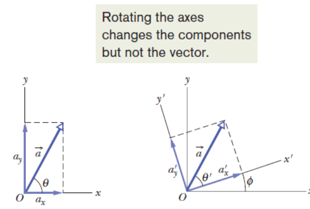
(c) average speed

Vectors and the Laws of Physics

Freedom of choosing a coordinate system

Relations among vectors do not depend on the origin or the orientation of the axes.

Relations in physics are also independent of the choice of the coordinate system.



$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x'^2 + a_y'^2}$$
$$\vartheta = \vartheta' + \phi$$

$$\vec{a} = \underbrace{(a\cos\vartheta)}_{a_x}\hat{i} + \underbrace{(a\sin\vartheta)}_{a_y}\hat{j}$$
$$= \underbrace{(a\cos\vartheta')}_{a_x'}\hat{i}' + \underbrace{(a\sin\vartheta')}_{a_y'}\hat{j}'$$

Multiplying vectors (dot product)

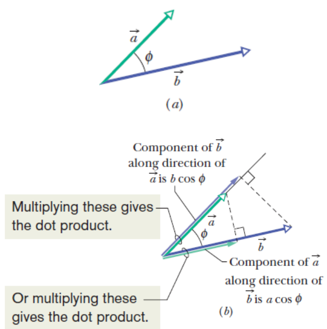
The dot product between two vectors is written as:

$\vec{a}\cdot\vec{b}$

It is defined as:

$\vec{a}\cdot\vec{b} = ab\cos\phi$

Here, a and b are the magnitudes of the vectors, and ϕ is the angle between them. The dot product is a scalar quantity.



Dot product by components

The dot product between two vectors in terms of components is:

$$\vec{a}\cdot\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$
$$= a_xb_x + a_yb_y + a_zb_z$$

We see that the magnitude of a vector can be written in terms of a dot product:

$$a^2 = |\vec{a}|^2 = \vec{a}\cdot\vec{a} = a_x^2 + a_y^2 + a_z^2$$

Problem 3.41

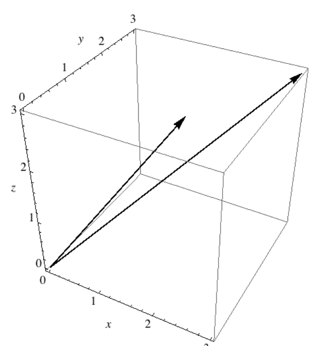
What is the angle between the vectors

$\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$

and

$\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$

Ans: 0.38 rad = 22°



Chapter 4

Motion in two and three dimensions

4-1 Position and Displacement

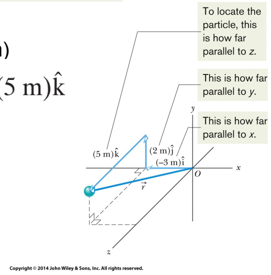
- A **position vector** locates a particle in space
  - Extends from a reference point (origin) to the particle

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}},$$

Example

- Position vector (-3m, 2m, 5m)

$$\vec{r} = (-3\ \mathrm{m})\hat{\mathrm{i}} + (2\ \mathrm{m})\hat{\mathrm{j}} + (5\ \mathrm{m})\hat{\mathrm{k}}$$



4-1 Position and Displacement

- Change in position vector is a **displacement**

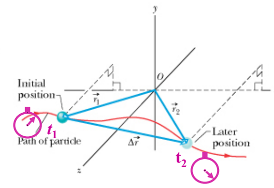
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

- We can rewrite this as:

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}},$$

- Or express it in terms of changes in each coordinate:

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}.$$



4-2 Average Velocity and Instantaneous Velocity

- **Average velocity** is
  - A displacement divided by its time interval

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

- We can write this in component form:

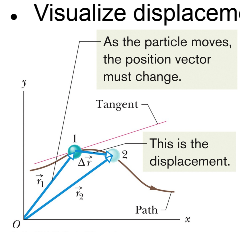
$$\vec{v}_{\mathrm{avg}} = \frac{\Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\hat{\mathrm{k}}.$$

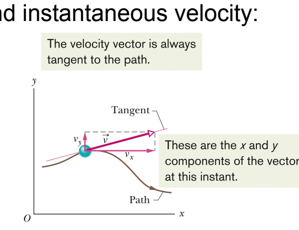
4-2 Average Velocity and Instantaneous Velocity

- **Instantaneous velocity** is
  - The velocity of a particle at a single point in time
  - The limit of avg. velocity as the time interval shrinks to 0

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

- Visualize displacement and instantaneous velocity:





4-2 Average Velocity and Instantaneous Velocity

> The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

- In unit-vector form, we write:

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

- Which can also be written:

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}.$$

- Note: a velocity vector does *not* extend from one point to another, only shows direction and magnitude