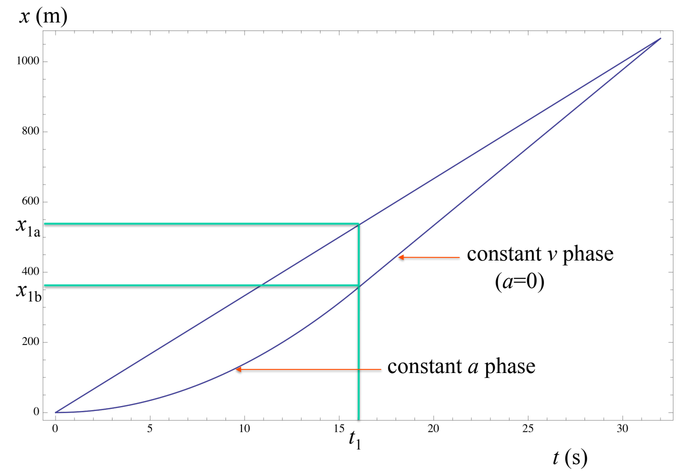
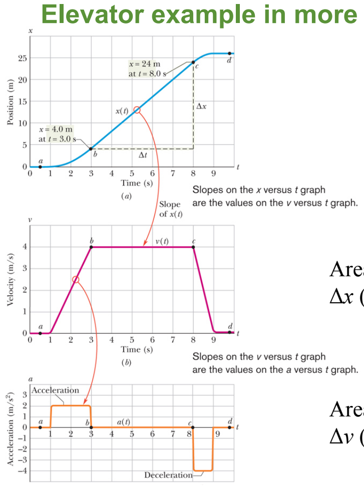
Example

Scenario 1: A car is speeding at 120 km/hr. It passes a police cruiser parked by the side of the road. At the moment the car passes, the cruiser accelerates at a constant rate of 10 km/hr/s. When and where does the cruiser overtake the car?

Scenario 2: As above, but the cruiser accelerates until it reaches its maximum speed of 160 km/hr, after which its speed remains constant.



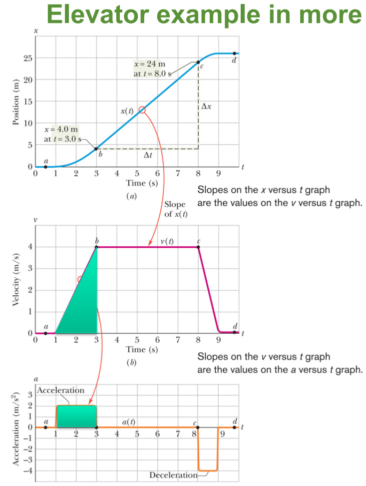
Elevator example in more detail



Area under plot of v vs. t gives Δx (change in x)

Area under plot of a vs. t gives Δv (change in v)

Elevator example in more detail



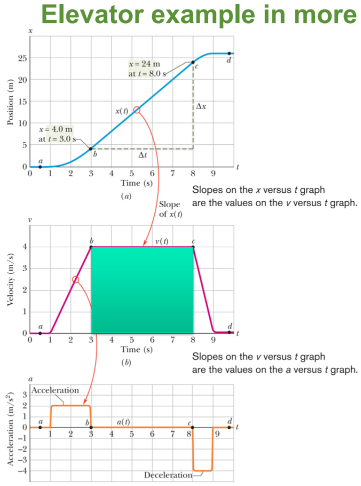
$\Delta x = \text{area} = \frac{1}{2} \times 4\,\frac{\text{m}}{\text{s}} \times 2\text{s} = 4\text{m}$

between t=1s and t=3s

$\Delta v = \text{area} = 2\,\frac{\text{m}}{\text{s}^2} \times 2\text{s} = 4\,\frac{\text{m}}{\text{s}}$

between t=1s and t=3s

Elevator example in more detail



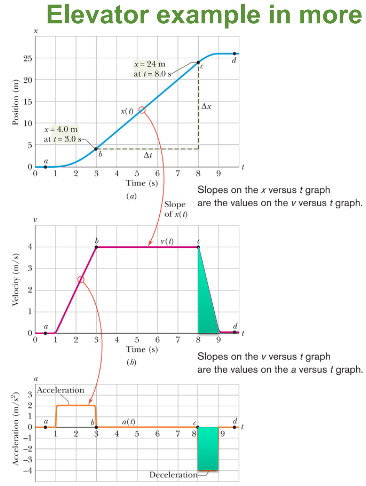
$\Delta x = \text{area} = 4\,\frac{\text{m}}{\text{s}} \times 5\text{s} = 20\text{m}$

between t=3s and t=8s

$\Delta v = \text{area} = 0$

between t=3s and t=8s

Elevator example in more detail



$\Delta x = \text{area} = \frac{1}{2} \times 4\,\frac{\text{m}}{\text{s}} \times 1\text{s} = 2\text{m}$

between t=8s and t=9s

$\Delta v = \text{area} = -4\,\frac{\text{m}}{\text{s}^2} \times 1\text{s} = -4\,\frac{\text{m}}{\text{s}}$

between t=8s and t=9s

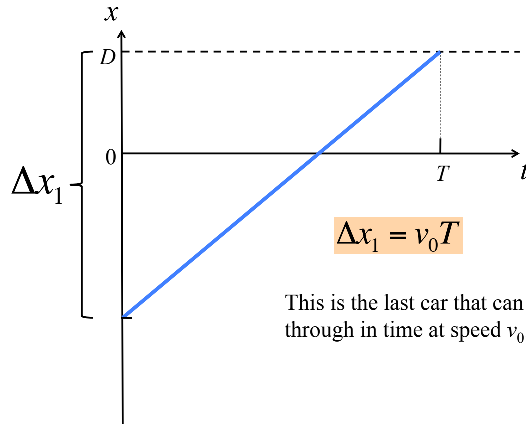
Example: The Dilemma Zone

Consider the design parameters of a traffic intersection on a road with speed limit v_0, assuming the braking acceleration is a ($a < 0$) under ideal conditions. The design parameters include:

- the time interval T of the yellow light;
- the length D of the intersection.

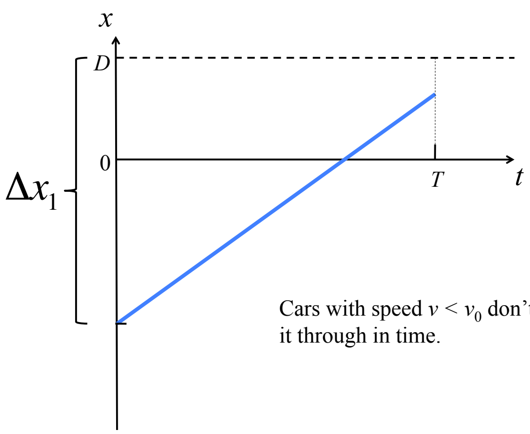
Answer the following questions:

a) What is the minimum braking distance from the intersection?
b) How should T be adjusted depending on v_0 and a?
c) What is the maximum distance beyond which a car cannot make it through the intersection before the light turns red?
d) What are the conditions for a dilemma zone, where a car can neither stop nor make it through?

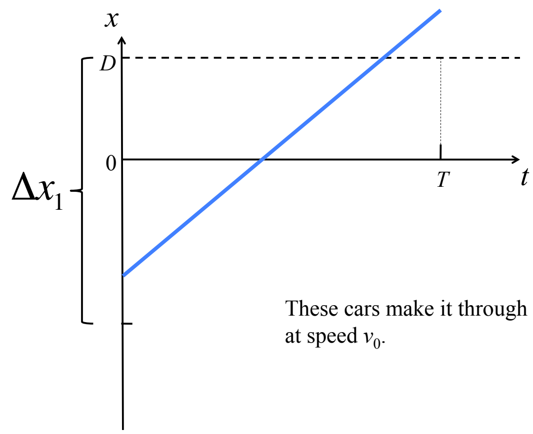


$$\Delta x_1 = v_0 T$$

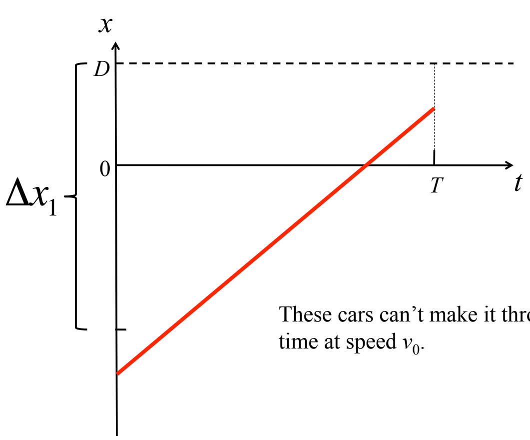
This is the last car that can make it through in time at speed v_0.



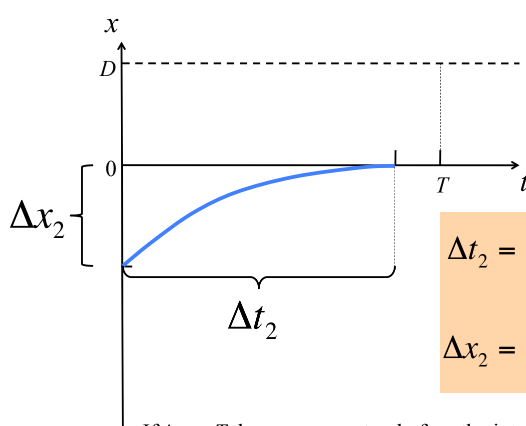
Cars with speed $v < v_0$ don't make it through in time.



These cars make it through in time at speed v_0.



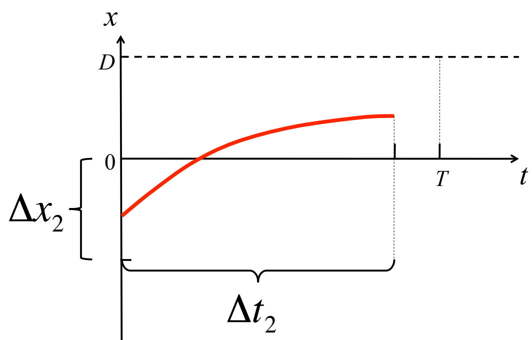
These cars can't make it through in time at speed v_0.



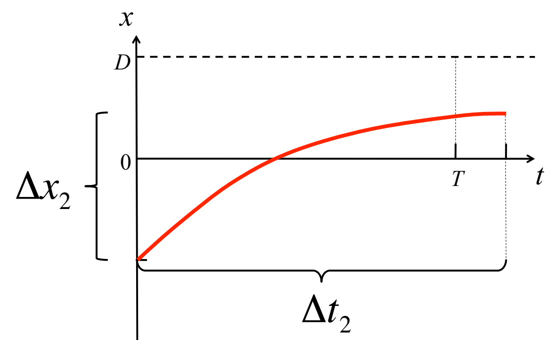
$$\Delta t_2 = \frac{v_0}{(-a)}$$

$$\Delta x_2 = \frac{v_0^2}{2(-a)}$$

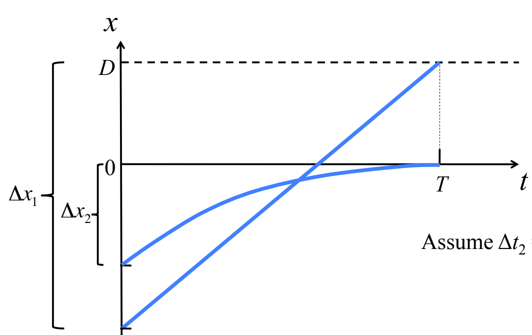
If $\Delta t_2 < T$ then cars can stop before the intersection.



Cars closer than Δx_2 can't stop in time.

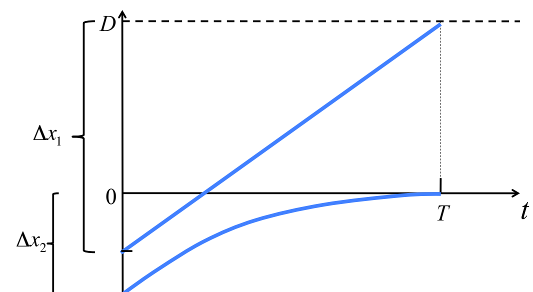


If $\Delta t_2 > T$ then cars can't stop before the intersection.

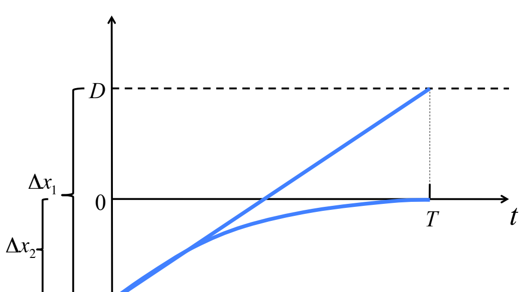


Assume $\Delta t_2 = T$.

If $(\Delta x_1 - D) > \Delta x_2$ then there is an overlapping zone where drivers have a choice to brake or go through.



If $(\Delta x_1 - D) < \Delta x_2$ then there is a dilemma zone where drivers can neither brake nor make it through in time.



An interesting case is if $(\Delta x_1 - D) = \Delta x_2$ (no overlap). Then

$$\Delta x_1 = v_0 T = \frac{v_0^2}{(-a)} = 2\Delta x_2$$
$$\therefore \Delta x_2 = D$$

i.e. the minimum stopping distance equals the length of the intersection!

Assumptions:

$$a = -20 \text{ km/h/s} = -5.6 \text{ m/s}^2$$
$$T = \Delta t_2 = \frac{v_0}{(-a)}$$
$$\Delta x_2 = \frac{v_0^2}{2(-a)}; \quad \Delta x_1 = v_0 T = \frac{v_0^2}{(-a)} = 2\Delta x_2; \quad \therefore D = \Delta x_2$$

v_0 (km/h)	v_0 (m/s)	Δt_2 (s)	Δx_2 (m)	Δx_1 (m)
40	11.1	2	11.1	22.2
60	16.7	3	25	50
80	22.2	4	44.4	88.9
100	27.8	5	69.4	138.9

The dilemma zone is more likely to occur at low speeds (small v_0), or under slippery driving conditions (large a).