Example

Suppose $x(t) = 3t^2$ (x in m, t in s)

Find $v(t)$. Answer: $v(t) = 6t$

General case:

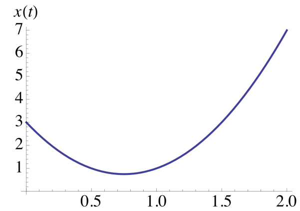
If $x(t) = A\,t^n$ (A,n constant)

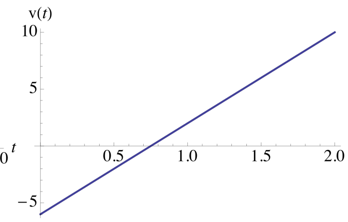
then $v(t) = \dfrac{dx(t)}{dt} = A\,nt^{n-1}$

Example

Suppose $x(t) = 4t^2 - 6t + 3$ (x in m, t in s)

Find $v(t)$. Answer: $v(t) = 8t - 6$





Minimum of $x(t)$ when $8t - 6 = 0$, so $t = 0.75$ s.

Average and instantaneous accelerations

Average acceleration is the change of velocity Δv over the change of time Δt:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

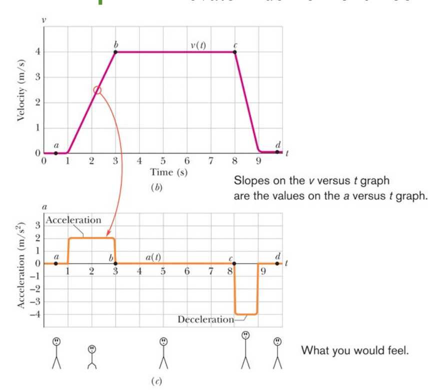
Here the velocity is $v_1 = v(t_1)$, and $v_2 = v(t_2)$.

The **instantaneous acceleration** is defined as:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

Example

Elevator ride from one floor up to another floor.



2.6.5. At one particular moment, a subway train is moving with a positive velocity and negative acceleration. Which of the following phrases best describes the motion of this train? Assume the front of the train is pointing in the positive x direction.

a) The train is moving forward as it slows down.

b) The train is moving in reverse as it slows down.

c) The train is moving faster as it moves forward.

d) The train is moving faster as it moves in reverse.

e) There is no way to determine whether the train is moving forward or in reverse.

Constant acceleration

When the acceleration is constant, its average and instantaneous values are the same.

$a = a_{\text{avg}} = \dfrac{v - v_0}{t - 0}$ means that $v = v_0 + at$ … (1)

Here, velocity at t=0 is v_o, *i.e.* $v_0 = v(0)$.

Similarly, $v_{\text{avg}} = \dfrac{x - x_0}{t - 0}$, which means that $x = x_0 + v_{\text{avg}}\,t$.

But $v_{\text{avg}} = \frac{1}{2}(v + v_0)$, so $x - x_0 = v_0 t + \frac{1}{2}at^2$ … (2)

Eliminating t from the Equations (1) and (2):

$$v^2 = v_0^2 + 2a(x - x_0) \quad \text{… (3)}$$

Kinematics for constant acceleration

$$v = v_0 + at \quad \ldots (1)$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \ldots (2)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \ldots (3)$$

Only valid when a is a constant!

Example

Scenario 1: A car is speeding at 120 km/hr. It passes a police cruiser parked by the side of the road. At the moment the car passes, the cruiser accelerates at a constant rate of 10 km/hr/s. When and where does the cruiser overtake the car?

Scenario 2: As above, but the cruiser accelerates until it reaches its maximum speed of 160 km/hr, after which its speed remains constant.