physics & astronomy **colloquia**

university of manitoba

Observation of gravitational waves from a binary black hole merger by LIGO

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FRIDAY

April 8, 2016

3:30 pm, 201 Armes Lecture Building

There will be coffee served at 3:00 pm in 316 Allen Building (Coffee Room) prior to the talk.

The twin paradox

The Set-up

Sally leaves for Vega 26 ly (light years) away at great speed, while her twin Sam decides to stay on earth.

The Problem

Upon Sally's return, Sam reasons that her clock measuring her age must run slow. Therefore she will be younger than him. However, Sally claims that the laws of physics are symmetric and that it is Sam who is moving away from her. Therefore his clock should run slow, and he should be the younger one.

The Paradox

Who is younger upon Sally's return? (They can't both be right.)

The resolution

1) Sam's clock is in an **inertial system** during the entire trip; however, Sally's clock is not. As long as Sally is travelling at constant speed away from Sam, both of them can argue that the other twin is aging less rapidly.

2) When Sally slows down to turn around, she leaves her original inertial system and eventually returns in a completely different inertial system.

3) Sally's claim is no longer valid, because she does not remain in the same inertial system. There is also no doubt as to who is in the inertial system. Sam feels no acceleration during Sally's entire trip, but Sally does. Therefore the situation is **not** symmetric.

37.4: The relativity of velocities (velocity addition in 1D)

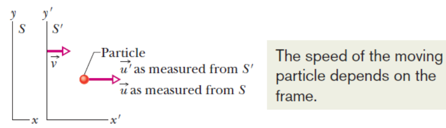


Fig. 37-11 Reference frame S' moves with velocity $\vec{v}$ relative to frame S. A particle has velocity $\vec{u}'$ relative to reference frame S' and velocity $\vec{u}$ relative to reference frame S.

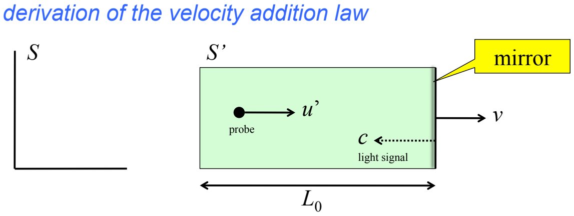
How to relate u' (measured by S') to u (measured by S)?

Galilean relativity: $u = u' + v$ (old notation $v_{PS} = v_{PS'} + v_{S'S}$)

Special relativity: $u = \dfrac{u' + v}{1 + u'v/c^2}$

v: velocity of frame S' wrt S
u': velocity of P wrt S'
u: velocity of P wrt S

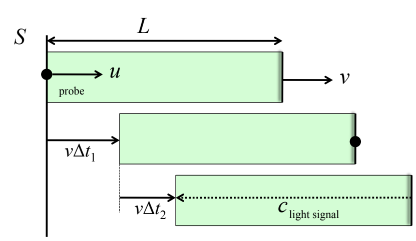
A derivation of the velocity addition law



- Frame S' is moving at speed v wrt frame S
- Observer at origin in S' sends out a probe at speed u' (wrt S')
  - probe reaches mirror in time $\Delta t_1' = L_0/u'$
  - light signal sent back to origin in time $\Delta t_2' = L_0/c$
  - time to receive light signal is

$$\Delta t_0 = \Delta t_1' + \Delta t_2' = L_0\left(\frac{1}{u'} + \frac{1}{c}\right) = L_0\left(\frac{u' + c}{u'c}\right)$$

- Δt_0 is the proper time



$u\Delta t_1 = v\Delta t_1 + L$
$\therefore \Delta t_1 = L/(u - v)$

$c\Delta t_2 = L - v\Delta t_2$
$\therefore \Delta t_2 = L/(c + v)$

- Frame S sees probe moving at speed $\underline{u}$
  - length of boxcar in frame S is L (not L_0)
  - probe reaches mirror in time Δt_1
  - light signal sent back to origin of S' in time Δt_2
  - time to receive light signal is

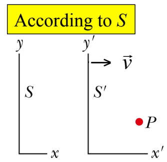
$$\Delta t = \Delta t_1 + \Delta t_2 = L\left(\frac{1}{u - v} + \frac{1}{c + v}\right) = L\left(\frac{c + u}{(u - v)(c + v)}\right)$$

But $\Delta t = \gamma \Delta t_0$, $L = L_0/\gamma$

$$\therefore \frac{L_0}{\gamma}\left(\frac{c+u}{(u-v)(c+v)}\right) = \gamma L_0\left(\frac{c+u'}{u'c}\right) \qquad \text{with} \quad \frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2}$$

Solve for u in terms of u' and v:

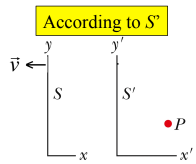
$$u = \frac{u'+v}{1+u'v/c^2}$$



OR solve for u' in terms of u and v:

$$u' = \frac{u-v}{1-uv/c^2} = \frac{u+(-v)}{1+u(-v)/c^2}$$

Note the complete symmetry here, changing v to $-v$.



Example

Car 1, moving at 0.90c wrt ground, observes car 2 passing it at a speed of 0.80c (in the same direction).

What is the velocity of car 2 wrt ground? 0.988c

$$v = 0.90c$$
$$u' = 0.80c$$
$$\therefore u = \frac{0.80c + 0.90c}{1+(0.8)(0.9)} = \frac{1.70c}{1.72} = 0.988c$$

Example

A particle at rest decays into two other particles, A and B. Particle A is moving at speed 0.6c in the positive x direction, and particle B at a speed of 0.4c in the opposite direction.

What is the velocity of particle B wrt particle A? -0.81c

$$v = -0.60c$$
$$u' = -0.40c$$
$$\therefore u = \frac{-0.40c - 0.60c}{1+(-0.4)(-0.6)} = \frac{-1.00c}{1.24} = -0.81c$$

Example

A space ship moving away from earth at a speed 0.4c reports back to earth that they observe a comet coming toward the ship at a speed of 0.8c.

What is the velocity of the comet as measured by an observer on earth? -0.59c

$$v = 0.40c$$
$$u' = -0.80c$$
$$\therefore u = \frac{0.40c - 0.80c}{1+(0.4)(-0.8)} = \frac{-0.40c}{0.68} = -0.59c$$