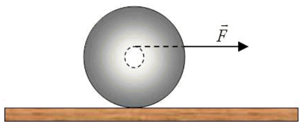
Example 2

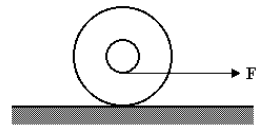
The drawing shows a yo-yo in contact with a tabletop. A string is wrapped around the central axle. How will the yo-yo behave if you pull on the string with the force shown?



a) The yo-yo will roll to the left.
b) The yo-yo will roll to the right.
c) The yo-yo will spin in place, but not roll.
d) The yo-yo will not roll, but it will move to the left.
e) The yo-yo will not roll, but it will move to the right.

Example 3

What happens if we apply the torque from below the centre?



a) The yo-yo will roll to the left.
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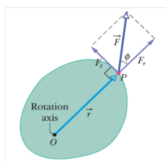
Angular Momentum of a Single Particle



- The torque for a single particle a distance r from the axis of rotation is $\tau = rF_\perp$
- Define the *angular momentum* as $\ell = rp_\perp$
- Then clearly $\dfrac{d\ell}{dt} = r\dfrac{dp_\perp}{dt} = rF_\perp = \tau$ (remember that r is constant)
- For a particle with angular velocity ω, we can also write

$$\ell = r(mv_\perp) = (mr^2)\omega$$

- Recall that mr^2 is the *moment of inertia* of the particle about the axis of rotation

Angular Momentum of a Rigid Body

- For a rigid body, we add the *angular momentum* of each particle to get the *total angular momentum*

$$L = \ell_1 + \ell_2 + \ldots + \ell_n = \sum_i \ell_i$$
$$= \sum_i (m_i r_i^2)\omega = I\omega$$

- Then clearly $\dfrac{dL}{dt} = I\dfrac{d\omega}{dt} = I\alpha = \tau_{\text{net}}$
- This is the angular counterpart to Newton's Law $\dfrac{dP}{dt} = F_{\text{net}}$

  where P is the total momentum of the system, $P = \sum_i p_i$
- Like torque, angular momentum is positive in the *ccw* direction.
- Note that the torque and angular momentum must be measured relative to the same origin.
- Note that *internal* torques do not contribute to the *net* torque.

Conservation of Angular Momentum

- For an isolated system, or a system with no net external torque acting on it, we have a new conservation law:

  L = a constant (isolated system)

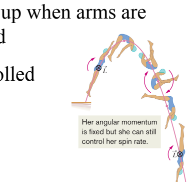
- The **law of conservation of angular momentum** states that

  (*net initial angular momentum*) = (*net final angular momentum*)

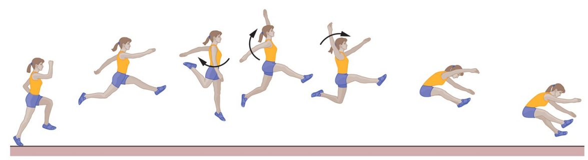
$$L_i = L_f$$
$$\therefore I_i\omega_i = I_f\omega_f$$

Examples of angular momentum conservation

- **A student spinning on a stool**: rotation speeds up when arms are brought in, slows down when arms are extended
- **A springboard diver**: rotational speed is controlled by tucking her arms and legs in, which reduces rotational inertia and increases rotational speed



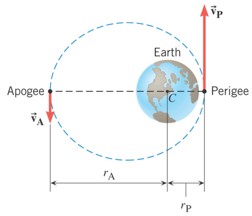
- **A long jumper**: the angular momentum caused by the torque during the initial jump can be transferred to the rotation of the arms, by windmilling them, keeping the jumper upright



Example 4 A satellite in an elliptical orbit

An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37×10^6 m from the center of the earth, and its point of greatest distance is 25.1×10^6 m from the center of the earth.

The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.



Example 5 Spinning

- A student sits on a stool that can rotate without friction. In position (a) his arms are carrying weights and are extended. He then pulls the weights in to position (b).
- There are no external torques on the system, so L is conserved.

$$\therefore L = I_i\omega_i = I_f\omega_f$$

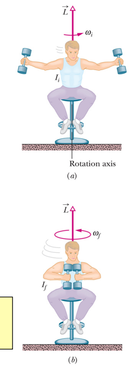
Since $I_f < I_i$, we must have $\omega_f = \dfrac{I_i}{I_f}\omega_i > \omega_i$

- Is kinetic energy conserved? No!

$$K_i = \tfrac{1}{2}I_i\omega_i^2 = \frac{L^2}{2I_i}, \text{ but } K_f = \tfrac{1}{2}I_f\omega_f^2 = \frac{L^2}{2I_f} \quad \therefore K_f > K_i$$

- Where does the extra kinetic energy come from?

Answer: From the work done by centripetal force supplied by the person in pulling the weights inward.



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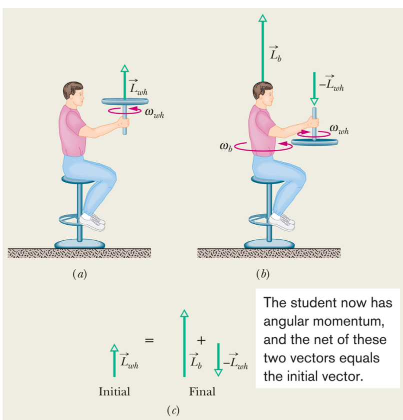
Example 6 Flipping over a spinning wheel

$$L_i = L_{\text{wheel}}$$
$$L_f = L_{\text{boy}} - L_{\text{wheel}}$$

The torque applied to flip the wheel over is *internal*, and therefore the total angular momentum is conserved.

$$L_i = L_f$$
$$\therefore L_{\text{boy}} = 2L_{\text{wheel}}$$



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