Chapter 11

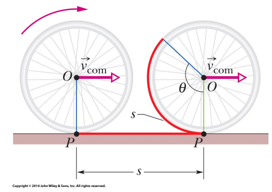
Rolling and Angular Momentum

Rolling as translation plus rotation

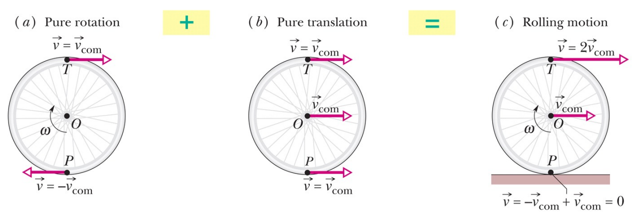
- We consider only objects that roll smoothly (no slipping)
- The centre-of-mass (*com*) of the object moves in a straight line parallel to the surface with speed v_{com}
- The object rotates around the *com* with angular speed ω
- Ignoring signs for now, the rotational motion is defined by:

$$s = \theta R$$

$$v_{\text{com}} = \frac{ds}{dt} = \frac{d\theta}{dt} R = \omega R$$

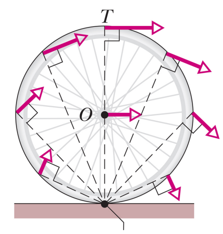


A different perspective: rotation + translation = rolling



- The figure shows how the velocities of translation and rotation combine at different points on the wheel.
- From figure (a), it is clear that $v_{\text{com}} = \omega R$ (in magnitude).

Another perspective: pure rotation about point of contact P



Rotation axis at P

- The wheel rotates instantaneously about point P with angular velocity ω, while point P moves to the right at speed v_{com}.
- For example, at the top, $v_T = \omega(2R) = 2v_{\text{com}}$ to the right, as we found in the previous slide.

Kinetic Energy of Rolling

- Combine translational and rotational kinetic energy:

$$K = \tfrac{1}{2} I_{\text{com}} \omega^2 + \tfrac{1}{2} m v_{\text{com}}^2$$

- If $I_{\text{com}} = \beta m R^2$, and $\omega = \dfrac{v_{\text{com}}}{R}$, then

$$K = \tfrac{1}{2} m v_{\text{com}}^2 (1 + \beta)$$

- This shows how the total kinetic energy is split between translational and rotational.
- For a few simple circular shapes:

hollow cylinder (hoop):	$\beta = 1$
solid cylinder (disc):	$\beta = 1/2$
hollow sphere:	$\beta = 2/3$
solid sphere:	$\beta = 2/5$

- We can also regard this as pure rotation about point P:

$$K = \tfrac{1}{2} I_P \omega^2$$

From the parallel-axis theorem, $I_P = I_{\text{com}} + mR^2$, so that

$$\begin{aligned} K &= \tfrac{1}{2}(I_{\text{com}} + mR^2)\omega^2 \\ &= \tfrac{1}{2} I_{\text{com}} \omega^2 + \tfrac{1}{2} m v_{\text{com}}^2 \\ &= \tfrac{1}{2} m v_{\text{com}}^2 (1 + \beta) \end{aligned}$$

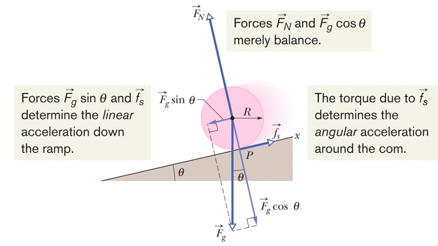
as we found in the previous slide, with $I_{\text{com}} = \beta\, mR^2$ and $v_{\text{com}} = \omega R$.

- If a wheel that rolls without slipping is accelerating at the rate a_{com}, then it must also have an angular acceleration α about point P, with

$$a_{com} = \alpha R$$

- Note that this relation is for the *magnitudes only*, with the proper sign determined by the actual setup.
- A force must act to prevent slipping. This is the static frictional force f_s of the surface acting **on** the wheel.
- The direction of f_s is to oppose the direction in which the wheel would slip.
- If the wheel rolls at a constant speed then f_s is 0.

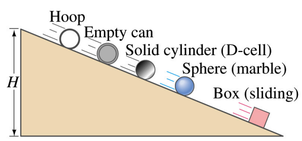
- If slipping occurs, then the motion is *not* smooth rolling!
- For smooth rolling down a ramp:
  1. The gravitational force is vertically down, and acts through the *com* at point O.
  2. The normal force is perpendicular to the ramp, and acts through point P. Its *line of action* therefore goes through both O and P.
  3. The force of friction points up the ramp, and acts through point P.



Example 1

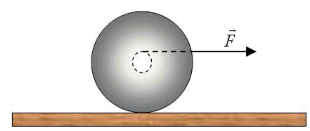
Consider various round objects with moment of inertia $I = \beta m R^2$ rolling down a ramp of length L (height $H = L\sin\theta$). Assuming the object starts from rest, find the velocity at the bottom of the ramp by

(a) using torque and Newton's Laws;
(b) using conservation of energy.
(c) Why is energy conserved if there is friction in the problem?
(d) What is the time to reach the bottom?



Example 2

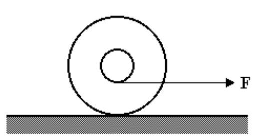
The drawing shows a yo-yo in contact with a tabletop. A string is wrapped around the central axle. How will the yo-yo behave if you pull on the string with the force shown?



a) The yo-yo will roll to the left.
b) The yo-yo will roll to the right.
c) The yo-yo will spin in place, but not roll.
d) The yo-yo will not roll, but it will move to the left.
e) The yo-yo will not roll, but it will move to the right.

Example 3

What happens if we apply the torque from below the centre?



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