10.8: Work and Rotational Kinetic Energy

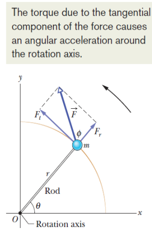
$$\Delta K = K_f - K_i = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2 = W$$

$$W = \int \vec{F} \cdot d\vec{s}$$

$ds = r\,d\theta$ in tangential direction

$$\therefore W = \int F_t (r\,d\theta) = \int (F_t r)\,d\theta$$

$$\therefore W = \int_{\theta_i}^{\theta_f} \tau\,d\theta$$



where τ is the torque doing the work W, and θ_i and θ_f are the body's angular positions before and after the work is done, respectively.

When τ is constant, $W = \tau(\theta_f - \theta_i) = \tau\Delta\theta$

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TABLE 10-3

Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

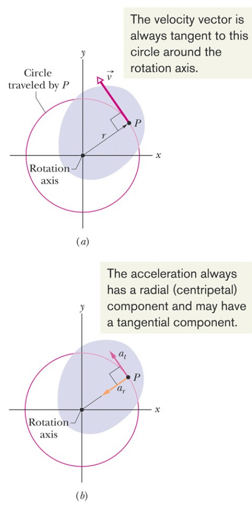
Recall the relationship between translational and rotational variables for a point P on the rim of a circular path of radius r around the axis of rotation:

arclength: $s = \theta r$

velocity: $v = \dfrac{ds}{dt} = \dfrac{d\theta}{dt} r = \omega r$

tangential acceleration:

$$a_t = \frac{dv}{dt} = \frac{d\omega}{dt} r = \alpha r$$



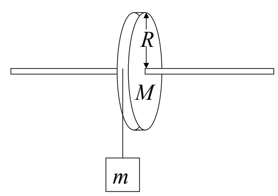
Example 7

A solid disc with radius $R = 0.20$ m, and mass $M = 2.5$ kg, is free to rotate about its axis. A string is wrapped around its circumference with a block of mass $m = 1.2$ kg attached. Static friction prevents the string from slipping on the disc. The block is released from rest. Using Newton's Law:

(a) Find the tension in the string.

(b) Find the acceleration of the block.

(c) Find the velocity after the block has fallen a distance of 0.5 m.

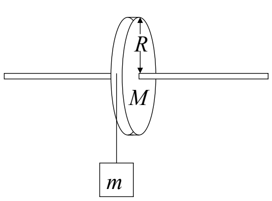


Example 8

We will repeat example 7, but this time using conservation of energy:

(a) Find the velocity after the block has fallen a distance h.

(b) Deduce the acceleration from the result in part (a).



Example 9

Two blocks of mass m_1 and m_2 are connected by a string passing over a pulley in the form of a solid disc of mass m_3. Static friction prevents the string from slipping.

Find the acceleration of the system. (This can be done either by using Newton's Law or by using conservation of energy.)