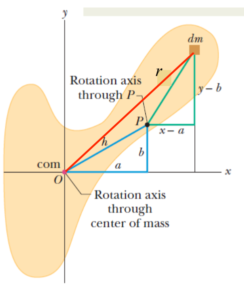
10.5: Calculating the Rotational Inertia

Parallel Axis Theorem:

If h is a **perpendicular distance** between a given axis and the axis through the center of mass (these two axes being **parallel**), then the rotational inertia I about the given axis is:

$$I_P = I_{\text{com}} + Mh^2 \quad \text{(parallel-axis theorem)}$$



Proof:

$$I_P = \sum_i m_i\left[(x_i - a)^2 + (y_i - b)^2\right]$$

$$= \sum_i m_i\left[\underbrace{x_i^2 + y_i^2}_{r_i^2} + \underbrace{a^2 + b^2}_{h^2} - 2ax_i - 2by_i\right]$$

$$= \underbrace{\sum_i m_i r_i^2}_{I_{\text{com}}} + h^2\underbrace{\sum_i m_i}_{M} - 2a\underbrace{\sum_i m_i x_i}_{0} - 2b\underbrace{\sum_i m_i y_i}_{0}$$

$$= I_{\text{com}} + Mh^2$$

Example 4

Calculate the moment of inertia for the figure below.

- Summing by particle:

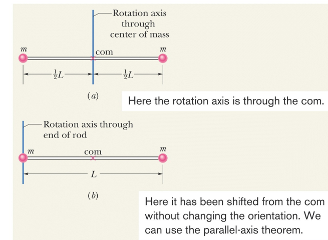
$$I = m(0)^2 + mL^2 = mL^2$$

- Using the parallel-axis theorem:

$$I = I_{\text{com}} + Mh^2$$

$$I_{\text{com}} = m(\tfrac{1}{2}L)^2 + m(\tfrac{1}{2}L)^2 = \tfrac{1}{2}mL^2$$

$$\therefore I = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2 = mL^2$$



Example 5

The moment of inertia about the centre of a rod of mass m, length L, is

$$I = \tfrac{1}{12}mL^2$$

(a) Use the parallel axis theorem to show that the moment of inertia of this rod about its end is

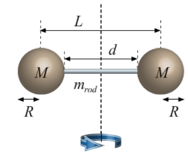
$$I = \tfrac{1}{3}mL^2$$

(b) If we put 2 such rods together back-to-back, show that the moment of inertia about the *com* is consistent with part (a).

Parallel Axis Theorem for composite systems

$$I_{\text{rod}} = \tfrac{1}{12}md^2$$

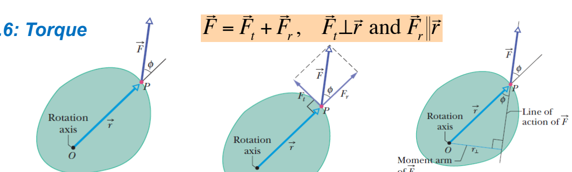
$$I_{\text{sphere}} = \tfrac{2}{5}MR^2$$



$$I_{Dumbbell} = \frac{1}{12}m_{rod}d^2 + 2\left[\frac{2}{5}MR^2 + M\left(\frac{L}{2}\right)^2\right]$$

10.6: Torque

$$\vec{F} = \vec{F}_t + \vec{F}_r, \quad \vec{F}_t \perp \vec{r} \text{ and } \vec{F}_r \parallel \vec{r}$$



The torque due to this force causes rotation around this axis (which extends out toward you).
(*a*)

But actually only the *tangential* component of the force causes the rotation.
(*b*)

You calculate the same torque by using this moment arm distance and the full force magnitude.
(*c*)

- Ability to cause rotation depends only on F_t and distance from the pivot point O to point of application.
- Radial component F_r cannot cause rotation.
- Define **torque**:

$$\tau = rF_t = (r)(F\sin\phi)$$

- This is equivalent to the full force F applied along a *line of action* a distance $r_\perp$ from the pivot point O.

$$\tau = (r\sin\phi)(F) = r_\perp F$$

- $r_\perp$ is sometimes called the *moment arm*

10.7: Newton's Law for Rotation

$$F_t = ma_t$$

$$\Downarrow$$

$$\tau = F_t r = ma_t r$$

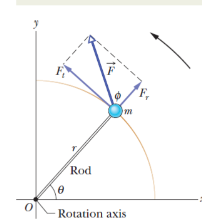
$$\Downarrow$$

$$\tau = m(\alpha r)r = (mr^2)\alpha$$

$$\Downarrow$$

$$\tau = I\alpha$$

The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.



For more than one force, we can generalize:

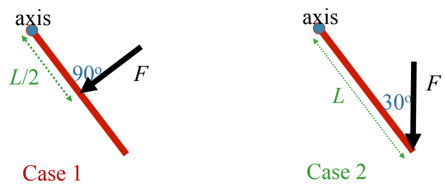
$$\tau_{\text{net}} = I\alpha$$

CheckPoint

In Case 1, a force F is pushing perpendicular on an object a distance $L/2$ from the rotation axis. In Case 2 the same force is pushing at an angle of 30 degrees a distance L from the axis.

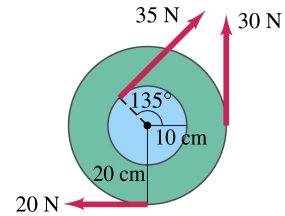
In which case is the torque due to the force about the rotation axis biggest?

A) Case 1 B) Case 2 C) Same



Example 6

Find the net torque on the wheel about the centre axle.



CheckPoint

Two hoops can rotate freely about fixed axles through their centers. The hoops have the same mass, but one has twice the radius of the other. Forces F_1 and F_2 are applied as shown.

How are the magnitudes of the two forces related if the angular acceleration of the two hoops is the same?

A) $F_2 = F_1$

B) $F_2 = 2F_1$

C) $F_2 = 4F_1$