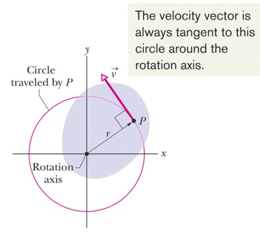
10.3: Relating Linear and Angular Variables



- Linear and angular variables are related by r, perpendicular distance from the rotational axis
- Position (θ *must* be in radians):

$$s = \theta r$$

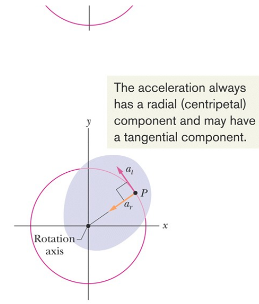
- Speed (ω *must* be in radian measure):

$$v = \frac{ds}{dt} = \frac{d\theta}{dt} r = \omega r \quad \text{or} \quad v = \frac{2\pi r}{T}$$

- We can express the period in radian measure:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{1}{f}$$

Acceleration of point P



- Tangential acceleration:

$$a_t = \alpha r$$

(positive in counter-clockwise direction)

- We can write the radial acceleration in terms of angular velocity:

$$a_r = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

(directed towards centre)

- Total acceleration is:

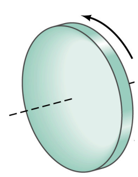
$$\vec{a} = \vec{a}_t + \vec{a}_r$$

Example 2

A cylinder of radius 12 cm starts from rest and rotates about its axis with a constant angular acceleration of 5.0 rad/s^2.

At $t = 2.0$ s, what is:

(a) Its angular velocity?
(b) The linear speed of a point on the rim?
(c) The angular displacement of that point?
(d) The radial and tangential components of acceleration of that point?
(e) The total acceleration at that point?



10.4: Kinetic Energy of Rotation for a Rigid Body

For an extended rotating **rigid body**, treat the body as a collection of particles with different speeds, and add up the kinetic energies of all the particles to find the total kinetic energy of the body:

$$K = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \ldots \tfrac{1}{2} m_n v_n^2 = \sum_{i=1}^{n} \tfrac{1}{2} m_i v_i^2$$

(m_i is the mass of the i^{th} particle, and $v_i = \omega\, r_i$ is its speed).

$$\therefore K = \sum_{i=1}^{n} \tfrac{1}{2} m_i (\omega r_i)^2 = \tfrac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2$$ (ω is the **same** for all particles).

The quantity in parentheses is called the **rotational inertia** (or **moment of inertia**) I of the body with respect to the axis of rotation. It is a constant for a particular rigid body and a particular rotation axis. (That axis must always be specified.)

Therefore, $I = \sum_{i=1}^{n} m_i r_i^2 \Rightarrow K = \tfrac{1}{2} I \omega^2$

I tells us how mass is distributed about the axis of rotation.

Example 3

Three equal point masses are rotating about the origin at 2 rad/sec.

The masses are located at (4,0), (0,4), and (4,4), (in units of m), and the masses are 1 kg, 2 kg, and 4 kg, respectively.

Find the moment of inertia, and the total kinetic energy.

If a rigid body consists of a great many adjacent particles (it is continuous), we consider an integral and define the rotational inertia of the body as:

$$I = \int r^2 dm \qquad \text{(rotational inertia, continuous body)}$$

Table 10-2 Some Rotational Inertias