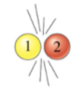
Announcements

1. Tutorial 4 next week covers chapters 8-9.5. The formula sheet is up already.
2. The module on rotational motion starts next Wednesday. We will be doing a “trial” of the **FlipIt** system. It includes Prelecture material and Problem Activities. You will be given 21 days of free access. Participation is voluntary.

Another perspective on conservation of momentum:



$\vec{f}_{12}(t)$: internal force of 2 on 1

$\vec{f}_{21}(t)$: internal force of 1 on 2

$\vec{f}_{21}(t) = -\vec{f}_{12}(t)$ at all times t

$$\Delta\vec{p}_1 = \vec{p}_{1f} - \vec{p}_{1i} = \vec{J}_1 = \int_{t_i}^{t_f} \vec{f}_{12}(t)\,dt$$

$$\Delta\vec{p}_2 = \vec{p}_{2f} - \vec{p}_{2i} = \vec{J}_2 = \int_{t_i}^{t_f} \vec{f}_{21}(t)\,dt = -\vec{J}_1$$

$\therefore \Delta\vec{p}_1 = -\Delta\vec{p}_2$

Change in momentum is equal in magnitude but opposite in direction.

$$(\vec{p}_{1f} - \vec{p}_{1i}) = -(\vec{p}_{2f} - \vec{p}_{2i})$$
$$\therefore \vec{P}_i = \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} = \vec{P}_f$$

Equivalently, total momentum is conserved.

9-6 Momentum and Kinetic Energy in Collisions

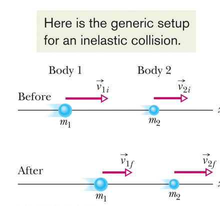
Types of collisions

- **Elastic collisions**:
  - Total kinetic energy is unchanged (conserved)
  - An idealization, since in real collisions some energy is always transferred to other forms of energy (*e.g.* thermal, sound)
- **Inelastic collisions**:
  - Total kinetic energy is lost from the system
  - Arises from non-conservative collisional forces
- **Completely inelastic collisions**:
  - The objects stick together
  - Maximum loss of kinetic energy from the system (as we will prove shortly)
  - This is an explosion in reverse!

For motion in one dimension (1D)

- General collision: momentum is always conserved

$$P_i = P_f$$
$$\therefore m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$



- we have one equation and two unknowns (v_{1f} and v_{2f})
- we cannot determine v_{1f} and v_{2f} without more information (*e.g.* loss of energy)
- total kinetic energy may or may not be conserved

There are two special cases:

a. Completely inelastic collision: $v_{1f} = v_{2f} = v_{com}$.

b. Completely elastic collision: kinetic energy is conserved.

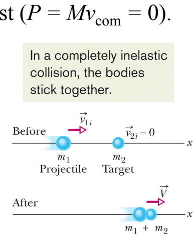
There are also two special reference frames in which we can analyze the motion:

1. The “lab” frame: the “target” is initially at rest, so $v_{2i} = 0$.
2. The “centre-of-mass” frame: the *com* is at rest ($P = Mv_{com} = 0$).

Since these are both inertial reference frames, Newton’s Laws are valid.

For simplicity, start by looking at a completely inelastic collision with target at rest ($v_{2i} = 0$). Therefore $v_{1f} = v_{2f} = v_{com}$, and

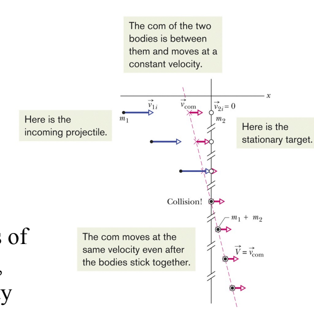
$$P = m_1 v_{1i} = (m_1 + m_2) v_{com}$$



- The centre-of-mass velocity remains unchanged throughout the collision process:

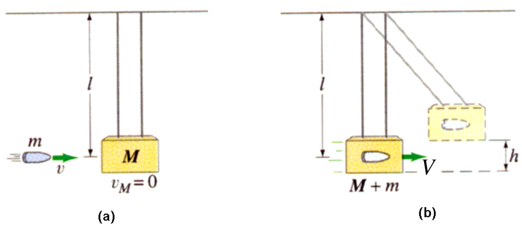
$$v_{com} = \frac{P}{M} = \frac{m_1 v_{1i}}{m_1 + m_2}$$

- The figure shows freeze frames of a completely inelastic collision, showing centre-of-mass velocity



Example – the ballistic pendulum

A 10.0 g bullet is fired horizontally into a 5.00 kg block of wood that is suspended by a pair of strings from the ceiling. The bullet comes to rest in the block. The block+bullet swing in an arc and rise a vertical distance h = 10.0 cm above the initial position. What is the velocity v of the bullet?



9-7 Elastic Collisions in One Dimension

- Total kinetic energy is conserved in elastic collisions

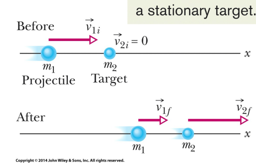
In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

- For a target at rest (lab frame), $v_{2i} = 0$, so we have

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{(linear momentum)}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \quad \text{(kinetic energy)}$$

Here is the generic setup for an elastic collision with a stationary target.



- With some algebra we can solve for v_{1f} and v_{2f}:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

- Results
  - Equal masses, $m_1 = m_2$: $v_{1f} = 0$, $v_{2f} = v_{1i}$
    First object stops, exchange of velocities (*e.g.* billiard balls)
  - Massive target, $m_2 >> m_1$: $v_{1f} = -v_{1i}$
    First object bounces backwards, but speed unchanged.
  - Massive projectile, $m_1 >> m_2$: $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$
    First object keeps going; target goes forward at twice its speed

Here is one way to do the algebra. Note that a difference of squares can be factored, so rewrite these 2 equations as:

$$m_1(v_{1i} - v_{1f}) = m_2 v_{2f} \quad (1)$$

$$\tfrac{1}{2} m_1(v_{1i}^2 - v_{1f}^2) = \tfrac{1}{2} m_2 v_{2f}^2 \quad (2)$$

$$\therefore \underbrace{m_1(v_{1i} - v_{1f})}_{m_2 v_{2f}}(v_{1i} + v_{1f}) = m_2 v_{2f}^2$$

$$\therefore v_{1i} + v_{1f} = v_{2f} \quad (3)$$

Put (3) into (1): $\quad m_1 v_{1i} - m_1 v_{1f} = m_2 v_{1i} + m_2 v_{1f}$

$$\therefore v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

Put this into (3): $\quad v_{2f} = \dfrac{2m_1}{m_1 + m_2} v_{1i}$