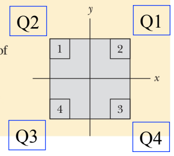
9-1 The Center of Mass: $\vec{r}_{com} = \frac{1}{V}\int \vec{r}\, dx\, dy\, dz$

- The centre of mass lies at a point of symmetry (if there is one)
- It lies on the line or plane of symmetry (if there is one)
- It need not be on the object (consider a doughnut)

Checkpoint 1

The figure shows a uniform square plate from which four identical squares at the corners will be removed. (a) Where is the center of mass of the plate originally? Where is it after the removal of (b) square 1; (c) squares 1 and 2; (d) squares 1 and 3; (e) squares 1, 2, and 3; (f) all four squares? Answer in terms of quadrants, axes, or points (without calculation, of course).



Answer: (a) at the origin (b) in Q4, along diagonal $y = -x$ (c) along the $-y$ axis

(d) at the origin (e) in Q3, along diagonal $y = x$ (f) at the origin

See the text for more examples.

9-3 Linear Momentum

- The **linear momentum** is defined as:

$$\vec{p} = m\vec{v}$$

- The momentum:
  - Points in the same direction as the velocity
  - Can only be changed by a net external force
- We can therefore write Newton's second law as:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Force is the time-rate of change of momentum.

For a system of n particles, the total momentum

$$\vec{P} = \sum_{i=1}^{n} \vec{p}_i = \sum_{i=1}^{n} m_i \vec{v}_i$$

$$= M\left(\frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i\right)$$

$$\therefore \vec{P} = M\vec{v}_{com}$$

Furthermore,

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

If $\vec{F}_{ext} = \vec{0}$, then $\frac{d\vec{P}}{dt} = \vec{0}$, so $\vec{P}$ is constant.

Corollary: If the component of $\vec{F}_{ext}$ in some direction is 0, then the component of $\vec{P}$ in that direction is constant.

9-4 Collision and Impulse

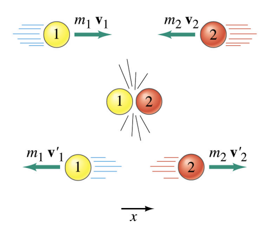
Collision: Two (or more) objects exert strong forces on each other over a finite (short) time interval $\Delta t = t_f - t_i$.

Stages of a collision

before ($t < t_i$): particles approach with initial momenta

during ($t_i < t < t_f$): particles exert equal but opposite forces on each other

after ($t > t_f$): particles leave with different momenta



The momentum of each particle has changed, but the momentum of the system has not!

- In a collision, the momentum of a particle can change:

$$\vec{F}(t) = \frac{d\vec{p}(t)}{dt} \quad \text{where } F(t) \text{ is the force on the particle.}$$

$$\therefore \int_{t_i}^{t_f} \vec{F}(t)\,dt = \int_{t_i}^{t_f} d\vec{p}(t) = \vec{p}(t_f) - \vec{p}(t_i) = \vec{p}_f - \vec{p}_i \equiv \Delta\vec{p}$$

- We define the **impulse** $\vec{J}$ acting on a particle during a collision:

$$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)\,dt$$

- This means that the applied impulse is equal to the change in momentum of the object during the collision:

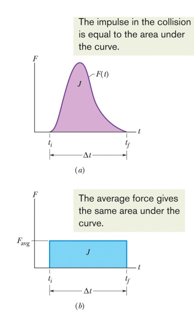
$$\therefore \vec{J} = \Delta\vec{p}$$

- This equation can be rewritten component-by-component, like other vector equations.

- Given $\vec{F}_{avg}$ and duration Δt:

$$\Delta\vec{p} = \vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)\,dt = \vec{F}_{avg}\,\Delta t$$

- We are integrating: we only need to know the area under the force curve
- $\vec{J}$ is a vector in the direction of $\Delta\vec{p}$, the same direction as the average force $\vec{F}_{avg}$



- The collision concept is most useful if Δt is short, so that the objects hardly move during the collision phase

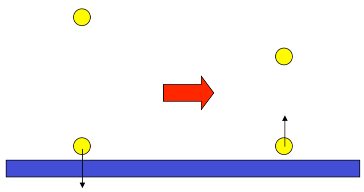
Example

A ball of mass 0.10 kg is released from 2.0 m above a floor, and rebounds to a height of 1.5 m.

(a) What is the impulse of the floor on the ball?

(b) How much mechanical energy is lost from the system due to the collision?

(c) From this deduce the speed of the ball immediately before and immediately after striking the floor.

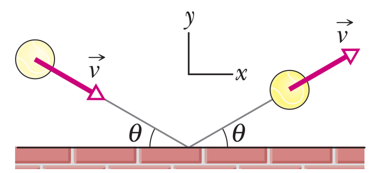


Problem 9.38

A ball of mass 300 g with a speed of $v = 6.0$ m/s strikes a wall at an angle $\theta = 30°$, and then rebounds with the same speed and angle, as shown. The ball is in contact with the wall for 10 ms.

(a) What is the impulse of the wall on the ball?

(b) What is the average force on the ball?



9-5 Conservation of Linear Momentum

- If there is no external force on the system, only the internal forces of each particle another, then

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system)}$$

which means that $\vec{P}_i = \vec{P}_f$.

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

- This is called the **law of conservation of linear momentum.**
- Check the components of the net external force to know if you can apply this.

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

- Internal forces can change momenta of parts of the system, but cannot change the linear momentum of the entire system
- Do not confuse momentum and energy

Checkpoint 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor, one of them in the positive *x* direction. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the *x* axis? (c) What is the direction of the momentum of the second piece?

Answer: (a) zero (b) no (c) the negative *x* direction