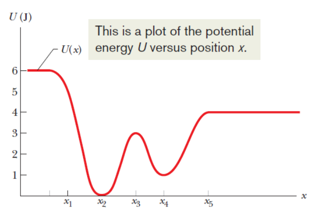
Reading a Potential Energy Curve (one dimensional motion)

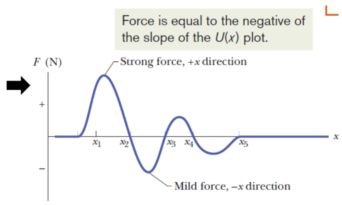
$$\Delta U(x) = -W = -F(x)\Delta x \Longrightarrow F(x) = -\frac{dU(x)}{dx}$$

or, mathematically more rigorously:

$$U(x) = -\int_{x_i}^{x} F(x')\,dx' \Leftrightarrow F(x) = -\frac{dU(x)}{dx}$$

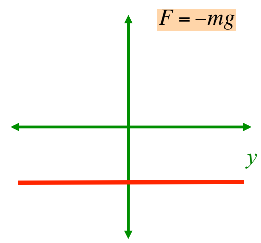


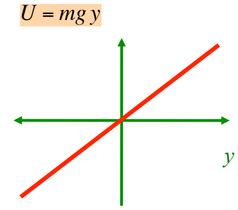
A plot of *U(x)* vs. *x* for a particle confined to move along an *x*-axis. There is no friction, so mechanical energy is conserved.



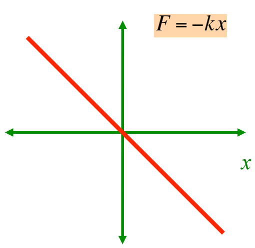
A plot of the force *F(x)* vs. *x* acting on the particle, derived from the potential energy plot by taking its slope at various points.

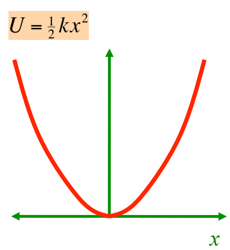
Graphs of F and PE for gravity



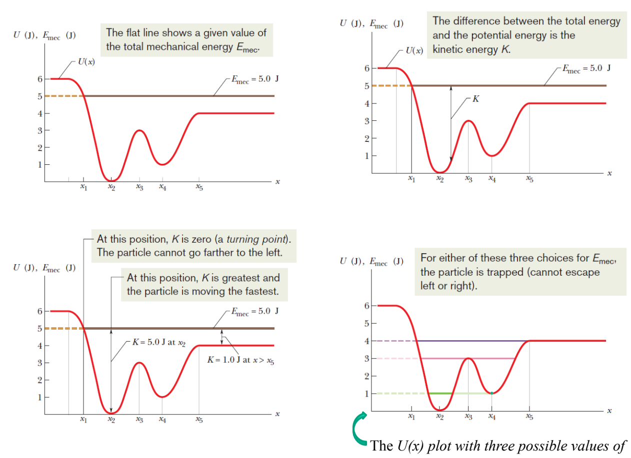


Graphs of F and PE for spring



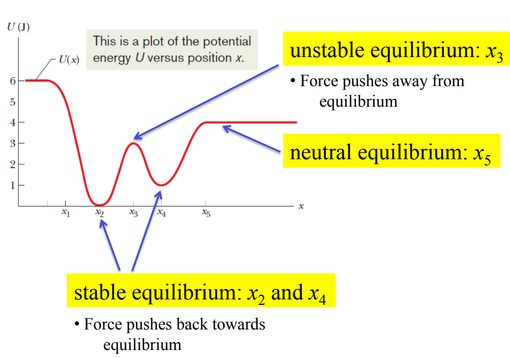


Force pushes object back towards equilibrium
(bottom of potential well)

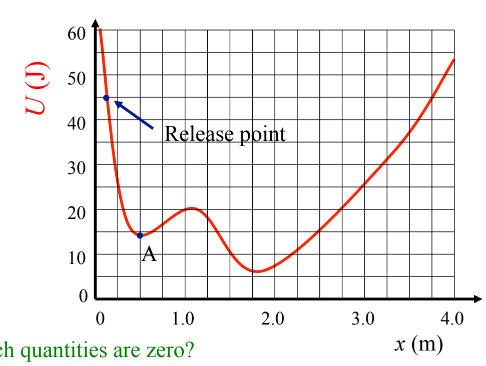


The U(x) plot with three possible values of E_{mec} shown.

Potential Energy Curve, Equilibrium Points



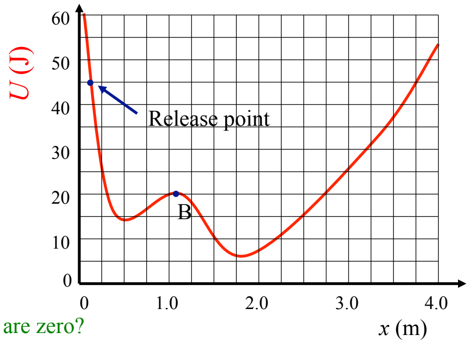
Example



At point 'A', which quantities are zero?

a) force
b) acceleration
c) force and acceleration
d) velocity

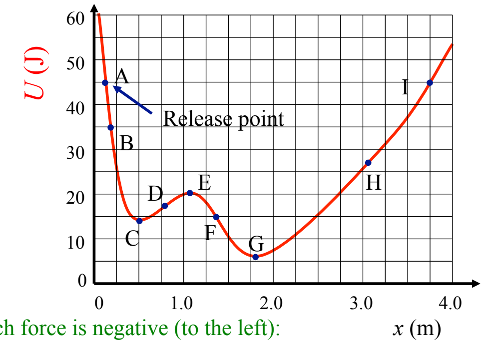
Example



At point 'B', which are zero?

a) force
b) acceleration
c) force and acceleration
d) velocity
e) kinetic energy

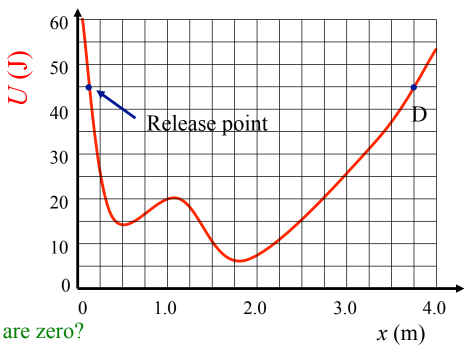
Example



All points for which force is negative (to the left):

a) C, E and G
b) B and F
c) A and I
d) D and H
e) D, H and I

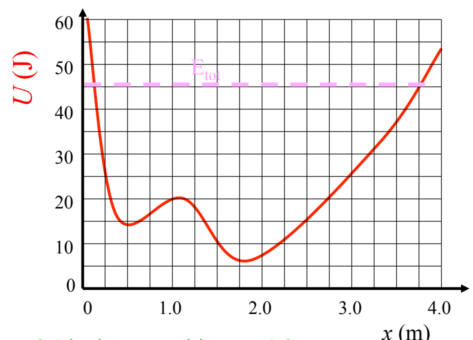
Example



At point 'D', which are zero?

a) force
b) acceleration
c) force and acceleration
d) velocity
e) velocity and kinetic energy

Example



A particle of mass m = 0.5 kg is at a position x = 1.0 m, and has a velocity of v = -10.0 m/s.
What are the furthest points (turning points) to the left and right that it will reach as it oscillates back and forth?

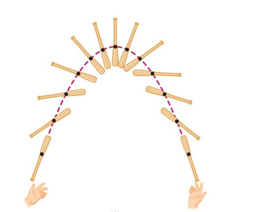
0.125 and 3.75 m

Chapter 9

Centre of Mass and Linear Momentum

9.1 The Center of Mass

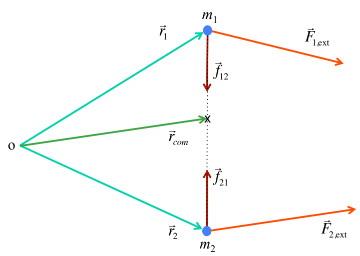
- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air)
- But there is a special point on the object for which the motion is simple, called the centre of mass (*com*).
- The centre of mass of the bat traces out a parabola, just as a tossed ball does
- All other points rotate around this point
- An object suspended from its *com* is completely balanced



9.1 The Center of Mass

Centre-of mass: a point on an object (or a system of objects) that moves as though all of the mass were concentrated at that point, and all external forces applied at that point.

Consider a two-particle system with both internal and external forces:



$\vec{F}_{1,ext}$: external force on 1

$\vec{F}_{2,ext}$: external force on 2

$\vec{f}_{12}$: internal force of 2 on 1

$\vec{f}_{21}$: internal force of 1 on 2

$\vec{f}_{21} = -\vec{f}_{12}$: by Newton's third law

$$\vec{F}_{1,ext} + \vec{f}_{12} = m_1\vec{a}_1$$

$$\vec{F}_{2,ext} + \vec{f}_{21} = \vec{F}_{2,ext} - \vec{f}_{12} = m_2\vec{a}_2$$

Adding these equations, we find the total external force on the system obeys the equation:

$$\vec{F}_{ext} = \vec{F}_{1,ext} + \vec{F}_{2,ext} = m_1\vec{a}_1 + m_2\vec{a}_2$$

$$= m_1\frac{d^2\vec{r}_1}{dt^2} + m_2\frac{d^2\vec{r}_2}{dt^2}$$

$$= \frac{d^2(m_1\vec{r}_1 + m_2\vec{r}_2)}{dt^2}$$

$$= (m_1 + m_2)\frac{d^2}{dt^2}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}\right) = M\frac{d^2\vec{r}_{com}}{dt^2}$$

where $M = m_1 + m_2$ is the total mass, and the centre-of-mass is defined as:

$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

9.1 The Center of Mass

- The **centre of mass** (*com*) of a system of particles:

> The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

- For two particles separated by a distance d, where the origin is chosen at the position of particle 1:

$$x_{com} = \frac{m_2}{m_1 + m_2}d$$

- For two particles, for an arbitrary choice of origin:

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

9.1 The Center of Mass

- The centre of mass is in the same location regardless of the coordinate system used
- It is a property of the particles, not the coordinates