Work-energy relation for non-conservative forces

W_C = work done by conservative forces
W_{NC} = work done by non-conservative forces (*e.g.* external, friction)

$$\Delta K = W_C + W_{NC}$$
$$\Delta U = -W_C$$
$$\therefore \Delta E_{mech} = \Delta K + \Delta U = W_{NC} \neq 0$$

So ΔE represents gain or loss of mechanical energy in the system due to the non-conservative forces.

Example

An object of mass m is raised through a height h at constant speed by an external force F.

What the change in mechanical energy of the system?

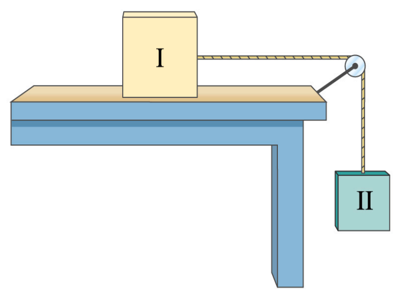
Example

A block is released from rest and slides a down a ramp of length L that is inclined at an angle θ to the horizontal.

If the coefficient of kinetic friction is μ_k, what is the speed of the block at the bottom of the ramp?

Example revisited (see lecture 14, which asked for acceleration of the system)

(a) What is the speed of the system after block I has travelled a distance L, assuming there is kinetic friction present?
(b) Deduce the acceleration from this result.



Power (rate of energy transfer)

The average rate at which energy is transferred to the system due to the work W_{NC} done by a force F over a time interval Δt, is defined as the **power**:

$$P_{avg} = \frac{W_{NC}}{\Delta t} = \frac{\Delta E}{\Delta t}$$

The instantaneous rate of energy transfer is therefore $P = \frac{dE}{dt}$

Power has units of Watts (1W = 1 J/s). Another common unit is:

1 horsepower = 1 hp = 746 W

Energy can also be expressed as $P \times t$:

$$1 \text{ kilowatt-hour} = 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) = 3.6 \times 10^6 \text{ J}$$

Power for constant force and velocity

The work done by a constant force $\vec{F}$ over a displacement $\Delta\vec{r}$ is

$$W = \vec{F} \cdot \Delta\vec{r}$$

The average power due to the force applied over a time interval Δt is therefore

$$P_{avg} = \frac{\vec{F} \cdot \Delta\vec{r}}{\Delta t} = \vec{F} \cdot \vec{v}_{avg}$$

The instantaneous power is therefore

$$P = \vec{F} \cdot \vec{v}$$

Problem 8.83

A 15kg block is accelerated at 2.0 m/s^2 along a horizontal frictionless surface, with the speed increasing from 10 m/s to 30 m/s. What are:

(a) The change in the block's mechanical energy?
(b) The average rate at which energy is transferred to the block?
(c) The instantaneous rate of that transfer when the block's speed is 10 m/s, and 30 m/s?

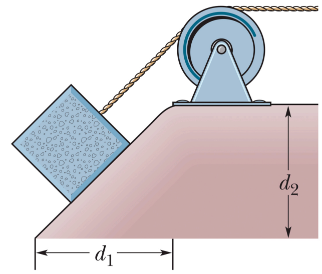
Answer:

ΔE_{mech}=6,000 J; P_{avg} = 600 W; P = 300 W and 900 W

Problem 8.80

A 1400 kg block is pulled up an incline at a constant speed v = 1.34 m/s by a cable and winch.
We have dimensions d_1 = 40 m, d_2 = 30 m, and μ_k = 0.40.

What is the power due to the force applied to the block by the cable?



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Answer:

Work done over a distance d along the ramp is:

$$W = mgd(\sin\theta + \mu_k \cos\theta)$$

Given $P = W/\Delta t$, and $v = d/\Delta t$, the power is:

$$P = mgv(\sin\theta + \mu_k \cos\theta)$$
$$= 1.69 \times 10^4 \text{ W}$$