Conservation of Mechanical Energy

Summary of last time:

For conservative forces, F depends on position only, not velocity or direction. Therefore

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

is independent of the path going from x_1 to x_2.

Introduce potential energy $U(x)$, where $\Delta K = W = -\Delta U$

Define $E_{\text{mech}} = K + U$, so that $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$

Equivalently, $E_{\text{mech}} = K_1 + U_1 = K_2 + U_2$ is conserved.

The potential energy $U(x)$ is defined relative to a reference point where it has the value 0. This reference point is arbitrary because only changes ΔU are relevant:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

(This is analogous to choosing an arbitrary origin for measuring position x, since only displacement Δx enters Newton's Laws.)

Gravity: $U(y) = mgy, \quad F = -\dfrac{dU}{dy} = -mg \qquad (U = 0 \text{ at } y = 0)$

Spring: $U(x) = \tfrac{1}{2}kx^2, \quad F = -\dfrac{dU}{dx} = -kx \qquad (U = 0 \text{ at } x = 0)$

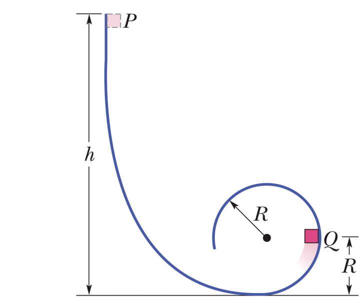
Problem 8.112

A 70.0 kg man jumping from a window lands in an elevated fire net 11.0 m below the window. His motion momentarily stops when he has stretched the net by 1.50 m.

(a) Assuming the net behaves like an idea spring, what is the spring constant k?
(b) What is the net force on the man at that point?

Problem

Find the minimum h needed to go around a loop of radius R without losing contact with the track.

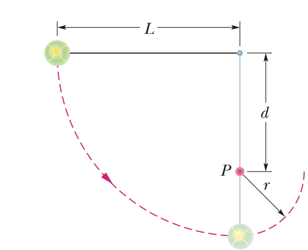


Answer:
$h = \tfrac{5}{2}R$

Problem 8.23 (This is a variation of the previous problem.)

A ball on a string of length L swings down and hits a peg a distance d below the pivot point, and subsequently moves in a circle of radius r.

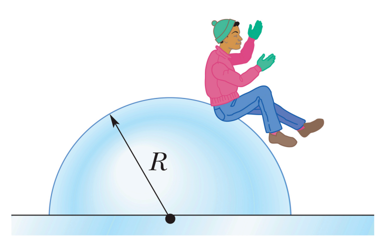
What is the speed of the ball at the lowest point?
What is the minimum distance d that the peg can be placed so that the ball reaches the top of the circle without the string going slack?



Answer:
$d = \tfrac{3}{2}r$

Problem 8.34

At what height does the boy lose contact with the dome?



Answer:
$h = \tfrac{2}{3}R$