Example

An ideal spring has a spring constant k and an equilibrium length L. Suppose we hang an object of mass m from this spring in a vertical orientation.

(a) Find the new equilibrium length.
(b) Show that Hooke's Law is obeyed for displacements from the new equilibrium position.

Chapter 8

Potential energy and conservation of energy

The potential energy concept

Recall that $$\Delta K = W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

- Suppose a system configuration changes.
  - A force does work W_1, representing transfer of kinetic energy to some other form of energy
  - If the configuration changes back, the force does work W_2, also representing transfer of kinetic energy

- We note that for some forces F, the work done is **reversible**, meaning that $W_1 = -W_2$
- ΔK depends only on the position of the object. Therefore $\Delta K = 0$ if we return to the same configuration
  - examples include gravity, spring forces, a pendulum
- What do these forces have in common?
  - Force depends only on position, not on velocity (*i.e.* direction of motion)

Potential energy

Potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

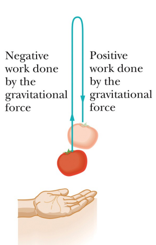
Some forms of potential energy:

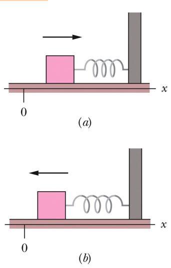
- Gravitational Potential Energy
- Elastic Potential Energy (*e.g.* in a spring)

Work and potential energy

The change ΔU in potential energy (gravitational, elastic, etc) is **defined** as the negative of the work done on the object by the force (gravitational, elastic, etc)

$$\Delta U = -W = -\Delta K$$





Conservative and non-conservative forces

Suppose:

1. A system consists of two or more objects.
2. A force acts between a particle-like object in the system and the rest of the system.
3. When the system configuration changes, the force does work (call it W_1) on the object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation in which $W_1 = -W_2$ is **always true**, the other type of energy is a potential energy, and the force is said to be a conservative force.

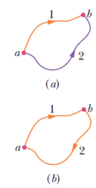
A force that is not conservative is called a non-conservative force. The kinetic frictional force and drag force are non-conservative.

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies

- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force, drag force
  - Nonconservative forces dissipate kinetic energy from the system
  - Kinetic energy of a moving particle is transferred to heat by friction
  - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy

Path Independence of Conservative Forces

The net work done by a conservative force on a particle moving around any closed path is zero.



The force is conservative. Any choice of path between the points gives the same amount of work.

And a round trip gives a total work of zero.

$W_{ab,1}$: work done from a to b along path 1

$W_{ab,2}$: work done from a to b along path 2

If the force is conservative,

$$W_{ab,1} + W_{ba,2} = 0, \quad \therefore W_{ab,1} = -W_{ba,2}$$

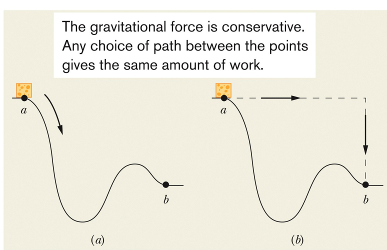
$$W_{ab,2} = -W_{ba,2} \quad \therefore W_{ab,1} = W_{ab,2}$$

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

$$W_{ab,1} = W_{ab,2}$$

This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved.

Example:



- For the general case, we calculate work as:

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

- So we calculate potential energy as:

$$\Delta U = -W = -\int_{x_i}^{x_f} F(x)\,dx$$

- For the gravitational force $F = -mg$, taking y as our variable, so

$$\Delta U = mg(y_f - y_i) = mg\Delta y$$

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$U(y) = mgy$$

- Use the same process to calculate elastic (spring) PE:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx = k\int_{x_i}^{x_f} x\,dx$$

$$\therefore \Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2$$

- With **reference point** $x_i = 0$ for a relaxed spring:

$$U(x) = \tfrac{1}{2}kx^2$$

Force and potential energy are related:

$$U(x) = -W = -\int_{x_i}^{x} F(x')\,dx'$$

$$\therefore F(x) = -\frac{dU(x)}{dx}$$

Gravity: $U(y) = mgy, \quad F = -\frac{dU}{dy} = -mg$

Spring: $U(x) = \tfrac{1}{2}kx^2, \quad F = -\frac{dU}{dx} = -kx$

Conservation of Mechanical Energy

- The mechanical energy of a system is the sum of its potential energy U and kinetic energy K:

$$E_{\text{mech}} = K + U$$

- Work done by conservative forces increases K and decreases U by that amount, so:

$$\Delta K = -\Delta U$$

- Using subscripts to refer to different instants of time:

$$(K_2 - K_1) = -(U_2 - U_1)$$
$$\therefore E_{\text{mech}} = K_1 + U_1 = K_2 + U_2$$

- In other words:

> In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

- This is the principle of the **conservation of mechanical energy**:

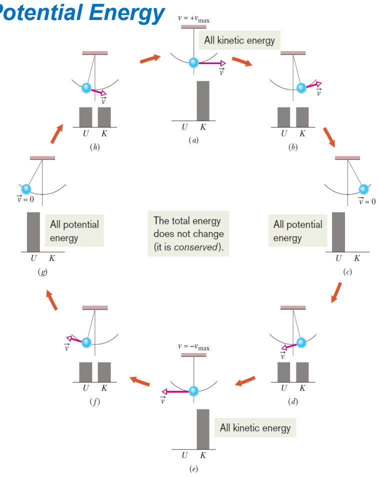
$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

- This is very powerful tool:

> When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

Kinetic Energy and Potential Energy

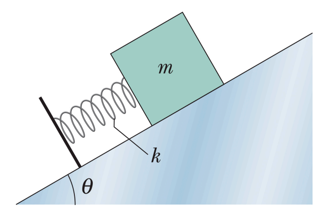
Energy is transferred back and forth between K and U as the object rises and falls.



Problem 8.31

A block of mass $m = 2$ kg is placed against a spring that is compressed by $x_0 = 20$ cm from its equilibrium position. The block sits on a frictionless ramp with $\theta = 30°$, and the spring constant is $k = 19.6$ N/cm.

How far will the block travel up the ramp from the release point?



Answer:

$$d = \frac{\Delta h}{\sin\vartheta} = \frac{kx_0^2}{2mg\sin\vartheta} = 4 \text{ m}$$